The Language of el Pleno: the constants of nature as geometric declensions of a single elastic medium

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Abstract

The vacuum is taken to be a real elastic medium with internal structure, described by a Lagrangian density with no free parameters. Its condensation into a diamond lattice breaks the rotation group down to its binary tetrahedral subgroup, and from these two faces of one symmetry the internal invariants of the programme are generated; two dimensional anchors dress them as physical constants. This work presents the system that carries out that reading —the Language of el Pleno— and surveys its coverage across four experimentally independent domains. In gravitation, the stiffness of the medium yields $c^4/G = \rho_P c^2 \ell_P^2 = 40\,\sigma_{\mathrm{Pleno}}$ and the $1/r$ potential. In quantum mechanics, $\hbar = E_{\mathrm{cell}}\tau_P$ reduces $\ell_P = \sqrt{\hbar G/c^3}$ to a tautology, Bose-Einstein statistics follow from counting over the modes of the equation itself —recovering Stefan-Boltzmann to thirteen figures— and the Tsirelson bound $2\sqrt{2}$ decomposes into a spinorial factor times the tetrahedral half-angle. In thermodynamics, the dissipation floor turns out to be a pure number, $2 \cdot 5^{3/4} 6^{7/8} \sqrt{\pi}/75$, with Boltzmann's constant cancelling out. In cosmology, the dark-to-baryonic density ratio is $\sqrt{3}\pi = 5.4414$, at 0.86σ from the Planck value, along two converging routes. The vocabulary supporting all four readings consists of six numbers with a single physical origin, and no target introduces additional parameters. The argument is that this coverage, and no single reading on its own, constitutes the evidence: the joint probability under coincidence decays exponentially with the number of independent targets, and the domains surveyed share no fitting mechanism. The limits are stated precisely: one calibrated state value in the cosmological sector, the inherited variational structure, and the points at which the reading ends.

Keywords: fundamental constants, elastic medium, lattice invariants, symmetry breaking, Bayesian analysis of coincidences

[1]

1 Introduction

1.1 Constants that are measured and not explained

Fundamental constants enter physics as data. The gravitational constant is measured in torsion experiments, the quantum of action in spectroscopy, and their values are inserted into the equations without the theory saying why they are those and not others. The Standard Model and general relativity describe with extraordinary precision the behaviour of systems whose parameters must be supplied from outside.

The question of where those values come from is legitimate and old. A theory that explained them —that produced them rather than receiving them— would have a content the current ones lack. This work presents a framework in which the internal constants of a physical programme turn out to be geometric relations of a single object, and examines what kind of evidence supports that claim.

1.2 The objection any such attempt must face

Attempts to explain constants through mathematical relations have an unfavourable record, and the reason is structural. The space of simple expressions constructible from π, a few square roots, the first integers and elementary operations is large. For a given numerical target and a tolerance of order one percent, that space contains an appreciable number of expressions that hit it. Reproducing a constant with a combination of roots and integers is therefore an empty exercise: the coincidence is guaranteed by the size of the search space, not by the structure of the object described.

The objection is correct and applies without mitigation to any isolated result of this class. A framework aspiring to explain constants must show why its case escapes it, and the answer cannot consist of displaying one more precise hit.

1.3 The thesis

There is a condition under which the objection ceases to apply: that many independent targets be reached with a fixed vocabulary and no additional parameters per target. The joint probability of hitting N targets by coincidence decays as p^N; the dependence is exponential, not linear. If the targets moreover belong to experimental domains that share no fitting mechanism, the statistical independence underwriting the exponent is guaranteed by the physics of the measurements rather than assumed.

This work argues that the PIU-Ψ programme satisfies that condition. From a single Lagrangian density with no free parameters, through an explicit reading system and with a vocabulary of six irreducible numbers, results follow in gravitation, quantum mechanics, thermodynamics and cosmology. The coverage is the result; none of the readings on its own is.

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1.4 The object and its description

The framework takes the vacuum to be a real elastic medium with internal structure, el Pleno. The complex field $\Psi = S\,e^{i\theta}$ is the mathematical description of that medium, not the medium itself: its modulus is the local amplitude, its phase the angular freedom. The numbers this work reads are not coefficients of an abstract Lagrangian but invariants of the internal geometry of a physical object, and that distinction conditions how the claim is to be assessed.

1.5 Route

Section 2 presents the single root: the equation, the two symmetry roots its condensation generates, and the two dimensional anchors. Section 3 defines the reading system —declension, family, rejection criterion— and the epistemic classification accompanying each result. Section 4 applies the full machinery to a single object, the configuration annulus of the field, including three rejections that show the criterion operating before any hit is counted. Section 5 surveys the harvest across the four domains. Section 6 quantifies the coverage argument and answers the objection of §1.2. Section 7 states the limits: what is calibrated, what is inherited, and what remains open. Section 8 recapitulates.

2 The single root

2.1 The object and its equation

The vacuum is taken to be a real elastic medium with internal structure: el Pleno. The complex field $\Psi = S\,e^{i\theta}$ is the mathematical description of that medium, not the medium itself. Its modulus S is the local amplitude of el Pleno; its phase θ, the angular freedom. The medium condenses into a diamond lattice of coordination $Z = 4$, and that lattice is the source of all the numerical structure that follows.

The dynamics is fixed by a single Lagrangian density, F1. For $\Psi = S\,e^{i\theta}$, F1 gathers a kinetic term, a potential, a flexural term and a quantum wall, and none of its coefficients is a free input: the flexural $1/40$ comes from evaluating F1 on the diamond lattice, the $3/8$ of the wall from the affine quantisation of the half-line $S \geq 0$, the kinetic coefficient from the identity c^4/G. F1 has no free parameters; all subsequent physics is its evaluation.

Expanding the kinetic term with $\Psi = S\,e^{i\theta}$ separates the radial sector from the angular one,

$$\tfrac{1}{2}\rho_{\mathrm{P}} c^2 \ell_{\mathrm{P}}^2\,|\partial\Psi|^2 = \tfrac{1}{2}\rho_{\mathrm{P}} c^2 \ell_{\mathrm{P}}^2\left[(\partial S)^2 + S^2(\partial\theta)^2\right], \tag{1}$$

and that separation organises the rest: every constant of el Pleno belongs either to the modulus sector or to the phase sector, and that membership —not an arbitrary combination— is what the language reads.

2.2 Pure ratios and anchors

Dimensional values —ρ_{P}, ℓ_{P}, c— do not report on el Pleno but on how many human yardsticks fit into the corresponding bound. In units of the medium itself ($c = 1$,

$\ell_{\mathrm{P}} = 1$) those numbers vanish and the bounds remain. The intrinsic content of el Pleno resides in its *pure ratios*: the dimensionless quantities the geometry of the lattice produces. A dimensional constant is a pure ratio dressed with a yardstick.

Two dimensional quantities suffice to dress any lattice invariant: the Planck length ℓ_{P} and the stiffness $K \equiv \rho_{\mathrm{P}} c^2$. These are the two *anchors*. Over them, a dimensionless invariant becomes a physical constant through powers of ℓ_{P}:

$$\text{energy} = K\ell_{\mathrm{P}}^3, \qquad \text{force} = K\ell_{\mathrm{P}}^2, \qquad \text{action} = Kc^{-1}\ell_{\mathrm{P}}^4. \tag{2}$$

This conversion is the *declension*, and it is the precise point at which the human yardstick enters: the invariant and its skeleton belong to el Pleno; the number it takes upon declension encodes our units, not the medium.

2.3 The two symmetry roots

Every internal invariant of the programme is a diamond-lattice invariant, generated from two symmetry roots that are, at bottom, one root with a breaking.

The root is the rotation group SO(3). Through the series of dimensions of its irreps, $2\ell + 1$, it generates the angular lineage: the spectral ratios $1/\langle\cos^{2n}\theta\rangle = 2n + 1$ —the 3 of the k^2 dispersion, the 5 of the k^4 flexure, the 7 confirmed in the k^6 term, whose lattice coefficient is $1/3360 = 1/(2^5 \cdot 3 \cdot 5 \cdot 7)$— and, with the Laplacian, the powers of π (π^4 as the Brillouin-zone volume). This is the continuous sector.

The diamond lattice breaks SO(3) down to its finite subgroup, the binary tetrahedral group 2T of coordination $Z = 4$. From that breaking come the integers of the symmetry lineage —$8 = Z(Z - 2)$, $12 = 3Z$, $16 = Z^2$— and the algebraic roots $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$ of the tetrahedral bond geometry. This is the finite sector.

The two lineages are the integer and half-integer sectors of the same SO(3) Casimir; the finite lineage is what remains when 2T splits the multiplets $2\ell + 1$ into its irreps. Thus π, $\sqrt{6}$ and $\sqrt{3}$ —the numbers that run through the rest of this work— are not primitive roots: they are the signatures of these two faces of a single symmetry. The angular family carries the mark of continuous SO(3); the bond family, that of 2T.

2.4 The two bounds as first invariants

The modulus sector has two walls, the two simplest declinable invariants of the programme. The lower bound, the affine-quantisation wall (Klauder regime), is

$$S_{\min} = \frac{12(\pi^2 - 4)}{\pi^4} \approx 0.723087, \tag{3}$$

an invariant of the angular family: the $12 = 3Z$ is dimension times coordination, the $(\pi^2 - 4)$ the first mode, the π^4 the Brillouin-zone volume. The upper bound, the saturation wall (Gent regime), is

$$S_{\max}^2 = \frac{8\sqrt{6}}{9} = \sin^2\theta_{\mathrm{tet}} \cdot \sqrt{6} \approx 2.177324, \tag{4}$$

an invariant of the bond family, with $\cos\theta_{\rm tet} = -1/3$ the tetrahedral bond angle and $\sin^2\theta_{\rm tet} = 1 - 1/9 = 8/9$. One bound carries the signature of SO(3), the other that of 2T: both roots already appear in the two walls of the modulus. From them the rest is declined.

3 The Language of el Pleno

3.1 What the language is

The Language of el Pleno is the observation, made systematic, that the internal constants of the programme, expressed dimensionlessly, are pure geometric ratios of the diamond lattice over the two anchors. Speaking it means translating between the human form (with units) and the form of el Pleno (pure ratios), composing constants out of ratios, and separating genuine structure from numerical coincidence.

The language adds no content to F1 and does not decide the truth of the programme; it gives its internal grammar. Riemann curvature added no force to gravity: it gave the language in which gravity is curvature. The language plays that role for the constants, which turn out to be declensions of the lattice geometry rather than numbers measured and explained afterwards.

3.2 Common origin and operations

Every derivation in the language starts from the same fixed origin: the field $\Psi = S\,e^{i\theta}$, the two anchors $\ell_{\rm P}$ and $K \equiv \rho_{\rm P} c^2$, the native operators $D_\mu = \ell_{\rm P}\partial_\mu$, the diamond lattice $Z = 4$, and the two symmetry roots. Three concepts operate on that origin.

Declension.

A dimensionless lattice invariant is declined when it is dressed with the anchors to give a physical constant, or —read in reverse— when a constant is stripped down to the pure ratio that generates it. Reading a quantity in the language means exhibiting its declension: its invariant, the symmetry sector it comes from, and the powers of $\ell_{\rm P}$ that dress it.

Family.

Invariants are grouped by the symmetry root that generates them: the angular family (SO(3), marked by π and the series $2n + 1$) and the bond family (2T, marked by $\sqrt{6}$, $\sqrt{3}$ and the integers 8, 12, 16, 40). Family is not a classification of convenience. Two invariants of the same family may share structure; two from disjoint families that agree in value share only the number.

False friend.

A numerical coincidence between invariants of disjoint families is rejected as a structural identity, however close. The rejection criterion is disjointness of families, not the size of the discrepancy. The identity criterion is finer still: two numbers equal in value are the same word only if they have the same atoms with the same origins. The spectral 9, $1/\langle\cos^8\theta\rangle$, and the rotational 9, 3×3, agree in value and are not grouped together.

3.3 Epistemic classification

Each reading carries a label declaring its strength, in increasing order:

- [calibrated] — value fitted to a datum, with the error stated.
- [language-derived] — closed by the rules of the language (composition of pure ratios, two anchors, grammar); the form forced, not merely admitted.
- [algebra-derived] — complete algebraic computation, zero free parameters, verifiable symbol by symbol.
- [AI-derived] — two independent routes converge on the same quantity, touching only common premises and anchors, or through a declared disjoint link.

Two status marks are added: [inherited], for structure common to any theory with an action principle, which the language uses without claiming it; and [positional], for a state value that depends on the epoch —like H_0— and admits no closed form without circularity. The class [AI-derived] carries the greatest evidential weight: a quantity reached by two routes that do not share their decisive link is not a coincidence.

4 The language on one object: the configuration annulus

4.1 The state space is not a value

In each cell of the medium, the field $\Psi = S\, e^{i\theta}$ does not take a value but inhabits a region of the complex plane. The two walls of the modulus fix its radial edges: the floor at $S_{\min} \approx 0.7231$ and the ceiling at $S_{\max} \approx 1.4756$. The argument θ remains free, by global U(1) invariance. The region is a circular annulus.

The origin is forbidden. The annulus has a hole at its centre and is topologically a ring, not a disc. That hole is the no-vacuum axiom written in geometry: Ψ does not vanish in any cell. Four axioms are inscribed in a single figure: two set the radial walls, one frees the angle, one punctures the centre.

The ontological reading follows from the figure. The entire theory is the field moving inside this ring: approaching the inner wall is cooling towards a vacuum never reached; touching the outer one is saturating against ρ_{P}; turning in the angle is phase —Noether charge, temperature, Goldstone mode—.

4.2 The area

The area of the annulus composes the two bounds into a single number:

$$A = \pi\left(S_{\max}^2 - S_{\min}^2\right) = \pi\left(\frac{8\sqrt{6}}{9} - S_{\min}^2\right) \approx 5.197669. \tag{5}$$

The subtracted term is not an algebraic residue: $S_{\min}^2 \approx 0.522855$ is exactly the occupation $\langle S^2 \rangle$, the same word that appears in the dictionary of the language with that meaning. The multiplying factor π is the spherical π —that of area in the plane— and is distinct from the π inside $S_{\min}$, which is the zone-boundary one. Two occurrences of

π with different origins within one expression: the discipline of families operates even at the level of a repeated symbol.

4.3 The third circle: the self-dual

Between the walls lies a third circle, distinguished by a symmetry of the medium itself: the fixed point of the conformal inversion $\Psi \to S^{*2}/\bar{\Psi}$, located at

$$S^* = \sqrt{S_{\min} S_{\max}} \approx 1.032942. \tag{6}$$

Three properties characterise it. It is the *logarithmic equator* of the ring,

$$\frac{S^*}{S_{\min}} = \frac{S_{\max}}{S^*} = \sqrt{\frac{S_{\max}}{S_{\min}}} \approx 1.428517, \tag{7}$$

so that it is equidistant from both walls on a logarithmic scale. It *exchanges the walls*: the inversion $S \to S^{*2}/S$ sends the floor to the ceiling, with $S^{*2}/S_{\min} = S_{\max}$ exactly, reflecting each interior point to an exterior one along the same radius and preserving θ. And it *splits the annulus into two physical regimes*: the inner sub-ring $[S_{\min}, S^*]$ is the Klauder domain —quantum, infrared— and the outer $[S^*, S_{\max}]$ the Gent domain —geometric, ultraviolet—.

The self-dual circle has independent physical meaning: it is crystallisation. Its density is $\rho(S^*) \approx 1.067\, \rho_{\mathrm{P}} \approx \rho_{\mathrm{Planck}}$, the scale at which U(1) breaks and the diamond lattice is born. The object geometry marks as the centre of symmetry is the one dynamics marks as the crystallisation point. [derived]

Self-duality is exact in the geometry and broken in the dynamical potential: the real infrared wall is Klauder's, not the one the inversion would generate, and the breaking is measured by $S_{\min}^2 - 3/8 = 0.148$.

4.4 Three rejections

The annulus offers three attractive numerical coincidences. All three are rejected, and the reason is in each case the same: disjoint families.

The ratio between the walls is $S_{\max}/S_{\min} \approx 2.0407$, within 1.9 % of 2. It is not 2: its closed form is $2^{3/4}3^{1/4}\pi^4/[18(\pi^2-4)]$, a composition of both families that does not collapse to an integer.

The annular fraction $1 - S_{\min}^2/S_{\max}^2 \approx 0.75986$ falls within 0.25 % of the dissipation floor $R_{\mathrm{floor}} \approx 0.75797$. The proximity is notable and the identity false: the annular fraction belongs to the amplitude family and the dissipation floor to the transport family.

The point $\sqrt{2}\, S_{\min} \approx 1.02260$ —the amplitude at which the late partition gives $\Omega_{\mathrm{DE}} = 1/2$— falls within 1.0 % of the self-dual $S^* \approx 1.03294$. The closed forms differ and so does the status: $\sqrt{2}\, S_{\min}$ comes from a reparametrisation of the partition, S^* from the geometry of the annulus.

The case sets the evidential standard for what follows. A hit counts insofar as it survives the criterion that here rejected three plausible candidates, one of them at 0.25 %.

5 The harvest

5.1 Two classes of target

The results that follow fall into two classes, and the distinction determines how each is to be read.

Class A — reconstruction. The target is a quantity science has already fixed, with precision far exceeding any margin of the programme: G, $\hbar$, the Stefan-Boltzmann law, the Tsirelson bound. Reproducing it is a pass-or-fail test against a value nailed down in advance, not a fit: there is no freedom to accommodate the prediction to the datum. A class A result inherits the validation of the original —a constant reached by a new route is the same constant, with the same experimental status—.

Class B — prediction. The target is a quantity measured with appreciable uncertainty, where the programme takes a risk: the dark-to-baryonic density ratio, the dark energy fraction. Here the confrontation is statistical and is stated with its deviation.

Both classes share the same pieces. The invariants that decline to G and to $\hbar$ come from the same two symmetry roots that decline to $\sqrt{3}\pi$. A choice of pieces made to hit class B would not reconstruct class A, because A was fixed beforehand and with greater precision.

5.2 Gravitation

G as the stiffness of the medium.

The kinetic coefficient of F1 is, in human form, $c^4/(16\pi G)$ —the same as Einstein-Hilbert—. In the language, the meaningful quantity is not G but its dimensional inverse, the gravitational stiffness of el Pleno:

$$\frac{c^4}{G} = \rho_{\mathrm{P}} c^2 \ell_{\mathrm{P}}^2 = K\ell_{\mathrm{P}}^2 \qquad \Longrightarrow \qquad G = \frac{c^4}{K\ell_{\mathrm{P}}^2} = \frac{c^2}{\rho_{\mathrm{P}}\ell_{\mathrm{P}}^2}. \tag{8}$$

With the surface tension of the medium $\sigma_{\mathrm{Pleno}} = K\ell_{\mathrm{P}}^2/40$,

$$\frac{c^4}{G} = K\ell_{\mathrm{P}}^2 = 40\,\sigma_{\mathrm{Pleno}} \approx 1.2\times 10^{44}\ \mathrm{N} \equiv F_{\mathrm{Planck}}. \tag{9}$$

G is small because the medium is stiff: the constant governing the weakest interaction is the inverse of the greatest stiffness in the programme. The 40 linking it to the surface tension is the angular molecule $40 = 2Z\cdot 5$ (bipartition $\times$ coordination $\times$ spectral average), the same one that fixes the flexural stiffness. [derived]

The $1/r$ potential.

In the static regime and at scales $\gg \ell_{\mathrm{P}}$, the modulus equation reduces to Poisson form,

$$K\ell_{\mathrm{P}}^2\,\nabla^2(\delta S) = -\rho_S, \tag{10}$$

whose Green function in three dimensions is $\mathcal{G} = -1/(4\pi r)$. The modulus perturbation then falls off as $\delta S \propto 1/r$, and the interaction energy of two perturbations takes the Newtonian form $U \propto q_1 q_2/r$. Comparison with $U_{\text{Newton}} = -Gm_1m_2/r$ recovers the same identity $c^4/G = K\ell_{\text{P}}^2$, with no residual factor. The logical order matters: G is not measured and inserted into F1; the kinetic coefficient of F1 is fixed by the anchors and *is* c^4/G. The force law and its constant emerge from the same computation. [derived]

The Einstein-Hilbert coefficient.

The 16π in the denominator decomposes: the 4π comes from the Green function (angular family, SO(3)); the $16 = Z^2$ is the molecule of the lattice operator (finite lineage, 2T). Two roots in a single coefficient. [derived]

Newton's laws.

The first law is the Euler-Lagrange solution without forcing; the second, the same equation in the low-energy limit with force as a gradient; the third, momentum conservation by Noether's theorem over the homogeneity of the medium. These three are [inherited] structure: any theory with an action principle contains them. What the programme adds is the origin of what Newton left primitive —the constant G and the nature of mass, identified with retained configuration energy—.

5.3 Quantum mechanics

$\hbar$ as cell per tick.

The declension of action over the anchors is $Kc^{-1}\ell_{\text{P}}^4$. The language reads it by composing the energy of one cell, $E_{\text{cell}} = \rho_{\text{P}}c^2\ell_{\text{P}}^3$, with the duration of one tick, $\tau_{\text{P}} = \ell_{\text{P}}/c$:

$$\hbar = E_{\text{cell}}\,\tau_{\text{P}} = \left(\rho_{\text{P}}c^2\ell_{\text{P}}^3\right)\left(\frac{\ell_{\text{P}}}{c}\right) = \rho_{\text{P}}c\,\ell_{\text{P}}^4. \tag{11}$$

The quantum of action is one cell of the medium acting for one tick. [derived]

The cross-closure with G.

Substituting (8) and (11) into the celebrated relation,

$$\frac{\hbar G}{c^3} = \frac{\left(\rho_{\text{P}}c\ell_{\text{P}}^4\right)\left(c^2/\rho_{\text{P}}\ell_{\text{P}}^2\right)}{c^3} = \ell_{\text{P}}^2, \tag{12}$$

that is, $\ell_{\text{P}} = \ell_{\text{P}}$. The formula was the artefact of slicing a single quantity into three separately measurable pieces. In el Pleno, every relation among human constants is a structural tautology or a definition. [derived]

Thermal statistics from F1.

Expanding F1 to quadratic order in the phase perturbation gives, for each Fourier mode k, a harmonic oscillator with

$$M_k = \rho_{\text{P}}\ell_{\text{P}}^2S_0^2, \qquad K_k = \rho_{\text{P}}c^2\ell_{\text{P}}^2S_0^2\left(k^2 + \frac{\ell_{\text{P}}^2k^4}{20}\right), \tag{13}$$

whence

$$\omega_k^2 = \frac{K_k}{M_k} = c^2 k^2 \left(1 + \frac{(k\ell_\mathrm{P})^2}{20}\right), \tag{14}$$

the dispersion relation of the programme, with the angular regulator $1/20 = 2\gamma_\kappa$. The levels of each mode are $E = m\hbar\omega_k$. Microcanonical counting of indistinguishable quanta over those oscillators gives, by Stirling's approximation,

$$\frac{\sigma(n)}{k_B} = (1+n)\ln(1+n) - n\ln n. \tag{15}$$

Imposing the definition of temperature $1/T = \partial S/\partial E$ with $dE = \hbar\omega\, dn$:

$$\frac{1}{\hbar\omega}\frac{\partial\sigma}{\partial n} = \frac{1}{k_B T} \implies \ln\frac{1+n}{n} = \frac{\hbar\omega}{k_B T} \implies \langle n\rangle = \frac{1}{e^{\hbar\omega/k_B T} - 1}. \tag{16}$$

The Bose-Einstein occupation follows from the counting; it is not assumed. Integrating over the three-dimensional mode measure with $g_* = 2$ polarisations recovers

$$s = \frac{2\pi^2}{45}\, g_*\, k_B \left(\frac{k_B T}{\hbar c}\right)^3, \tag{17}$$

with the coefficient $2\pi^2/45 = 0.4386490845$ reproduced to thirteen figures. The only external ingredient is the equally spaced spectrum of the harmonic oscillator, already used in the U(1) rotor quantisation that fixes $\hbar$. The bosonic character is not imported: the phase mode is a spin-0 Goldstone with $p_\theta = n\hbar$, $n \in \mathbb{Z}$ unbounded, hence of unlimited occupation. [algebra-derived]

The Tsirelson bound.
The quantum bound of the CHSH inequality, $|S|_\mathrm{max} = 2\sqrt{2} = 2.828427$, reads as a product of two words: the 2 is the spinorial nature (a physical rotation α acts on the spinor as $\alpha/2$, through the covering $2\mathrm{T} \to A_4$), and the $\sqrt{2}$ is the invariant of the tetrahedral half-angle, $\tan(\theta_\mathrm{tet}/2)$. The $\sqrt{2}$ that maximises CHSH ($\cos\pi/4$) coincides with the $\sqrt{2}$ of the lattice L point. [derived]

The generalised uncertainty principle.
The coefficient of the corrective term,

$$\Delta x\, \Delta p = \frac{\hbar}{2}\sqrt{1 + \frac{3}{80}\left(\frac{\ell_\mathrm{P}}{\sigma}\right)^2}, \qquad \frac{3}{80} = \underbrace{\frac{1}{20}}_{\text{flexural}} \cdot \underbrace{3}_{\text{quartic}} \cdot \underbrace{\frac{1}{4}}_{\text{2nd moment}} = 0.0375, \tag{18}$$

decomposes into three factors of separate origin. The GUP is native and its coefficient derived, not fitted. [derived]

The Born rule.

$P(R) = Q_R/Q_{\mathrm{total}}$: the probability is the fraction of U(1) Noether charge localised in the region, with density $\rho_Q = \rho_\mathrm{P}\ell_\mathrm{P}^2\omega S^2$. It is not postulated; it is a ratio of conserved charges. [derived]

5.4 Thermodynamics and transport

The dissipation floor as a pure number.

In the relaxation-time approximation, $\eta/s = (1/5)\tau_{\mathrm{col}}T$. Measured against the Kovtun-Son-Starinets bound, $\hbar/(4\pi k_B)$:

$$R \equiv \frac{\eta/s}{\hbar/(4\pi k_B)} = \frac{4\pi}{5}\,\frac{\tau_{\mathrm{col}}k_BT}{\hbar}. \tag{19}$$

In the transverse regime the collision time reaches the lattice floor $\tau_{\mathrm{II}} = \ell_\mathrm{P}/(\sqrt{20}\,c) = \sqrt{5}\,\ell_\mathrm{P}/(10c) \approx 0.223607\,\tau_\mathrm{P}$. Evaluating at maximum temperature and substituting $\hbar = \rho_\mathrm{P}c\ell_\mathrm{P}^4$, all dimensional quantities cancel and there remains

$$R_{\mathrm{floor}} = \frac{2\cdot 5^{3/4}\cdot 6^{7/8}\sqrt{\pi}}{75} = 0.757973, \tag{20}$$

a pure number: zero free symbols after the substitution. Its families are visible in the closed form —the 5 and the $\sqrt{\pi}$ of the angular family, the $6^{7/8}$ of the bond family—. The physical value in the transverse regime lies in the window $1 \lesssim R \lesssim 2.5$: the medium respects the bound with margin and does not saturate it. [algebra-derived]

The maximum temperature.

By saturation of the upper wall,

$$k_BT_{\max} = \left(\frac{40\sqrt{6}}{3\pi^2}\right)^{1/4}\rho_\mathrm{P}c^2\ell_\mathrm{P}^3 = \frac{5^{1/4}\,6^{7/8}}{3\sqrt{\pi}}\,\rho_\mathrm{P}c^2\ell_\mathrm{P}^3, \qquad \frac{5^{1/4}6^{7/8}}{3\sqrt{\pi}} = 1.348741. \tag{21}$$

The two forms are identical. [derived]

k_B at three levels.

Boltzmann's constant does not appear in (20): it enters the definition $T = \varepsilon/k_B$ and the bound $\hbar/(4\pi k_B)$, and cancels between them. This places it at the lowest level of the hierarchy: ρ_P and ℓ_P enter by dimension; G, $\hbar$ and γ_κ by substitution; k_B by self-annihilation. The temperature of el Pleno is the energy of one cell, $T_\mathrm{P} = E_{\mathrm{cell}}/k_B$, and k_B only converts units. [SLD]

The bound as a no-vacuum principle.

In a dilute gas nothing prevents $\eta/s \to 0$: it suffices to dilute. In el Pleno this is impossible, because the medium is always full and mode-to-mode coupled and no mode isolates itself. The existence of a positive floor for η/s is the impossibility of vacuum read as transport. Two limits bound the claim: the medium does not saturate the bound, and the coefficient $1/(4\pi)$ is not derived from the axiom. [SLD]

Convergence of the 20 across three sectors.

The angular word $20 = 40/2$ appears declined in three independent places: the minimum localisability $\sigma_{\min} = \ell_{\mathrm{P}}/\sqrt{20} = 0.223607\,\ell_{\mathrm{P}}$ (flexural); the relaxation time τ_{Π}, whose dimensionless form $\tau_{\Pi} k_B T_{\mathrm{P}}/\hbar = 1/\sqrt{20} = 0.223607$ (dissipative); and the elastic ratio $\xi/20 = 0.985$ (elastic). The exact agreement between the flexural and the dissipative shows that the viscosity coefficient is not free: it is the same angular word declined in another sector. [derived]

5.5 Cosmology

The dark-to-baryonic ratio.

$\Omega_{\mathrm{DM}}/\Omega_{\mathrm{B}}$ reads as a ratio of thermodynamic weights per sector, proportional to the boundary area of each defect class: for dark matter, a spherical Q-ball of area $A_S = 4\pi R^2$; for baryonic matter, a tetrahedral soliton of area $A_T = (8\sqrt{3}/3)R^2$. With the spinorial factor 1/2 of the fermionic sector:

$$\frac{\Omega_{\mathrm{DM}}}{\Omega_{\mathrm{B}}} = \frac{A_S \cdot 1}{A_T \cdot \frac{1}{2}} = \frac{2A_S}{A_T} = \frac{2\,(4\pi R^2)}{(8\sqrt{3}/3)R^2} = \frac{8\pi \cdot 3}{8\sqrt{3}} = \frac{3\pi}{\sqrt{3}} = \sqrt{3}\,\pi = 5.441398. \tag{22}$$

The π is the signature of the sphere; the $\sqrt{3}$, that of tetrahedral geometry. Against the Planck datum $\omega_{\mathrm{dm}}/\omega_{\mathrm{b}} = 5.375 \pm 0.077$:

$$\frac{5.441398 - 5.375}{0.077} = 0.86\,\sigma. \tag{23}$$

The ratio of physical densities is the most robust quantity in the sector: it is read from peak heights in a nearly model-independent way and the factor h^2 cancels. [AI-derived]

The [AI-derived] status follows from two routes reaching the same number: the geometric one (ratio of pure areas) and the dynamical one (thermodynamic weights per sector). They share the geometry A_S/A_T, so the independence is not total; the disjoint link is the reason why the weight scales with area —derived in the second route, postulated in the first—.

Robustness against defect counting.

The ratio is a quotient of partition functions per sector integrated over freeze-out configurations, not a product of defect number times individual weight: the counting sits inside each partition function. The ground-state density is common to both sectors, $\varphi_{\mathrm{DM}}/\varphi_{\mathrm{B}} = 1$. The subleading correction does not separate them either: the curvature term of the log-determinant is $\int K\,dA = 2\pi\chi$ by Gauss-Bonnet, and sphere and tetrahedron are both topologically S^2 with $\chi = 2$, hence $\int K\,dA = 4\pi$ for both. The curvature is distributed differently —smooth versus concentrated at four vertices of deficit π— but the invariant that weighs is the same. [derived]

The Hubble bound.
$H_{\max} = \sqrt{8\pi S^2_{\max}/3}\,(c/\ell_{\rm P})$, a product of the angular family (the $8\pi/3$ of Friedmann) and the bond family (the hard bound $S^2_{\max}$). The corresponding minimum horizon is $R_{H,\min} = c/H_{\max} \approx 0.234\,\ell_{\rm P}$. [derived]

The dark energy fraction.
$f_c = S^2_{\max,\rm eff}/S^2_{\max}$ is the only calibrated ingredient of the sector. $S^2_{\max,\rm eff}$ is the upper turning point of the radial oscillation of the modulus, fixed by the present oscillation energy: a state quantity, analogous to H_0, which grows towards 1 as the medium relaxes and admits no closed form without circularity, because time in the programme *is* the descent of the modulus. [positional]

5.6 The inventory

The readings above share a vocabulary of small size. The atoms of the language — the numbers with a single physical origin— are the 2 of the bipartition, the $Z = 4$ of tetrahedral coordination, the odd numbers 3, 5 and 7 of the SO(3) irreps, and the π of the continuous spectrum. Everything else are molecules the language presents decomposed: $8 = Z(Z-2)$, $12 = 3Z$, $16 = Z^2$, $40 = 2Z\cdot 5$, $9 = 3\cdot 3$.

The same molecule reappears across domains. The 40 links the gravitational stiffness to the surface tension (§5.2) and fixes the flexural stiffness $\gamma_\kappa = 1/40$, whence come the regulator 1/20 of the dispersion (§5.3), the localisability $\sigma_{\min}$ and the relaxation time (§5.4). The $\sin^2\theta_{\rm tet} = 1-(-1/3)^2 = 8/9$ appears in the Maxwell coefficient and in the upper bound of the modulus, $S^2_{\max} = (8/9)\sqrt{6} = 2.177324$: the same word in electromagnetism and in cosmology.

The elastic sector reduces to a single non-trivial number. With $\tilde\mu = 1$ exactly — shear is the anchor, which is why the transverse mode travels at c—, the bulk modulus is

$$\frac{K}{\mu} = \frac{1}{4}\,\xi\,S^2_{\min} = \frac{1}{4}(19.6899)(0.723087)^2 = 2.5737, \tag{24}$$

and thence $\tilde\lambda = K - 2\mu/3 = 1.907$, $\tilde E = 9K\mu/(3K+\mu) = 2.656$ and $\nu = (3K - 2\mu)/[2(3K+\mu)] = 0.328$. All moduli are of order $\rho_{\rm P}c^2 \sim 10^{113}$ Pa. El Pleno is the stiffest possible medium, and that stiffness is why gravity is the weakest interaction.

The curvature of the well, $\xi = V''_{\rm eff}(S_{\min})/(\rho c^2) \approx 19.6899$, has no elementary closed form, and the language states as much. Its magnitude has a testable physical consequence —the existence condition for the underdamped radial mode is $\xi < 20$, satisfied by 1.55 %—. The two masses of the mode are two readings of the same object, roots of $W^2 + 20W - 20\xi = 0$ with $W \equiv (\omega\ell_{\rm P}/c)^2$:

$$m_S = \sqrt{\xi} = 4.4373\,E_{\rm P}, \qquad m_{\rm prop} = \sqrt{-10 + \sqrt{100 + 20\xi}} = 3.4959\,E_{\rm P}, \tag{25}$$

where the 20 is again the flexural regulator. [derived]

Four domains, a vocabulary of six atoms, zero free parameters per target. None of these readings on its own constitutes the evidence; their conjunction does, and the next section quantifies why.

6 Coincidence and structure

6.1 The objection

The space of simple expressions constructible from π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$, the first integers and elementary operations is large. For a given numerical target and a tolerance of order one percent, that space contains an appreciable number of expressions that hit it. Reproducing *one* constant through a combination of roots and integers is therefore an exercise without content: the coincidence is guaranteed by the size of the search space, not by the structure of the object described. This objection applies without mitigation to any isolated result of the programme taken on its own.

6.2 The exponential dependence

The objection reverses when targets multiply under fixed constraint. Let p be the probability that an expression from the available space hits a target within tolerance. For N independent targets, reached without enlarging the vocabulary or introducing new parameters, the joint probability under the coincidence hypothesis is

$$P(N \text{ hits} \mid \text{chance}) = p^N. \tag{26}$$

The dependence is exponential in N, not linear. With $p = 0.05$ —one in twenty expressions hits a given target— the joint probabilities and the odds against are those of Table 1. Doubling the number of targets does not double the evidence: it squares it.

Table 1 Joint probability under the coincidence hypothesis, with $p = 0.05$.

N	p^N	odds against
1	5.0×10^{-2}	2.0×10^{1}
2	2.5×10^{-3}	4.0×10^{2}
4	6.3×10^{-6}	1.6×10^{5}
6	1.6×10^{-8}	6.4×10^{7}
8	3.9×10^{-11}	2.6×10^{10}

The exact value of p depends on the space of expressions admitted and is not the point. The point is the form of the dependence. A programme that hits one target says nothing; one that hits eight with the same vocabulary and no additional parameters per target occupies a region of probability space that coincidence does not reach.

6.3 The independence condition

The preceding argument rests on an assumption that must be examined: that the N targets be independent. Were they all from one domain, they could be correlated projections of a single fit, and the exponent N would overstate the evidence.

The harvest of §5 satisfies that condition by construction. Its domains share no fitting mechanism: gravitational stiffness is measured by torsion experiments and orbital dynamics; the quantum of action, by spectroscopy and interferometry; the Stefan-Boltzmann law, by radiometry; the Tsirelson bound, by correlation experiments in quantum optics; the dark-to-baryonic density ratio, by the acoustic peak heights of the cosmic background. A fit able to accommodate the density ratio has no route by which to alter the Tsirelson bound. The disparity of the domains turns the sum of hits into a product of probabilities.

The Bayes factor computed in the corpus of the programme reflects this structure: $K_c \approx 6.7 \times 10^5$ for the cosmological sector, and $K_{\text{global}} \sim 10^{11}$ once the remaining sectors are included. On the conventional interpretation scale, a factor above 10^2 counts as decisive evidence; the global figure is seven orders of magnitude larger.

6.4 Reconstruction and prediction

The distinction of §5.1 closes the objection on its other flank. The class A targets —G, $\hbar$, Stefan-Boltzmann, Tsirelson— were fixed by science before the programme existed and with precision exceeding any margin of the programme. There is no freedom to accommodate a prediction to such a value: the expression either hits or misses. A class A hit is not a fit but a test passed, and the constant reached inherits the experimental status of the original.

The same pieces produce both classes. The molecule 40 of the gravitational stiffness is the one that fixes γ_κ and, with it, the regulator of the dispersion; the $\sin^2\theta_{\text{tet}} = 8/9$ appears in the Maxwell coefficient and in the upper bound of the modulus. A vocabulary chosen to hit class B would not reconstruct class A, because class A targets admit no accommodation. The agreement of both classes over the same reduced vocabulary is what distinguishes a structure from a lucky search.

6.5 The rejection criterion

A programme accepting every numerical coincidence would have an effectively unlimited search space, and the preceding argument would not apply to it. The language operates with an explicit rejection criterion prior to the hits: two quantities from disjoint families that agree in value are not the same word, however close the coincidence. Section 4.4 showed three rejections, one of them at $0.25\,\%$, and Appendix B collects the full register. This criterion reduces the space of admissible expressions before any hit is counted.

7 Boundary

7.1 What is calibrated

The cosmological sector contains one calibrated ingredient, f_c, and its status is stated precisely. $S^2_{\text{max,eff}}$ is the upper turning point of the radial oscillation of the modulus, fixed by the present oscillation energy of the medium. It is a state quantity, analogous to H_0: it grows towards unity as the medium relaxes, and its current value encodes the epoch, not a constant of the theory. Asking for a closed form would be asking for a closed expression for the present moment, and the programme measures time by the descent of the modulus. The structure of the sector is geometric; a state value remains positional.

7.2 What is inherited

The laws of motion, Noether's theorem and the variational machinery are structure common to any theory formulated with an action principle. The programme uses them without claiming them. What it contributes is the origin of their constants and primitive quantities: the stiffness that fixes G, the identification of mass with retained energy, the homogeneity that conserves momentum as a consequence of the uniqueness of the medium.

7.3 What is open

Four limits bound the claims of §5, and their nature differs in each case.

The coefficient of the dissipation bound. What is derived is that a positive floor for η/s exists —a consequence of the medium being full and mode-to-mode coupled, so that no mode isolates itself and dissipation cannot be switched off by dilution— and its value in the transverse regime, $R_{\text{floor}} = 0.757973$. The coefficient $1/(4\pi)$ of the Kovtun-Son-Starinets bound does not follow from the no-vacuum axiom. The programme reproduces the order of magnitude and the sign of the inequality, not the holographic prefactor.

The independence of the two routes to $\sqrt{3}\pi$. The geometric route computes the area ratio $2A_S/A_T$; the dynamical route derives that the thermodynamic weight of each sector scales with boundary area and obtains the same quotient. Both share the geometry A_S/A_T. The independence is not total but link-wise: what one postulates — the proportionality of weight to area— the other derives. The [AI-derived] classification is applied with that qualification explicit.

The curvature of the well. The value $\xi \approx 19.6899$ admits no elementary closed form. It is not $2\pi^2 = 19.7392$, from which it differs by 0.25 %, nor 20, from which it differs by 1.55 %; both proximities are of the kind that §4.4 and Appendix B reject by criterion, and accepting either here would contradict the method. The quantity has a testable physical consequence —the condition $\xi < 20$ guarantees that the radial mode is underdamped— but its algebraic origin remains open.

The background trajectory. The value of f_c at the present epoch requires integrating the background modulus equation of motion, $\ddot{S} + 3H\dot{S} + (\rho_{\text{P}}\ell_{\text{P}}^2)^{-1}V'_{\text{eff}} = 0$. The integration is numerical and yields no closed form; the positional cosmological readings depend on it.

None of these limits affects the thesis. The claim is that the internal relations of the medium are geometric and readable, and that their joint coverage resists explanation by coincidence. The four points mark where that reading ends today.

7.4 What the language does not decide

The language describes the internal structure of the programme; it does not decide its truth. That the constants of el Pleno are declensions of lattice invariants and that their conjunction resists explanation by coincidence are claims about internal coherence and about the improbability of chance. The truth of the programme is decided by data: gravitational wave observatories, spectroscopic surveys of large-scale structure, gamma-ray astronomy. A high Bayes factor indicates that coincidence is a poor explanation; it does not establish that the proposed explanation is the correct one.

8 Conclusion

A Lagrangian density with no free parameters describes a real elastic medium whose condensation into a diamond lattice breaks the rotation group down to its binary tetrahedral subgroup. From these two faces of one symmetry —continuous SO(3) and finite 2T— the internal invariants of the programme are generated, and two dimensional anchors, ℓ_{P} and $K = \rho_{\mathrm{P}} c^2$, dress them as physical constants. The Language of el Pleno is the system that carries out that reading: it declines invariants into constants, groups them by the root that generates them, and rejects coincidences between disjoint families.

The reading spans four domains. In gravitation it gives the stiffness $c^4/G = K\ell_{\mathrm{P}}^2 = 40\sigma_{\mathrm{Pleno}}$, the $1/r$ potential through the Green function of the modulus, and the $16\pi = Z^2 \cdot 4\pi$ of the Einstein-Hilbert coefficient. In quantum mechanics it gives $\hbar = E_{\mathrm{cell}}\tau_{\mathrm{P}}$ as cell per tick, with $\ell_{\mathrm{P}} = \sqrt{\hbar G/c^3}$ reduced to a tautology; Bose-Einstein statistics derived from counting over the modes of the equation itself, with Stefan-Boltzmann to thirteen figures; the Tsirelson bound $2\sqrt{2}$ as spinor times tetrahedral half-angle; and the coefficient $3/80$ of the generalised uncertainty principle decomposed into its three factors. In thermodynamics it gives the dissipation floor as a pure number, $2{\cdot}5^{3/4}6^{7/8}\sqrt{\pi}/75$, with k_B self-annihilating, and the convergence of the 20 across three independent sectors. In cosmology it gives $\sqrt{3}\pi = 5.4414$ for the dark-to-baryonic density ratio, at 0.86σ from the Planck datum, along two converging routes.

The vocabulary supporting that coverage consists of six atoms: the 2 of the bipartition, the $Z = 4$ of the coordination, the odd numbers 3, 5 and 7 of the SO(3) irreps, and the π of the continuous spectrum. Everything else is presented decomposed into them. No target introduces a new parameter.

The coverage is the result. An isolated hit with roots and integers has no content, and the programme acknowledges as much; a sequence of hits across domains with no common fitting mechanism, with fixed vocabulary and no parameters per target, occupies a region of probability space that coincidence does not reach, with a Bayes factor of order 10^{11} against the chance hypothesis. That the same pieces reconstruct constants fixed by science long before and with greater precision, and predict others confronted with current observation, is the difference between a structure and a lucky search.

What lies outside is stated: one calibrated state value in the cosmological sector, the inherited variational structure, four open points in the reading. And the decision on the truth of the programme, which belongs not to this work but to the data.

The complete material of the programme, including the derivations this work summarises and the register of rejected coincidences, is publicly deposited. The reading is reproducible: every declension can be recomputed, every family checked, every rejection reviewed.

Declarations. DOI of this work. 10.5281/zenodo.21514411 (https://doi.org/10.5281/zenodo.21514411).

Competing interests. The author declares no competing interests.

Funding. This work received no funding.

Data and materials availability. No new datasets were generated or analysed. All numerical values quoted come from the referenced sources or follow from recomputing the closed forms given in the text. Supporting material for the PIU-Ψ programme —the complete mathematical compendium containing the derivations this work summarises— is deposited on Zenodo, 10.5281/zenodo.21205881, and at piuniversal.com.

Appendix A Inventory of declensions

A.1 Atoms and molecules of the vocabulary

Table A1 Vocabulary of the language. An atom has a single physical origin and appears directly; a molecule is presented decomposed.

Number	Class	Physical origin
2	atom	A/B bipartition of the lattice
$Z = 4$	atom	tetrahedral coordination
3	atom	dimension / first SO(3) irrep
5	atom	$1/\langle\cos^4\theta\rangle$, second odd number
7	atom	$1/\langle\cos^6\theta\rangle$; confirmed in k^6: $1/3360 = 1/(2^5 \cdot 3 \cdot 5 \cdot 7)$
π	atom	continuous spectral
8	molecule	$Z(Z-2)$; $\sin^2\theta_{\text{tet}} = 8/9$
12	molecule	$3 \times Z$ (dimension × coordination)
16	molecule	Z^2 (lattice operator)
40	molecule	$2 \times Z \times 5$ (symmetry × spectral)
9	molecule	3×3 (tripartition × Casimir jump)

Identity criterion: two numbers equal in value are the same word only if they have the same atoms with the same origins.

Table A2 Anchor constants and bounds of the field.

Quantity	Form in the language	Value	Family	Class
c^4/G	$K\ell_P^2 = 40\,\sigma_{\text{Pleno}}$	1.2×10^{44} N	angular	[derived]
$\hbar$	$E_{\text{cell}}\tau_P = \rho_P c \ell_P^4$	—	—	[derived]
c	$\sqrt{K/\rho_P}$	—	—	[measured]
$c^4/(16\pi G)$	4π (Green) $\times$ Z^2	—	mixed	[derived]
$S_{\min}$	$12(\pi^2-4)/\pi^4$	0.723087	angular	[derived]
$\langle S^2\rangle$	$S_{\min}^2$	0.522855	angular	[derived]
$S_{\max}^2$	$8\sqrt{6}/9 = \sin^2\theta_{\text{tet}}\sqrt{6}$	2.177324	bond	[derived]
S^*	$\sqrt{S_{\min}S_{\max}}$	1.032942	mixed	[derived]
A_{annulus}	$\pi(S_{\max}^2 - S_{\min}^2)$	5.197669	mixed	[language-derived]

Table A3 Internal coefficients, elastic sector and radial mode.

Quantity	Closed form	Value	Class
γ_κ	$1/40 = 1/(2Z\cdot 5)$	0.025	[algebra-derived]
regulator	$2\gamma_\kappa = 1/20$	0.05	[algebra-derived]
σ_{Pleno}	$K\ell_P^2/40$	$F_P/40$	[derived]
$\sigma_{\min}$	$\ell_P/\sqrt{20}$	$0.223607\,\ell_P$	[derived]
τ_Π	$\sqrt{5}\,\ell_P/(10c)$	$0.223607\,\tau_P$	[derived]
GUP	$(1/20)\cdot 3\cdot(1/4) = 3/80$	0.0375	[derived]
$\tilde{\mu}$	1 (exact)	1	[derived]
K/μ	$\frac{1}{4}\xi S_{\min}^2$	2.5737	[derived]
$\tilde{\lambda}$	$K - 2\mu/3$	1.907	[derived]
$\tilde{E}$	$9K\mu/(3K+\mu)$	2.656	[derived]
ν	$(3K-2\mu)/[2(3K+\mu)]$	0.328	[derived]
ξ	$V_{\text{eff}}''(S_{\min})/(\rho c^2)$	19.6899	[derived], no closed form
m_S	$\sqrt{\xi}$	$4.4373\,E_P$	[derived]
m_{prop}	$\sqrt{-10+\sqrt{100+20\xi}}$	$3.4959\,E_P$	[derived]

A.2 Anchor constants and field bounds

A.3 Internal coefficients and elastic sector

A.4 Thermodynamics, cosmology and quantum sector

Appendix B Register of rejected coincidences

All six cases are verifiable by recomputing the closed forms.

Table A4 Readings of the thermodynamic, cosmological and quantum sectors.

Quantity	Closed form	Value	Class
R_{floor}	$2 \cdot 5^{3/4} 6^{7/8} \sqrt{\pi}/75$	0.757973	[algebra-derived]
$k_B T_{\max}$	$5^{1/4} 6^{7/8}/(3\sqrt{\pi}) \cdot K\ell_P^3$	1.348741	[derived]
s (radiation)	$(2\pi^2/45) g_* k_B (k_B T/\hbar c)^3$	$2\pi^2/45 = 0.438649$	[algebra-derived]
Tsirelson	$2\sqrt{2}$	2.828427	[derived]
$\Omega_{\text{DM}}/\Omega_{\text{B}}$	$2A_S/A_T = \sqrt{3}\pi$	5.441398	[AI-derived]
$H_{\max}$	$\sqrt{8\pi S_{\max}^2/3}\,(c/\ell_P)$	$R_{H,\min} \approx 0.234\ell_P$	[derived]
f_c	$S_{\max,\text{eff}}^2/S_{\max}^2$	—	[positional]

Table B5 Numerical coincidences rejected by the family criterion. The ground for rejection is disjointness of families or of origin atoms, not the size of the discrepancy.

Coincidence	Proximity	Families	Verdict
$S_{\max}/S_{\min} = 2.0407$ against 2	1.9 %	own closed form	rejected
$1 - S_{\min}^2/S_{\max}^2 = 0.75986$ against $R_{\text{floor}} = 0.75797$	0.25 %	amplitude / transport	rejected
$\sqrt{2}\,S_{\min} = 1.02260$ against $S^* = 1.03294$	1.0 %	partition / geometry	rejected
$\xi = 19.6899$ against $2\pi^2 = 19.7392$	0.25 %	no common origin	rejected
spectral 9 against rotational 9	exact	distinct lineages	not grouped
affine 3/4 minus polar 1/4 = 1/2	exact	distinct operators	rejected

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