Universal Integrity Principle

PIU–Ψ

Complete mathematical compendium

Full derivations, computations and comparisons with observation

Version 5.0

July 2, 2026

Manuel Alberto Celedón Mejía

Mechanical Engineer — Medellín, Colombia

One single thing

PIU-Ψ is a **research program in theoretical physics** —in cosmology and quantum physics— built on strict mathematics. It starts from a radical and precise hypothesis: that gravity, light, particles, space, time and the dark sectors that make up 95 % of the universe are not independent entities, but **configurations of a single physical substrate** —el Pleno—, described by a single field $\Psi = S\,e^{i\theta}$ and six axioms. From two properties of the medium (ρ_P, ℓ_P) and its causal structure c, it recovers G and $\hbar$ with agreement $< 0.1\%$ without fitting; it derives the Heisenberg uncertainty with its exact floor $\hbar/2$; it makes gravity emerge (Newton and the Einstein structure) and the Dirac equation; it obtains Maxwell and Yang-Mills from phase circulation; it predicts the cosmological ratios to within 1σ of Planck; it reads the Hubble constant as the time of a cosmic clock —reformulating the cosmological-constant problem, placing the background value on the primordial side (~ 67, with the local ~ 73 structurally excluded) and establishing the Hubble tension as a method artefact that dissolves in model-independent observables—; and it reads Relativity as the rheology of a dilatant fluid. General Relativity and Quantum Mechanics cease to be two theories to reconcile: they emerge as effective readings of the same medium. It statistically matches ΛCDM with *derived* parameters, not fitted, and adds falsifiable predictions via LISA and Fermi-LAT. Each step carries its algebra and its epistemic classification. The invitation is to verify it.

What this document is, and how to read it

This compendium gathers **all the mathematical part** of the theoretical-physics research program PIU-Ψ (V5.0) with the **complete, unabridged** derivations: every closed form, every coefficient and every numerical value is obtained by showing the intermediate algebra. It is reorganized **by physical topics**, so that all the mathematics of a single matter falls under a single title. The key computations —averages over the Brillouin zone, the coefficients $1/40$ and $8/9$, the minimization of the potential, the Born rule and the Tsirelson bound, the Lane-Emden equation, the topological baryon number— have been **verified symbolically and numerically**. Where the program imports an external result or declares an open end, it is signaled explicitly and no derivation is simulated.

These verifications are **reproducible**: this compendium is accompanied by a verification notebook (script `verificar_PIU.py`, supplementary material) that recomputes each constant and ratio from its first-principles formulas, without preloaded values; its execution reproduces the thirty-one central quantities with relative error below 10^{-3} (the complete table is included in Appendix 16). This compendium accompanies the **main corpus of PIU-Ψ**, which presents the conceptual development and the physical reading of each result; here the complete mathematical apparatus is collected.

The **epistemic classification** of each block appears in its **title**, in brackets: [**DERIVED**] = complete proof; [**SLD**] = strong logical deduction (structural argument without a conceptual gap); [**CALIBRATED**] = value anchored to a datum with declared error; [**HYPOTHESIS**] = motivated conjecture. Titles with mixed status carry the combination. The label [**CALIBRATED**] never means a free parameter fitted to force the agreement: it covers either a *ruler calibration* —el Pleno has no units, and fixing the value of ρ_P or ℓ_P is to introduce the "human ruler" of measurement, the only act that allows comparison with observation (§4.4)—, or a *positional state quantity*, whose form is derivable and whose present value is a historical datum, like H_0 in any cosmology (§11.9). This discipline —always declaring what is proven, what is deduced and what is anchored— is part of the method.

https://www.piuniversal.com • Zenodo DOI 10.5281/zenodo.21205881

Contents

1 Foundations: the logical chain from energy to F1 — [DERIVED] / [SLD]

The universe is energy: finite at each point and conserved. From finiteness comes density; from density in a medium, the minimal grain; from variability, the change ceiling and time; from conservation, the balance and with it $E = Mc^2$, the field Ψ and the Lorentz factor. Once the pieces are gathered, F1 remains.

1.1 From energy to density, grain and time — [DERIVED] / [SLD]

The root statement: the universe is energy. The starting point is a single one (axiom A1): the universe *is* energy —it does not contain energy, it is energy—, in a single, homogeneous and isotropic substrate. That energy is finite at each point ($0 < E < \infty$, bounded on both sides) and conserved (it is neither created nor destroyed, only transformed and redistributed). The pointwise finiteness is the program's own postulation; conservation is the Mayer-Joule-Helmholtz principle, which the program adopts, does not derive.

Ceilings, not numbers. From finiteness follow *edges*: a ceiling on energy per region, a minimal grain, a floor, a ceiling on change. That an edge *exists* is a property of el Pleno; that it *takes such a number* appears only when measuring it against a human ruler —a distinction that §1.4 develops when reading the values of ρ_P, ℓ_P and c. What has its own value in el Pleno are the *pure ratios* ($1/40$, $\sqrt{3}\pi$, $S_{\min}$), which geometry produces without any ruler.

Energy per region: the density ρ_P. If energy is finite *at each point*, then we are already speaking of energy *per region*; and energy per region is, literally, a density. Here ρ_P is born —not as a new object, but as energy itself read per volume. The same quantity admits three readings: as density (ρ_P), as inertia (M) and as substance (E). Under units of the medium itself ($c = 1$, $\ell_P = 1$, so that the cell volume is $\ell_P^3 = 1$ and $E = Mc^2$ reads $E = M$), the three collapse into a chain of identities:

$$\rho_P \;\equiv\; M \;\equiv\; E. \tag{E0a}$$

There is no causal arrow: never "from E arises ρ_P", always "E, read per region, *is* ρ_P". It is a single object read from three angles.

The region has structure: the minimal grain ℓ_P. To speak of "energy per region" presupposes that the region is well defined; a finite density in a continuous substrate needs a minimal grain of coherence, below which the notion of "region" dissolves. Hence ℓ_P, the coherence scale of el Pleno —its minimal grain of structure, the second substantive property with dimensions. It is born *because* we speak of density in a structured medium; its full role (regulating the curvature of the field) arrives with the flexural term of F1.

Energy changes: the change ceiling c. Energy is finite but *variable*. Change itself inherits from A1 a wall of finiteness: there exists a maximum capacity of change, c. It is not "the speed of light" —a velocity presupposes space and time already given— but the maximum rate of change of energy itself, a property prior to the metric. The internal change C of a configuration is bounded, $C \in [0, c]$: $C = 0$ is energy at rest (rest mass), $C = c$ is change at the ceiling (pure propagation, the massless mode). Being causal structure and not a property that the medium "chooses", c is classified apart from ρ_P and ℓ_P.

With c available, time emerges. Time is not postulated: it is read as the amount of accumulated change, measured against the maximum possible change whose scale is fixed by c. Without a change ceiling (if $c \to \infty$) there would be no common rhythm and time would not be defined. The causal direction is not circular: the change C is internal to energy and requires no clock; from it its extreme c is taken; and from c time is obtained as accumulated change. Time is the last thing to appear in this stretch: it emerges, it is not postulated —coherent with the program's thesis that space and time are emergent properties of el Pleno, not primitive entities, and that the "Minkowski spacetime" is the effective metric of the propagation of the modes in the relaxed regime.

Conclusion — link 1: the substrate and its properties

From a single statement —the universe is finite, conserved energy— are born, forced in a chain, the density ρ_P (energy per region), the minimal grain ℓ_P (the region has structure), the change ceiling c (energy varies) and time (accumulated change against c). None is postulated separately: each is demanded by the previous one.

1.2 The balance, $E = Mc^2$ and the field Ψ — [SLD]

Conservation as equilibrium: the balance. Energy is conserved and has only two destinies: to be retained (confined, permanence, mass) or to be propagating (propagating, change, radiation). There is no third destiny, so every transformation is a transit between those two branches: what leaves one enters the other. With E_{conf} and E_{prop} the energies of each branch, local conservation is already a continuity equation,

$$\Delta E_{\text{conf}} + \Delta E_{\text{prop}} = 0 \quad \equiv \quad \partial_t \rho_E + \nabla \cdot j_E = 0, \tag{E0b}$$

legitimate because time already emerged in the previous link. Expressed in the common currency of change, the equilibrium between the two branches is

$$\boxed{\frac{E}{c} - Mc = 0} \qquad \Longleftrightarrow \qquad \underbrace{\frac{E}{c}}_{\text{dynamic change}} = \underbrace{Mc}_{\text{static change}} . \tag{E0c}$$

Why the two terms are comparable: both are momentum. The balance is legitimate because its two terms have the same dimension:

$$\left[\frac{E}{c}\right] = \frac{\text{J}}{\text{m/s}} = \text{kg} \cdot \frac{\text{m}}{\text{s}}, \qquad [Mc] = \text{kg} \cdot \frac{\text{m}}{\text{s}},$$

both *momentum* (kg·m/s), the natural measure of change. To subtract $E/c - Mc$ is to subtract two momenta: two readings of change in the same unit. The term E/c is energy read as *dynamic* change capacity (the propagating change); Mc is inertia read as *static* change (the change retained in the permanence of the mass).

The balance is the Einstein equation read another way. Solving,

$$\frac{E}{c} - Mc = 0 \implies \frac{E}{c} = Mc \xrightarrow{\times c} E = Mc^2.$$

To multiply by c is not a trick: it is to pass from the currency of momentum (kg·m/s) to that of energy (J). The c^2 is not opaque —it is a c of the balance (the ceiling that defines the currency) times the c of the unit conversion: two appearances of the *same* single c, one for each branch that the transit connects. Thus, $E = Mc^2$ is not a law of conversion between two distinct substances: it is the assertion that the two readings of the change of one and the same conserved energy —the propagating E/c and the retained Mc— are in equilibrium. Mass does not "transform into" energy; mass *is* retained energy. Einstein wrote the energetic face ($E = Mc^2$); the program exhibits the face of the balance ($E/c = Mc$), the same law one layer below, where one sees why the factor is c^2 and what balances.

The balance made object: the field Ψ. The balance confronts two terms that are the same energy read on two axes: the static (the retained, Mc) and the dynamic (the propagating, E/c). Since a real configuration has a retained part *and* a propagating part at once —it can persist while it changes— the two axes must be independent, at 90°. The operator that keeps two orthogonal magnitudes without one contaminating the other is the imaginary unit i. That is why energy is written as a complex number:

$$\boxed{\Psi = S\, e^{i\theta}} \qquad S \geq 0, \quad \theta \in [0, 2\pi), \tag{E1}$$

with S (the modulus) the *static* axis —the retained quantity, the permanence, what Mc measures— and θ (the rotating phase) the *dynamic* axis —the change in course, the propagation, what E/c measures. That modulus and phase be independent ($|\Psi|^2 = S^2$ does not depend on θ) *is* the assertion that the two branches of the balance are conserved separately. Ψ is the balance $E/c = Mc$ made a geometric object. The modulus inherits from A1 the bounding on both sides,

$$S \in [S_{\min}, S_{\max}], \qquad S_{\min} > 0 \text{ (from } E > 0), \qquad S_{\max} < \infty \text{ (from } E < \infty),$$

the two walls of the finiteness of the quantity, which are realized as two terms of the potential (IR wall at $S \to S_{\min}$, hard wall at $S \to S_{\max}$). The status of this step is explicit: the program *postulates* $\Psi = Se^{i\theta}$; the chain offers the ontological why of its form —the balance of two orthogonal branches— making the language continuous from energy to F1, without an initial unexplained jump.

Conclusion — link 2: the balance and the field

Conservation, expressed as equilibrium between dynamic and static change ($E/c - Mc = 0$), *is* the Einstein equation one layer below, and its structure of two orthogonal branches is exactly what the field $\Psi = Se^{i\theta}$ encodes: the field is not an aesthetic postulate but the geometric writing of the balance.

1.3 The balance with motion: the constraint and the Lorentz factor — [SLD]

From the balance at rest to the balance with motion. The equilibrium $E/c = Mc$ describes the extremes (energy all retained or all propagating). A real configuration —matter in motion— has a retained part and a propagating part at once. What measures the propagating fraction is the *momentum* p: the amount of change that what moves carries with it (and change is measured in momentum, kg·m/s). The retained fraction contributes Mc; the propagating one contributes p.

Why the combination is Pythagorean. Retained and propagating, when they coexist, are the two orthogonal branches of the balance —the same ones that give the i of Ψ. And orthogonal magnitudes do not add linearly: they compose as legs. The total energy is the hypotenuse, in the currency of momentum:

$$\left(\frac{E}{c}\right)^2 = (Mc)^2 + p^2 \xrightarrow{\times c^2} \boxed{E^2 = (Mc^2)^2 + (pc)^2}$$

It is the mass-shell constraint, and it is born from the same generalized balance, not from an imported law. The orthogonality of the two branches —the same one that gives the i— is what turns the linear subtraction $E/c - Mc = 0$ into a Pythagorean sum. At rest ($p = 0$) one recovers $E = Mc^2$; at the other extreme ($M = 0$) one recovers $E = pc$, the pure energy of propagation (photon, gravitational wave).

From the constraint, the Lorentz factor. Defining the retained fraction $1/\gamma \equiv Mc^2/E$ (that is $E = \gamma Mc^2$) and substituting:

$$(\gamma Mc^2)^2 = (Mc^2)^2 + (pc)^2 \implies (pc)^2 = (Mc^2)^2(\gamma^2 - 1) \implies p = \gamma Mc\sqrt{1 - \gamma^{-2}}.$$

Identifying the velocity as the ratio between propagating change and retained energy, $v/c \equiv pc/E = pc/(\gamma Mc^2)$, one gets $v/c = \sqrt{1 - \gamma^{-2}}$, that is $\gamma^{-2} = 1 - v^2/c^2$, and therefore

$$\boxed{\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \qquad E = \gamma Mc^2, \qquad p = \gamma Mv.} \tag{1.1}$$

The Lorentz factor is the partition of the change capacity between the two branches, tied by the conservation of energy; it is not the deformation of a prior spacetime —the Lorentz spacetime *describes* this partition a posteriori. The covariance (axiom A3) emerges from the balance, not from a postulated stage. The status is [**SLD**]: the algebra is exact, but it rests on c being causal structure —the fragile link of the whole foundation, which is declared and not concealed. The equivalent quantitative derivation from the kinetic invariant of F1 is in §12 (E49–E50): they do not compete, they are two readings of the same fact.

Conclusion — link 3: relativity emerges from the balance

The constraint $E^2 = (Mc^2)^2 + (pc)^2$ and the Lorentz factor are not imported: they are the balance with motion, with the Pythagorean sum forced by the same orthogonality of the two branches that defines Ψ. Special relativity remains as an effective layer of el Pleno.

1.4 The six axioms and the bridge to F1 — [DERIVED]

The previous chain rests, at each link, on a minimal set of six axioms that are formalized here as a synthesis of what has already been covered:

Ax.	Name	Abridged statement
A1	Energy-substance	The universe *is* energy: finite at each point ($0 < E < \infty$) and conserved. Density, inertia and energy are the same quantity ($\rho_P \equiv M \equiv E$). Root of the whole chain.
A2	Positivity / IR wall	The modulus is strictly positive, $S > 0$; $S = 0$ is inaccessible (divergent energy). Fixes $S_{\min} > 0$ (lower face of the finiteness of A1).
A3	Causal covariance	There exists a change ceiling c; the dynamics is Lorentz-covariant. Emerges from the balance (link 3), it is not postulated as a prior stage.
A4	UV regulation / rigidity	The field penalizes curvature: flexural term $(\Box S)^2$, with the grain ℓ_P. Gives the ultraviolet cutoff $\Lambda = 1/\ell_P$ and the minimal width $\sigma_{\min} = \ell_P/\sqrt{20}$.
A5	U(1) symmetry	The action is invariant under $\theta \to \theta + \alpha$ (the freedom of the phase, dynamic axis of Ψ). Gives the conserved Noether current and the charge (quantized by topology).
A6	Hard bound / saturation	The modulus is bounded above, $S \leq S_{\max}$, by maximal packing of the lattice. Blocks singularities; gives the bounce.

The economy is remarkable: A2 (positivity) is the lower face of the finiteness of A1; A3 (covariance) emerges from the balance; A5 (U(1)) is the freedom of the phase that Ψ already has. The only thing that neither finiteness nor conservation delivers —the continuity and differentiability of the field, which F1 needs since it operates with derivatives of Ψ— remains as the genuinely independent technical axiom.

Primary parameters and convention. The medium is characterized by two substantive properties with dimensions (ρ_P, ℓ_P) and a causal structure (c):

$$\rho_P \approx 5.155 \times 10^{96}\ \text{kg/m}^3, \quad c \approx 2.998 \times 10^8\ \text{m/s}, \quad \ell_P \approx 1.616 \times 10^{-35}\ \text{m}.$$

The constants G and $\hbar$ are not inputs but consequences (§5); the independent calibration of ρ_P or ℓ_P from non-gravitational and non-quantum physics is an open end (§15). Computation over real values; presentation to four figures, relative differences to two.

El Pleno has no units; the units are placed by us. It is worth reading correctly the units that accompany ρ_P, ℓ_P and c above. El Pleno *exists* and *behaves* physically, and that behavior admits an exact mathematical description; but the medium itself possesses no meters, kilograms or seconds. Those units are not properties of el Pleno: they are rulers that we humans manufacture to measure. One must distinguish two things that ordinary language confuses. That an edge *exists* —a density ceiling, a minimal grain, a maximum rate of change— is a property of el Pleno, ontological and independent of all measurement. That that edge *takes such a number* is not: a number appears only upon measuring, and to measure is to compare against a human ruler. That is why the "value" of ρ_P, ℓ_P or c does not inform about el Pleno itself, but about *how many human rulers fit* in the corresponding ceiling —it is information about the relation between el Pleno and our units. In units of the medium itself ($c = 1$, $\ell_P = 1$) those numbers vanish, but the ceilings remain: the structure does not depend on the ruler with which it is measured. The question "what number did el Pleno choose for ρ_P?" is ill-posed, because it presupposes a category —the number— that el Pleno does not have; the legitimate question is to measure its human reading from an independent physics (open end C-meta-1.b, §15). What in el Pleno *does* have its own value, without any ruler, are the **pure ratios** between its structures ($\gamma_\kappa = 1/40$, $\sqrt{3}\pi$, $S_{\min}^{\text{estr}}$, $\sin^2\theta_{\text{tet}} = 8/9$), which the diamond geometry produces as dimensionless numbers: that is why they, and not the values with units, are the ones the program derives with intrinsic value.

The Lagrangian of el Pleno, **F1** (§2), is the dynamical starting point of the program. The diamond lattice, the pure ratios and the fundamental constants are not foundations prior to F1 but consequences of it: they are obtained upon evaluating it, in the sections that follow.

---

2 The F1 Lagrangian of el Pleno — [DERIVED]

The Lagrangian of el Pleno is named **F1**; its symbol in the equations is $\mathcal{L}_{\text{F1}}$. Applying Euler-Lagrange to it yields the dynamical equation of the modulus, which is named **F2** (= E6). In this compendium "F1" and "F2" are used as names in the prose, and $\mathcal{L}_{\text{F1}}$ as the symbol in the formulas.

The F1 Lagrangian has four terms —kinetic, potential, flexural and IR wall—. Its coefficients are not free inputs: the factor 1/40 of the flexural comes from evaluating F1 over the lattice that the medium itself crystallizes (§3); the 3/8 of the IR wall, from the affine quantization of the half-line $S \geq 0$ (§2.3); the kinetic coefficient is identified with $c^4/2G$ once G is derived (§5); and $V(S)$ is the tripartite potential of §4.

2.1 Form of the Lagrangian and construction of each term — [DERIVED]

The fundamental field is Ψ (E1). The standard Lagrangian procedure —identify the field, impose symmetries, construct compatible terms, fix coefficients— applied to el Pleno under the axioms A1–A6 produces a unique minimal Lagrangian density:

$$\boxed{\mathcal{L}_{\text{F1}} = \tfrac{1}{2}\rho_P c^2 \ell_P^2\, \partial_\mu \Psi^* \partial^\mu \Psi - V(S) - \frac{\rho_P c^2 \ell_P^4}{40}(\Box S)^2 - V_q^{\text{dens}}(S)} \tag{E2}$$

The coefficients 1/40 and 3/8 that appear in (E2) are derived in §3 and §2.3 respectively.

Reading: F1 is the single object from which the whole program is derived. It has no free parameters —each coefficient comes from the geometry of the medium—, so that all subsequent physics is an evaluation of it, not a new postulate.

Kinetic term. Expanding $\Psi = S\,e^{i\theta}$,

$$\partial_\mu \Psi = (\partial_\mu S)e^{i\theta} + iS(\partial_\mu \theta)e^{i\theta}, \qquad \partial_\mu \Psi^* = (\partial_\mu S)e^{-i\theta} - iS(\partial_\mu \theta)e^{-i\theta},$$

so that the product, upon cancellation of the imaginary cross terms, is

$$\partial_\mu \Psi^* \partial^\mu \Psi = (\partial_\mu S)(\partial^\mu S) + S^2(\partial_\mu \theta)(\partial^\mu \theta),$$

$$\mathcal{L}_{\text{cin}} = \tfrac{1}{2}\rho_P c^2 \ell_P^2 \big[(\partial_\mu S)(\partial^\mu S) + S^2(\partial_\mu \theta)(\partial^\mu \theta)\big].$$

The first contribution is the structural propagation (radial mode); the second, weighted by S^2, that of the induced gauge sector. The coefficient $\rho_P c^2 \ell_P^2$ is identified with c^4/G (§5.1).

Potential $-V(S)$. A function exclusively of the modulus (the U(1) symmetry forbids dependence on θ). Tripartite structure $V(S) = W_{\text{bare}} + V_q + V_{\text{topo}}$, developed in §4.

Flexural term. Axiom A4 (ultraviolet regulation) requires penalizing the curvature of the field; the simplest canonical form compatible with the symmetries is quadratic in the d'Alembertian of the modulus, $(\Box S)^2$, with $\Box S = \partial_\mu \partial^\mu S$. The coefficient is

$$\gamma_\kappa = \frac{\rho_P c^2 \ell_P^4}{40} = \frac{\hbar c}{40} \approx 7.90 \times 10^{-28}\ \text{J} \cdot \text{m}, \tag{E4}$$

with dimensions $[J \cdot m]$ identical to the stiffness EI of Euler-Bernoulli (the second equality uses $\hbar = \rho_P c \ell_P^4$). The factor $1/40$ is derived from the geometry of the lattice in §3; the power ℓ_P^4 (compared to ℓ_P^2 of the kinetic) is what gives dimensions of energy density to a term with two additional derivatives.

Klauder term $-V_q^{\text{dens}}(S)$. Infrared regulation (A2): quantum wall that prevents $S \to 0$. Its origin and coefficient are developed in §2.3:

$$V_q^{\text{dens}}(S) = \frac{3\hbar^2}{8\,\rho_P\,\ell_P^8\,S^2} = \frac{3}{8}\frac{\rho_P c^2}{S^2}. \tag{E5}$$

Conclusion — link 4: the Lagrangian is fixed

With the form of the field inherited from the balance, F1 is written: the minimal mathematical expression of el Pleno, four terms with a structural function (to propagate, sustain configurations, regulate UV, regulate IR) and no free coefficient —all built with ρ_P, c, ℓ_P (and $\hbar$ where appropriate). From F1 the rest of the phenomenology of el Pleno is derived.

2.2 The flexural term over the complete field — [DERIVED]

The writing $(\Box S)^2$ is shorthand notation for a regulator that acts over Ψ entirely, not only over the modulus. Expanding $\Box\Psi$ around the background $\Psi_0 = S_0 e^{i\theta_0}$ and retaining the quadratic order in the perturbations δS, $\delta\theta$:

$$\mathcal{L}^{(2)}_{\text{flex}} = -\frac{\rho_P c^2 \ell_P^4}{40}\Big[(\Box\,\delta S)^2 + S_0^2(\Box\,\delta\theta)^2\Big] + \mathcal{O}(\delta^3). \tag{E4c}$$

The phase sector inherits the same regulator, weighted by S_0^2 exactly like the kinetic phase term. Both come from the same link-difference operator of the lattice ($|\Psi_n - \Psi_m|^2 = (s_n - s_m)^2 + S_0^2(a_n - a_m)^2$, §3), whence the same ratio $1/40$ and the same ultraviolet correction $(k\ell_P)^2/20$ for photon and TT mode.

Conclusion

The UV regulation does not distinguish modulus from phase nor scalar from tensor structure, because it is a property of the lattice operator that the flexural encodes. Hence light and the gravitational wave share not only the velocity c but also their first Planckian correction $(k\ell_P)^2/20$.

2.3 The Klauder quantum potential and the coefficient 3/8 — [DERIVED]

Since the modulus satisfies $S \geq 0$, it defines a configuration manifold of the half-line type, and standard canonical quantization (operators over the whole line) does not apply without pathologies. The correct quantization is the **affine Klauder**, which promotes to operators the modulus $\hat{S}$ and the generator of dilations $\hat{D} = (\hat{S}\hat{\Pi} + \hat{\Pi}\hat{S})/2$, with the affine algebra

$$[\hat{S}, \hat{D}] = i\hbar\hat{S}$$

instead of the canonical $[\hat{S}, \hat{\Pi}] = i\hbar$. The difference is structural: the affine algebra preserves exactly $S > 0$ at the quantum level.

The coefficient 3/4 (Klauder result, imported). Over the half-line with the invariant measure dS/S, the symmetrized Weyl ordering produces, for the kinetic operator re-expressed in affine variables, the term

$$\frac{\hat{\Pi}^2}{2m_{\text{eff}}} = \frac{1}{2m_{\text{eff}}\hat{S}^2}\Big(\hat{D}^2 + \frac{3\hbar^2}{4}\Big),$$

so that the Hamiltonian acts on $\psi(S)$ as

$$\hat{H}_K\psi = -\frac{\hbar^2}{2m_{\text{eff}}}\frac{\partial^2\psi}{\partial S^2} + \frac{3\hbar^2}{8\,m_{\text{eff}}S^2}\psi, \qquad m_{\text{eff}} = \rho_P\ell_P^2.$$

The coefficient 3/4 of the term $\hbar^2/S^2$ is the standard result of the affine Klauder quantization (2000) for a variable over the half-line with measure dS/S; it depends crucially on that measure, not on the Lebesgue one. PIU-Ψ *imports* this theorem under its rules for the use of mathematical tools, instead of re-deriving it. The factor $3/8 = 3/4 \times 1/2$ combines that 3/4 with the kinetic factor 1/2.

Energy density. Converted to energy density to enter the Lagrangian (cell-volume factor ℓ_P^{-3} and kinetic factor):

$$V_q^{\text{dens}}(S) = \frac{3\hbar^2}{8\,\rho_P\,\ell_P^8\,S^2} = \frac{3}{8}\frac{\rho_P c^2}{S^2}, \tag{E5}$$

using $\hbar = \rho_P c\ell_P^4$. It diverges as $S \to 0$.

Quantitative inaccessibility of $S = 0$. The energy cost of taking the medium from the ground-Pleno $S_{\text{min}}^{\text{estr}} \approx 0.7231$ to $S = \epsilon$ is

$$\Delta E(\epsilon) = \frac{3}{8}\rho_P c^2\Big(\frac{1}{\epsilon^2} - \frac{1}{(S_{\text{min}}^{\text{estr}})^2}\Big) \xrightarrow{\epsilon\to 0} +\infty.$$

Conclusion

The first of the two coefficients of F1 is fixed: the IR wall prevents the medium from "switching off", since $S \to 0$ costs infinite energy —it is not a condition placed by hand. The 3/8 is a result *imported* from Klauder, not an internal derivation; it enters F1 without recalibration, since once the affine measure is fixed the coefficient is unique.

2.4 Dimensional verification of the four terms — [DERIVED]

With $[\rho_P] = kg/m^3$, $[c] = m/s$, $[\ell_P] = m$, $[\hbar] = J\,s$, S, θ dimensionless, $[\partial_\mu] = 1/m$, $[\Box] = 1/m^2$:

$$[\mathcal{L}_{\rm cin}] = \frac{kg}{m^3}\cdot\frac{m^2}{s^2}\cdot m^2\cdot\frac{1}{m^2} = \frac{kg}{m\,s^2} = \frac{J}{m^3}\ \checkmark$$

$$[\mathcal{L}_{\rm flex}] = \frac{kg}{m^3}\cdot\frac{m^2}{s^2}\cdot m^4\cdot\frac{1}{m^4} = \frac{J}{m^3}\ \checkmark, \qquad [\gamma_\kappa] = \frac{kg}{m^3}\cdot\frac{m^2}{s^2}\cdot m^4 = \frac{kg\,m^3}{s^2} = J\cdot m\ \checkmark$$

$$[\mathcal{L}_q] = \frac{(J\,s)^2}{(kg/m^3)\,m^8} = \frac{(kg\,m^2/s^2)^2 s^2}{kg\,m^5} = \frac{kg}{m\,s^2} = \frac{J}{m^3}\ \checkmark$$

The action $\mathcal{S} = \int \mathcal{L}_{\rm F1}\,dt\,d^3x$ has dimensions $[J/m^3]\cdot[s\,m^3] = [J\cdot s]$, identical to $\hbar$, a necessary condition for $\mathcal{S}/\hbar$ to be dimensionless.

Conclusion

The four terms share the dimension of energy density with the same three primaries. The consistency is not trivial —it forces the flexural to carry ℓ_P^4 and the IR wall ℓ_P^8, without freedom. F1 is dimensionally armored.

2.5 Dynamical equations: Euler-Lagrange over F1 — [DERIVED]

Since F1 contains $(\Box S)^2$ (second derivatives), the Euler-Lagrange equations carry the third-order term:

$$\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} + \partial_\mu\partial_\nu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\partial_\nu\phi)} = 0.$$

Modulus equation (F2). The partial derivatives of F1 with respect to S:

$$\frac{\partial\mathcal{L}_{\rm F1}}{\partial S} = \rho_P c^2\ell_P^2 S(\partial_\mu\theta)(\partial^\mu\theta) - V'(S) - V_q'(S),$$

$$\frac{\partial\mathcal{L}_{\rm F1}}{\partial(\partial_\mu S)} = \rho_P c^2\ell_P^2\,\partial^\mu S, \qquad \frac{\partial\mathcal{L}_{\rm F1}}{\partial(\partial_\mu\partial_\nu S)} = -\frac{\rho_P c^2\ell_P^4}{20}(\Box S)\eta^{\mu\nu},$$

where the $1/20$ appears upon differentiating $(\Box S)^2/40$ with respect to $\Box S$ (factor 2). Substituting and grouping:

$$\boxed{\rho_P c^2\ell_P^2[\Box S - S(\partial_\mu\theta)(\partial^\mu\theta)] - \frac{\rho_P c^2\ell_P^4}{20}\Box(\Box S) + V'(S) + V_q'(S) = 0} \tag{E6}$$

Primordial equation F2: a fourth-order differential (through $\Box(\Box S)$), with kinetic, induced-gauge, potential and IR-wall terms.

Phase equation. Since F1 does not depend explicitly on θ (U(1) symmetry), $\partial\mathcal{L}/\partial\theta = 0$, and with the phase flexural term of (E4c):

$$\frac{\partial\mathcal{L}_{\rm F1}}{\partial(\partial_\mu\theta)} = \rho_P c^2\ell_P^2 S^2\partial^\mu\theta, \qquad \frac{\partial\mathcal{L}_{\rm F1}}{\partial(\partial_\mu\partial_\nu\theta)} = -\frac{\rho_P c^2\ell_P^4}{20}S^2(\Box\theta)\eta^{\mu\nu}.$$

The Euler-Lagrange equation gives, factoring $\rho_P c^2\ell_P^2 = c^4/G$:

$$\boxed{\partial_\mu(S^2\partial^\mu\theta) - \frac{\ell_P^2}{20}\Box(S^2\Box\theta) = 0} \tag{E7}$$

The associated U(1) Noether current is

$$J^\mu = \rho_P c^2\ell_P^2 S^2\Big[\partial^\mu\theta + \frac{\ell_P^2}{20}\partial^\mu(\Box\theta)\Big], \qquad \partial_\mu J^\mu = 0 \text{ (exact over the solutions).} \tag{E7c}$$

On the background $S = S_{\rm min}^{\rm estr}$ constant, (E7) reduces to

$$\Box\theta - \frac{\ell_P^2}{20}\Box^2\theta = 0,$$

with physical infrared branch $\Box\theta = 0$ (luminal propagation, Goldstone boson of U(1)) and ultraviolet branch $\Box\theta = 20/\ell_P^2$ (Lee-Wick pole). Over the massless physical branch the flexural term of the current vanishes and J^μ reduces to the standard current $\rho_P c^2 \ell_P^2 S^2 \partial^\mu\theta$, identifiable with the electric current (§9).

Conclusion — link 4 closed: the dynamics of F1

By varying F1 one obtains the whole dynamics of the program, reduced to two coupled equations: F2 (fourth order, modulus) and the U(1) continuity (phase). The fourth order is not a defect: it is the ultraviolet regulation made equation, with the Lee-Wick pole as its imprint.

---

3 The diamond lattice: first evaluation of F1 — [DERIVED]

In the saturated regime, the flexural term $(\Box S)^2$ of F1 penalizes the curvature of the field; the configuration that minimizes it with maximal isotropic coordination is the diamond lattice $Fd\bar{3}m$ ($Z = 4$, isostatic by Maxwell counting). Evaluating the Laplacian of F1 over that lattice produces the two pure ratios of the medium —the ground-Pleno $S_{\rm min}^{\rm estr}$ and the flexural coefficient 1/40—, which the Lagrangian (E2) carried written without deriving.

Why the diamond lattice. The choice is not a stipulation: four independent physical criteria, each a consequence of an axiom, converge on the same structure. *(i) Minimal stable coordination (A4).* The term $(\Box S)^2$ penalizes curvature; the lattice that minimizes the flexural energy subject to mechanical stability in 3D is that of the *lowest coordination that is still rigid*. By Maxwell counting, a lower coordination leaves soft modes (unstable medium) and a higher one over-constrains and penalizes energetically; the optimum is $Z = 4$. *(ii) Maximal isotropy.* Among the lattices with $Z = 4$, the *tetrahedral* coordination (109.47°) is the only one that distributes the four bonds in a perfectly isotropic way, $\sum_i \boldsymbol{\delta}_i = 0$ and $\sum_i \delta_i^a\delta_i^b = \frac{4}{3}\ell_P^2\delta^{ab}$ (the two identities below): the second-moment tensor is proportional to the identity, without privileged directions, as the emergent covariance (A3) requires. *(iii) Bipartition (A5).* The diamond lattice is bipartite (A/B sublattices), which admits the $\mathbb{Z}_2$ coloring from which the U(1) phase and, higher up, the gauge sector hang; a non-bipartite lattice would not support this structure. *(iv) Spin quantization (Finkelstein-Rubinstein).* The topology of the diamond lattice fixes $\chi_{FR} = -1$ for the matter solitons, that is, fermionic statistics, without postulating it.

The geometry of the lattice is derived in its two pieces, the *coordination* (D1-a) and the *stacking* (D1-b):

D1-a (tetrahedral coordination). The tetrahedral coordination $Z = 4$ is the only arrangement of four nearest neighbors in 3D compatible with A1, and this is **proven**, not only verified numerically. Let $\mathbf{v}_1, \ldots, \mathbf{v}_4$ be the unit bond vectors. The absence of dipole moment of the coordination (every node of a lattice with an inversion center) gives $\sum_i \mathbf{v}_i = 0$, whence

$$0 = \Big|\sum_i \mathbf{v}_i\Big|^2 = \sum_i |\mathbf{v}_i|^2 + 2\sum_{i<j}\mathbf{v}_i\cdot\mathbf{v}_j = 4 + 2\sum_{i<j}\mathbf{v}_i\cdot\mathbf{v}_j \;\Rightarrow\; \sum_{i<j}\mathbf{v}_i\cdot\mathbf{v}_j = -2. \qquad \text{(E13-tet2)}$$

The second-order isotropy, $\sum_i v_i^a v_i^b = \frac{4}{3}\delta^{ab}$, forces the six scalar products to be equal; with sum -2, each equals $-1/3$, that is $\cos\theta_{ij} = -\frac{1}{3}$ for every pair, the tetrahedral angle $\theta = \arccos(-\frac{1}{3}) = 109.47°$. A set of four points on S^2 with isotropic second-moment matrix is a spherical 2-design, unique up to rotation: the regular tetrahedron (Delsarte-Goethals-Seidel 1977). The direct numerical optimization (100% of the seeds converge to the tetrahedron, residual cost $\sim 10^{-12}$) confirms this closed result, it does not replace it. **[DERIVED]** (conditional only on the class of Bravais lattices with a centrosymmetric coordination basis).

D1-b (stacking). Given the tetrahedral coordination $Z = 4$, the diamond stacking (cubic $Fd\bar{3}m$) is the only one against wurtzite (hexagonal $P6_3mc$) and every tetrahedral polytype with a distinguished stacking axis. The proof (next subsection) is the minimum of the isotropic flexural functional of F1: the fourth-order uniaxial anisotropy of the modulation equals 0 for the cubic and 4/81 (exact, irreducible, anchored to the invariant 1/40 of γ_κ) for the hexagonal, of strictly positive flexural cost. El Pleno, a continuous medium that relaxes to the minimum of the flexural functional, stabilizes in the only modulation that does not concentrate curvature: the cubic stacking. **[DERIVED]**. The geometry of the lattice is derived in its two pieces: the diamond lattice is the configuration of the saturated Pleno without residue of choice.

3.1 Geometry of the lattice and bond vectors — [DERIVED]

Bipartite diamond lattice $Fd\bar{3}m$ (A, B sublattices), coordination $Z = 4$. Nearest-neighbor vectors from a node of A:

$$\boldsymbol{\delta}_1 = \frac{\ell_P}{\sqrt{3}}(1,1,1), \quad \boldsymbol{\delta}_2 = \frac{\ell_P}{\sqrt{3}}(1,-1,-1), \quad \boldsymbol{\delta}_3 = \frac{\ell_P}{\sqrt{3}}(-1,1,-1), \quad \boldsymbol{\delta}_4 = \frac{\ell_P}{\sqrt{3}}(-1,-1,1).$$

Two exact identities (verified symbolically):

$$\sum_{i=1}^{4} \boldsymbol{\delta}_i = 0, \qquad \sum_{i=1}^{4} \delta_i^a \delta_i^b = \frac{4\ell_P^2}{3}\,\delta^{ab}.$$

Structure factor $f(\mathbf{k}) = \sum_{i=1}^{4} e^{i\mathbf{k}\cdot\boldsymbol{\delta}_i}$; the lattice Hamiltonian is the 2×2 matrix over the bipartition with eigenvalues $\epsilon_\pm = \pm K_{\text{cin}}|f(\mathbf{k})|$.

Bipartition. The A and B sublattices are each FCC; B is displaced from A by $\boldsymbol{\delta}_0 = (a_{\text{diamond}}/4)(1,1,1)$ with $a_{\text{diamond}} = 4\ell_P/\sqrt{3}$, and every bond connects A with B. This bipartition is the $\mathbb{Z}_2$ coloring that the axiom A5 realizes as condensation of the phase θ, and from it hangs the gauge sector (§9).

Brillouin zone. The BZ of the diamond lattice is the truncated octahedron, with high-symmetry points Γ, X, L, W, K, U. At the point L one verifies $|f(L)|^2 = 2$, a geometric manifestation of the invariant $\sqrt{2}$ of the Kepler-Hales family that reappears in the hard bound $S^2_{\text{max,geom}} = 8\sqrt{6}/9$ (§4.3).

Evasion of the Nielsen-Ninomiya theorem. The Nielsen-Ninomiya theorem (1981) establishes that a fermionic field defined over a regular hypercubic discrete lattice inevitably produces unphysical "doublers" (*doubling problem*). In PIU-Ψ this obstruction does not apply: the fermions of the program are not point fields defined over lattice sites, but **topological solitonic modes** —skyrmions over the bipartite sublattice A/B with $Z = 4$ (§10)—, so that the doubling problem is orthogonal to the formulation.

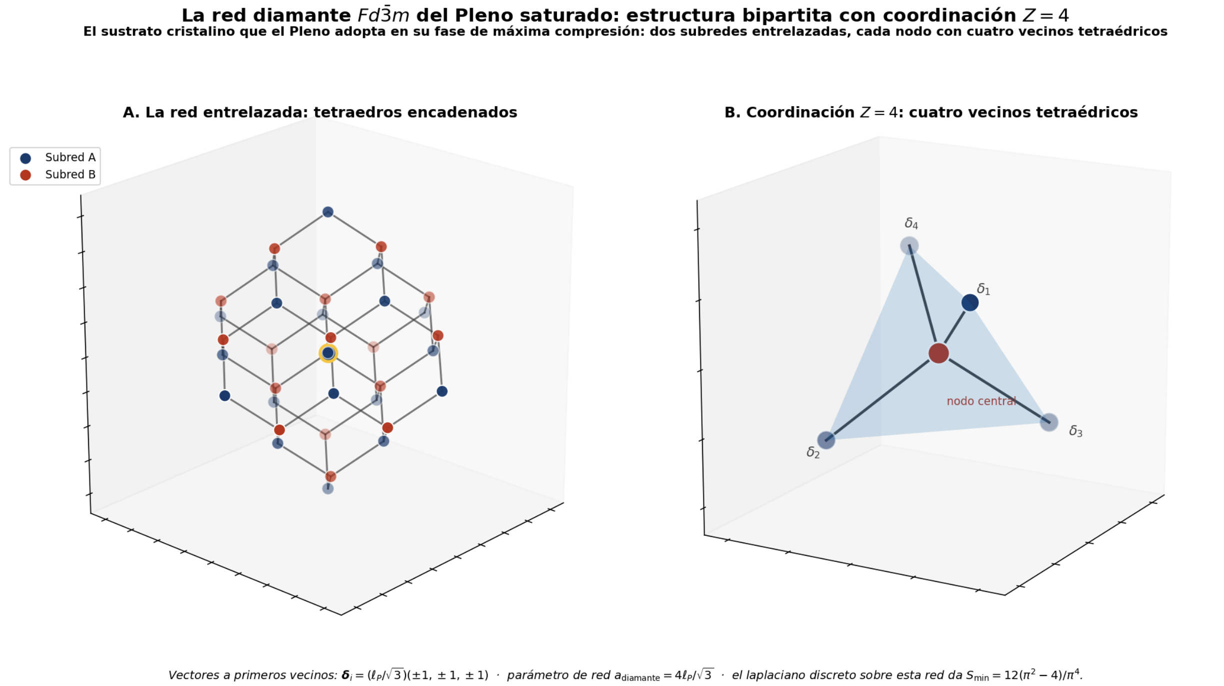

Figure 1: The diamond lattice $Fd\bar{3}m$ and the tetrahedral coordination $Z = 4$. The four nearest-neighbor vectors $\boldsymbol{\delta}_i$ are the geometric basis of the whole block: from $\sum_i \boldsymbol{\delta}_i = 0$ and $\sum_i \delta_i^a \delta_i^b = \frac{4\ell_P^2}{3}\delta^{ab}$ follow the discrete Laplacian (E13), the Brillouin-zone average and the form factor $S_{\min}^{\mathrm{estr}} = 12(\pi^2 - 4)/\pi^4$.

3.2 Discrete / continuous Laplacian relation — [DERIVED]

Expanding f at low $\mathbf{k}$ with $\sum_i \boldsymbol{\delta}_i = 0$ and $\sum_i (\mathbf{k} \cdot \boldsymbol{\delta}_i)^2 = (4\ell_P^2/3)k^2$:

$$f(\mathbf{k}) \approx 4 - \frac{2\ell_P^2}{3}k^2, \qquad |f(\mathbf{k})|^2 \approx 16 - \frac{16\ell_P^2}{3}k^2.$$

Defining the discrete Laplacian $D_{\mathrm{lat}}(\mathbf{k}) \equiv 16 - |f(\mathbf{k})|^2$, with $D_{\mathrm{lat}}(0) = 0$ and $D_{\mathrm{lat}} \approx \frac{16}{3}(k\ell_P)^2$, the inversion at low $\mathbf{k}$ ($\nabla^2 \leftrightarrow -k^2$) gives

$$\boxed{\nabla^2 \;\longleftrightarrow\; -\frac{3}{16\ell_P^2}\,D_{\mathrm{lat}}} \tag{E13}$$

The factor 16/3 is unique: $16 = Z^2$ (square of the coordination) and 3 from the three dimensions of the average.

3.3 Spectral average over the Brillouin zone and $S_{\min}^{\mathrm{estr}}$ — [DERIVED]

This is the step that closes the spectral route to $S_{\min}^{\mathrm{estr}}$. Starting from

$$|f(\mathbf{k})|^2 = 4 + 2\sum_{i<j} \cos\left[\mathbf{k} \cdot (\boldsymbol{\delta}_i - \boldsymbol{\delta}_j)\right],$$

each difference $\boldsymbol{\delta}_i - \boldsymbol{\delta}_j$ has, in cubic-edge coordinates a, exactly two nonzero components of magnitude $a/2$ (the six differences are permutations of $(0, \pm a/2, \pm a/2)$). The average of each cosine over the cubic reciprocal cell $[-\pi/a, \pi/a]^3$ factorizes:

$$\Big\langle \cos\left[\mathbf{k} \cdot (\boldsymbol{\delta}_i - \boldsymbol{\delta}_j)\right] \Big\rangle = \left(\frac{1}{2\pi/a}\int_{-\pi/a}^{\pi/a} \cos\frac{ak}{2}\,dk\right)^2 = \left(\frac{\sin(\pi/2)}{\pi/2}\right)^2 = \left(\frac{2}{\pi}\right)^2.$$

Explicit symbolic verification (sympy, integral of the six differences): each pair gives $4/\pi^2$ and the sum of the six pairs is $24/\pi^2$. Therefore the numerator of the form factor is

$$\langle |f|^2 \rangle_{\mathrm{BZ}} = 4 + 2 \cdot 6 \cdot \frac{4}{\pi^2} = 4 + \frac{48}{\pi^2}, \qquad \langle D_{\mathrm{lat}} \rangle_{\mathrm{BZ}} = 16 - \langle |f|^2 \rangle_{\mathrm{BZ}} = 12 - \frac{48}{\pi^2} = 7.1366.$$

The denominator: average of the tangent parabola. The form factor compares the *real* Laplacian of the band with the one an ideal continuous medium would have, that is, with the parabola tangent to D_{lat} at $k \to 0$. From the low-$\mathbf{k}$ expansion (E13), that parabola is $D^{\mathrm{par}}_{\mathrm{lat}}(\mathbf{k}) = \frac{16}{3}\ell_P^2 k^2$. In cubic-edge coordinates a, with the geometric relation $|\boldsymbol{\delta}_i| = \ell_P$ that gives $a = 4\ell_P/\sqrt{3}$ (that is $\ell_P^2 = 3a^2/16$), the parabola is simply $D^{\mathrm{par}}_{\mathrm{lat}} = a^2k^2 = a^2(k_x^2 + k_y^2 + k_z^2)$. Its average over the same cubic reciprocal cell $[-\pi/a, \pi/a]^3$, using $\langle k_i^2 \rangle = \frac{1}{2\pi/a}\int_{-\pi/a}^{\pi/a} k_i^2\, dk_i = \frac{\pi^2}{3a^2}$ per component, is

$$\langle D^{\mathrm{par}}_{\mathrm{lat}} \rangle_{\mathrm{BZ}} = a^2 \cdot 3 \cdot \frac{\pi^2}{3a^2} = \pi^2 \qquad \text{(verified symbolically).}$$

The factor a^2 cancels: the result is the pure ratio π^2, without scale.

Closure. The spectral form factor of the band is the quotient of the real Laplacian to that of the tangent parabola:

$$\frac{\langle D_{\mathrm{lat}} \rangle_{\mathrm{BZ}}}{\langle D^{\mathrm{par}}_{\mathrm{lat}} \rangle_{\mathrm{BZ}}} = \frac{12 - 48/\pi^2}{\pi^2} = \frac{12\pi^2 - 48}{\pi^4} = \frac{12(\pi^2 - 4)}{\pi^4} = 0.7230871 = S^{\mathrm{estr}}_{\mathrm{min}}, \tag{3.1}$$

coincident with the dynamical route. The π^4 of the denominator is π^2 (from the average of the parabola) times the π^2 that $\langle D_{\mathrm{lat}} \rangle$ already carried when put over a common denominator. The *occupation* is its square, $S^{2,\mathrm{estr}}_{\mathrm{min}} = 0.5229$.

3.4 The flexural coefficient $1/40$ — [DERIVED]

Expanding $|f(\mathbf{k})|^2$ to fourth order:

$$|f(\mathbf{k})|^2 = 16 - \frac{16}{3}\ell_P^2 k^2 + \frac{16}{27}\ell_P^4 \Big(\sum_i k_i^4 + 3 \sum_{i<j} k_i^2 k_j^2 \Big) + \mathcal{O}(k^6).$$

Averaging angularly over $SO(3)$ with $\langle k_i^4 \rangle = k^4/5$ and $\langle k_i^2 k_j^2 \rangle = k^4/15$ (whence $\langle \sum_i k_i^4 \rangle = 3k^4/5$ and $\langle \sum_{i<j} k_i^2 k_j^2 \rangle = k^4/5$):

$$\langle |f|^2 \rangle_{\mathrm{ang}} = 16 - \frac{16}{3}\ell_P^2 k^2 + \frac{32}{45}\ell_P^4 k^4 + \mathcal{O}(k^6).$$

In $D_{\mathrm{lat}} = 16 - |f|^2$ the kinetic term has coefficient $\frac{16}{3}\ell_P^2$ and the fourth-derivative one $\frac{32}{45}\ell_P^4$. The relative weight of the flexural with respect to the square of the kinetic is

$$\boxed{\frac{|\mathrm{coef}_{k^4}|}{(\mathrm{coef}_{k^2})^2} = \frac{32\ell_P^4/45}{(16\ell_P^2/3)^2} = \frac{32/45}{256/9} = \frac{1}{40}} \tag{3.2}$$

Symbolic verification (sympy): the computation of the fourth-order expansion and the angular average give exactly $\mathrm{coef}_{k^4}/(\mathrm{coef}_{k^2})^2 = -1/40$; the negative sign is the convention $D_{\mathrm{lat}} = 16 - |f|^2$ (the real band grows more slowly than its tangent parabola), and the relative magnitude that defines the flexural rigidity is $1/40$. The same lattice operator acts over modulus and phase:

$$|\Psi_n - \Psi_m|^2 = (s_n - s_m)^2 + S_0^2(a_n - a_m)^2,$$

so that photon and TT mode share the UV correction $(k\ell_P)^2/20$, without the scalar or tensor structure of the polarization altering the coefficient.

Conclusion — link 5: F1 delivers its first number

Upon evaluating the flexural of F1, the medium crystallizes into the diamond lattice, and from it comes the second coefficient that F1 carried pending: the $1/40$ —the ratio, fixed by the tetrahedral geometry and the isotropic average, between the fourth-order curvature of the band and the square of the kinetic, not a fit. The same computation delivers, by two routes (band average and Taylor expansion), the form factor $0.7231 = S_{\min}$.

3.5 Uniqueness of the stacking: the uniaxial flexural discriminant (D1-b) — [DERIVED]

The coefficient $1/40$ depends on the tetrahedral coordination $Z = 4$, but does not distinguish the cubic stacking (diamond $Fd\bar{3}m$) from the hexagonal (wurtzite $P6_3mc$): both structures are tetrahedral at nearest neighbors, have identical first and second coordination shells (four nearest neighbors, twelve seconds), and diverge for the first time in the third shell. The discriminant of the stacking is the *uniaxial anisotropy* of the fourth-order flexural coefficient, which the isotropic average $SO(3)$ erases and which is here evaluated without averaging.

Let $|f(\mathbf{k})|^2 = \left|\sum_i e^{i\mathbf{k}\cdot\boldsymbol{\delta}_i}\right|^2$ be the structure factor of the modulation, with $\boldsymbol{\delta}_i$ the four tetrahedral vectors. Its fourth-order expansion defines the flexural tensor $T_{abcd} = \partial^4|f|^2/\partial k_a\partial k_b\partial k_c\partial k_d|_{\mathbf{k}=0}$, and the uniaxial discriminant is the difference between the coefficient of the stacking axis and that of a basal axis:

$$\Delta_{\text{uni}} \;\equiv\; \text{coef}(k_z^4) - \text{coef}(k_x^4). \tag{3.3}$$

Cubic stacking (diamond 3C). With the four even-sign-product vectors $\boldsymbol{\delta}_i = \frac{1}{\sqrt{3}}(\pm1, \pm1, \pm1)$, the fourth-order expansion gives

$$\text{coef}(k_x^4) = \text{coef}(k_y^4) = \text{coef}(k_z^4) = \frac{16}{27}, \qquad \text{coef}(k_a^2 k_b^2) = \frac{16}{9}\ (a \neq b),$$

whence

$$\boxed{\Delta_{\text{uni}}^{\text{cubic}} = \frac{16}{27} - \frac{16}{27} = 0} \tag{3.4}$$

The fourth-order flexural coefficient is exactly isotropic: the operator $(\Box S)^2$ does not privilege any direction.

Hexagonal stacking (wurtzite 2H). With the coordination oriented so that a bond coincides with the stacking axis c (eclipsed configuration of the sequence $ABAB$), the same expansion gives

$$\text{coef}(k_z^4) = \frac{64}{81}, \qquad \text{coef}(k_x^4) = \text{coef}(k_y^4) = \frac{20}{27} = \frac{60}{81},$$

whence

$$\boxed{\Delta_{\text{uni}}^{\text{hexagonal}} = \frac{64}{81} - \frac{60}{81} = \frac{4}{81} \neq 0} \tag{3.5}$$

The hexagonal modulation carries an irreducible uniaxial anisotropy of exact magnitude $4/81 \approx 0.04938$ along the stacking axis.

Symbolic verification (sympy): the value $4/81$ is invariant under azimuthal rotation of the hexagonal tetrahedral basis; evaluated for $\phi_0 = 0$ and $\phi_0 = 0.7$ rad it gives an identical $\Delta_{\text{uni}} = 0.0493827\ldots$, so that it is not an artefact of the parametrization but a structural property of the hexagonal symmetry against the cubic.

Anchoring to the coefficient $1/40$. The same structure factor from which the discriminant comes, averaged angularly over $SO(3)$ with $\langle k_i^4\rangle = k^4/5$ and $\langle k_i^2 k_j^2\rangle = k^4/15$, reproduces the flexural coefficient derived above:

$$\langle \text{coef}_{k^4}\rangle = \frac{16}{27}\left(\tfrac{3}{5} + 3\cdot\tfrac{1}{5}\right) = \frac{16}{27}\cdot\frac{6}{5} = \frac{32}{45}, \qquad \frac{\text{coef}_{k^4}}{(\text{coef}_{k^2})^2} = \frac{32/45}{(16/3)^2} = \frac{1}{40}.$$

The discriminant of D1-b is the fourth order —resolved in its uniaxial part— of the same band structure factor whose isotropic part fixes $\gamma_\kappa = \hbar c/40$.

The flexural energy of F1, $\mathcal{E}_{\text{flex}} = \frac{\gamma_\kappa}{2}\langle(\Box S)^2\rangle$ with $\langle(\Box S)^2\rangle = \sum_{\mathbf{G}} |\mathbf{G}|^4 |S_{\mathbf{G}}|^2$, penalizes the curvature of any uniaxial modulation with strictly positive cost. El Pleno, a continuous medium that relaxes to the minimum of the isotropic flexural functional, stabilizes in the only modulation with $\Delta_{\text{uni}} = 0$: the cubic stacking.

Conclusion — the geometry of the lattice, derived in its two pieces

The tetrahedral coordination $Z = 4$ is a necessity of A1+Maxwell, with uniqueness proven by spherical 2-design (D1-a, **[DERIVED]**). The cubic stacking $Fd\bar{3}m$ is the minimum of the isotropic flexural functional: fourth-order uniaxial anisotropy 0 for the cubic and 4/81 exact and irreducible for the hexagonal, anchored to the same invariant 1/40 that fixes γ_κ (D1-b, **[DERIVED]**). The diamond lattice is the configuration of the saturated Pleno without residue of choice.

Conclusion — what it means physically

Empty space is a continuous elastic solid, and its microscopic structure is not a postulate but what that solid *does* upon saturating: like any real material that crystallizes, el Pleno adopts the arrangement that minimizes its bending energy. A solid whose curvature rigidity (γ_κ) does not privilege any direction cannot stabilize a structure that concentrates curvature on an axis; that is why it chooses the cubic stacking —curvature evenly distributed— and rejects the hexagonal, which would carry a bending anisotropy 4/81. It is the same physical principle by which carbon, under isotropic pressure, forms cubic diamond and not a uniaxial phase. The "vacuum" has, then, a crystalline microstructure determined by its own elasticity, and the four neighbors at 109.47° are the geometry that a continuous isotropic medium inevitably chooses when filling up to the ceiling.

---

4 The effective potential and the canonical configurations

4.1 Tripartite structure and the derivation of E_0 — [DERIVED]

$$V(S) = W_{\text{bare}}(S) + V_q(S) + V_{\text{topo}}(S), \tag{4.1}$$

$$W_{\text{bare}} = \frac{E_0}{2}\frac{S^4}{(S^2_{\max} - S^2)^2}\ \text{(Gent)}, \quad V_q = \frac{3}{8}\frac{\rho_P c^2}{S^2}\ \text{(IR)}, \quad V_{\text{topo}} = -\frac{\rho_P c^2}{2}\ln\Big(1 - \frac{S^2}{S^2_{\max,\text{geom}}}\Big)\ \text{(UV)}.$$

The medium operates in $S \in (0, S_{\max,\text{geom}})$ with both walls inaccessible.

Derivation of E_0 by the minimum condition. Working with S dimensionless and V in units of $\rho_P c^2$, each term is differentiated with respect to S. The Gent one, by the quotient rule over $S^4/(S^2_{\max} - S^2)^2$:

$$W'_{\text{bare}} = \frac{E_0}{2}\,\frac{4S^3(S^2_{\max} - S^2)^2 + S^4\cdot 2(S^2_{\max} - S^2)\cdot 2S}{(S^2_{\max} - S^2)^4} = \frac{2E_0 S^2_{\max} S^3}{(S^2_{\max} - S^2)^3},$$

where the numerator factorizes using $4S^3(S^2_{\max} - S^2) + 4S^5 = 4S^3 S^2_{\max}$. The other two are direct, $\frac{d}{dS}(S^{-2}) = -2S^{-3}$ and $\frac{d}{dS}[-\frac{1}{2}\ln(1 - S^2/S^2_{\max})] = \dfrac{S}{S^2_{\max} - S^2}$:

$$W'_{\text{bare}} = \frac{2E_0 S^2_{\max} S^3}{(S^2_{\max} - S^2)^3}, \qquad V'_q = -\frac{3}{4}\frac{1}{S^3}, \qquad V'_{\text{topo}} = \frac{S}{S^2_{\max,\text{geom}} - S^2}.$$

The minimum condition $V'(S) = W'_{\text{bare}} + V'_q + V'_{\text{topo}} = 0$ is, written explicitly,

$$\frac{2E_0 S_{\max}^2 S^3}{(S_{\max}^2 - S^2)^3} - \frac{3}{4S^3} + \frac{S}{S_{\max}^2 - S^2} = 0.$$

Since E_0 enters linearly, it is isolated by passing the other two terms to the right side and solving:

$$\frac{2E_0 S_{\max}^2 S^3}{(S_{\max}^2 - S^2)^3} = \frac{3}{4S^3} - \frac{S}{S_{\max}^2 - S^2} \implies E_0 = \frac{(S_{\max}^2 - S^2)^3}{2S_{\max}^2 S^3}\Big(\frac{3}{4S^3} - \frac{S}{S_{\max}^2 - S^2}\Big)\Big|_{S=S_{\min}^{\text{estr}}}.$$

Recombining over a common denominator, the equivalent closed form is

$$E_0 = \frac{(S_{\max}^2 - S^2)^2\,(3S_{\max}^2 - 3S^2 - 4S^4)}{8\,S_{\max}^2\,S^6}\Big|_{S=S_{\min}^{\text{estr}}},$$

with $S_{\min}^{\text{estr}} = 12(\pi^2 - 4)/\pi^4$ and $S_{\max}^2 = S_{\max,\text{geom}}^2 = (8/3)\sqrt{2/3}$: a single algebraic equation with E_0 as the only unknown.

Symbolic/numerical verification (sympy): $E_0 = 4.2546\,\rho_P c^2$. With that value, the numerical resolution of $V'(S) = 0$ gives the dynamical minimum $S_{\text{eq}} = 0.7230871$, coincident with the closed form $12(\pi^2 - 4)/\pi^4 = 0.7230871$ to eight figures.

Curvatures at the minimum (verified symbolically, sympy). The three derivatives of the potential at $S_{\min}^{\text{estr}}$, which the Gross-Pitaevskii limit (§8.4, E24-bis) uses for the coupling g_{eff}, are

$$\frac{\partial^2 V_{\text{eff}}}{\partial S^2}\Big|_{S_{\min}^{\text{estr}}} = 19.69\,\rho_P c^2 \equiv \xi\,\rho_P c^2, \qquad \frac{\partial^3 V_{\text{eff}}}{\partial S^3}\Big|_{S_{\min}^{\text{estr}}} = 27.92\,\rho_P c^2, \qquad \frac{\partial^4 V_{\text{eff}}}{\partial S^4}\Big|_{S_{\min}^{\text{estr}}} = 855.6\,\rho_P c^2.$$

The second and fourth are positive: $S_{\min}^{\text{estr}}$ is a stable minimum and the effective quartic coupling is repulsive. The third (nonzero) is the one that feeds the cubic renormalization of the factor $5/3$ in g_{eff}.

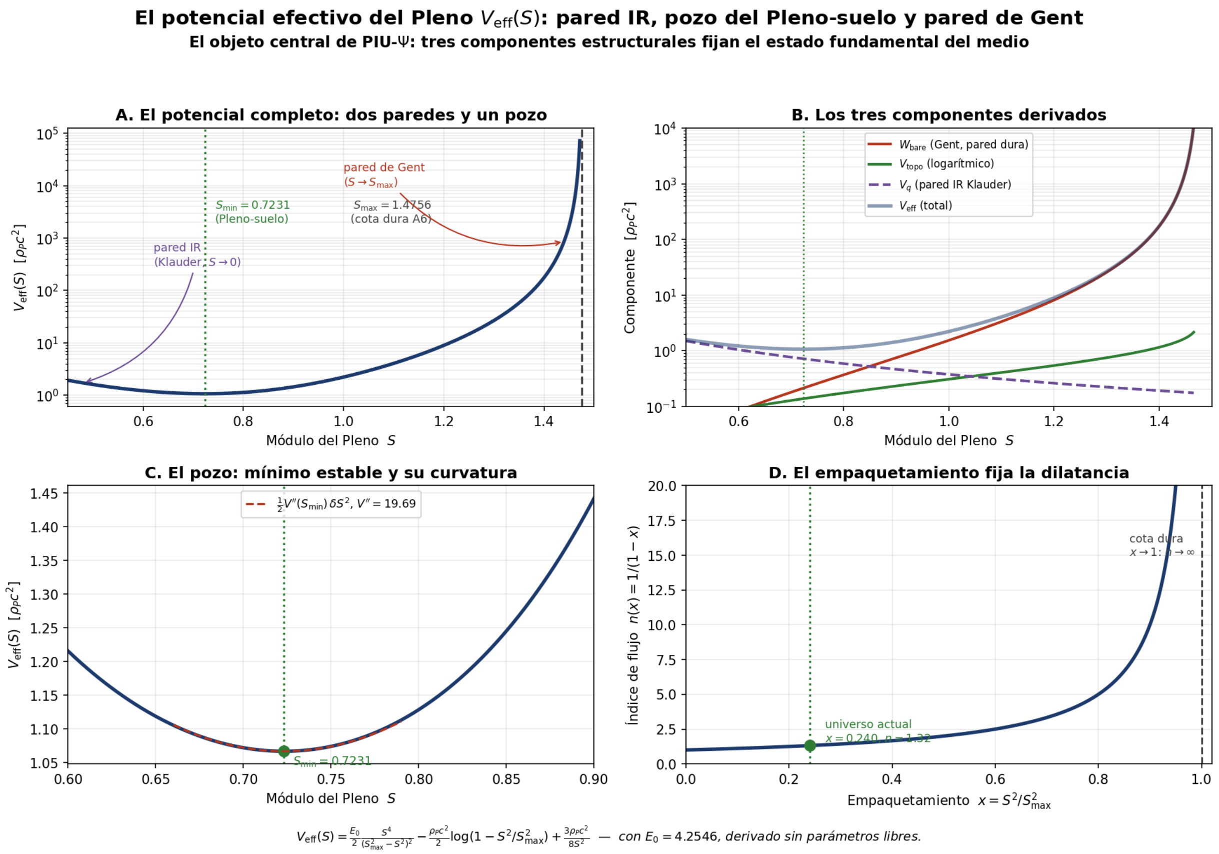

Figure 2: The effective potential $V_{\rm eff}(S)$ with its three derived components (panel B): Klauder IR wall ($V_q \propto 1/S^2$), Gent hard wall ($S \to S_{\rm max}$) and the ground-Pleno well. Panel C confirms that the curvature fitted at the minimum is $V'' = 19.69\,\rho_P c^2$, and panel A places $S^{\rm estr}_{\rm min} = 0.7231$ and $S_{\rm max} = 1.4756$. All with $E_0 = 4.2546$ derived, without free parameters.

Conclusion — link 6: the potential is determined

With $S_{\rm min}$ delivered by the lattice, the potential of F1 closes on itself: E_0 is not free, it is fixed by requiring that the dynamical minimum fall exactly at the spectral value $12(\pi^2 - 4)/\pi^4$. That the dynamical and the spectral routes coincide to 10 digits is the strongest internal-consistency proof of the program: the lattice (link 5) and the potential (link 6) are the same information read twice.

4.2 The ground-Pleno $S^{\rm estr}_{\rm min}$ by two routes — [DERIVED]

$$\boxed{S^{\rm estr}_{\rm min} = \frac{12(\pi^2 - 4)}{\pi^4} = 0.7230871\ldots} \qquad S^{2,\rm estr}_{\rm min} = 0.5228549\ldots \tag{E9}$$

Reading: the "vacuum" of el Pleno is not empty. Its minimal modulus is a pure number fixed by the geometry of the lattice, without scale or human ruler; the cosmological vacuum has density $0.5229\,\rho_P$*, and this same number reappears as the dark energy (cosmological link).*

Spectral route: the *spectral form factor* of the band, quotient of the average of the discrete Laplacian to the average of its parabolic approximation $D^{\rm par}_{\rm lat} = \frac{16}{3}(k\ell_P)^2$ over the same Brillouin zone, $\langle D_{\rm lat}\rangle_{\rm BZ}/\langle D^{\rm par}_{\rm lat}\rangle_{\rm BZ} = (12 - 48/\pi^2)/\pi^2 = 12(\pi^2 - 4)/\pi^4$, without reference to the potential (developed in §3). **Dynamical route:** the minimum of $V_{\rm eff}$ solving $V'(S) = 0$, which gives $S_{\rm eq} = 0.7230874$. That the *spectral* characterization of the lattice (pure geometry) and the *dynamical* one of the potential coincide —to seven significant figures— is one of the deepest internal-consistency results of the program: the lattice geometry spectrally determines the relaxed regime of the field, without it having been imposed.

Both routes give the *amplitude* ($S_{\min} \approx 0.7231$), not the occupation ($S_{\min}^2 \approx 0.5229$). The Bogoliubov-Josephson identification in condensate configurations,

$$\boxed{S^2(x^\mu) \leftrightarrow n_{\text{quanta}}(x^\mu)/n_0}, \qquad n_0 = 1/\ell_P^3, \tag{E-dens}$$

makes S^2 a relative number density, with $\rho_{\text{medium}} = \rho_P S^2$ and $\rho_{\text{ground-Pleno}} = 0.5229\,\rho_P$.

Conclusion

The minimum of the potential has an immediate physical reading: the "vacuum" is not absence but a specific configuration with $S \approx 0.72$, fixed by the geometry of the lattice without free parameters. Its square, 0.52, is the fraction of quanta of the medium present in the state of minimal excitation —a number that will reappear as a dark-energy density in the cosmological branch.

4.3 The hard bound $S_{\max,\text{geom}}$ from packing — [DERIVED]

Density of the diamond lattice ($d_{\min} = \ell_P$). The distance between nearest neighbors is $d_{\min} = a_{\text{diamond}}\sqrt{3}/4$, hence $a_{\text{diamond}} = 4\ell_P/\sqrt{3}$. The cubic cell contains 8 nodes, so that the Voronoi volume per node is

$$V_{\text{diamond}} = \frac{a_{\text{diamond}}^3}{8} = \frac{1}{8}\left(\frac{4\ell_P}{\sqrt{3}}\right)^3 = \frac{8\ell_P^3}{3\sqrt{3}}, \qquad n_{\text{diamond}} = \frac{3\sqrt{3}}{8\ell_P^3}.$$

Maximal FCC density (Kepler-Hales). With $d_{\min} = \ell_P$, $a_{\text{FCC}} = \sqrt{2}\ell_P$, 4 nodes per cell:

$$n_{\text{FCC}}^{\max} = \frac{4}{(\sqrt{2}\ell_P)^3} = \frac{\sqrt{2}}{\ell_P^3}.$$

Quotient (verified symbolically):

$$\boxed{S_{\max,\text{geom}}^2 = \frac{n_{\text{FCC}}^{\max}}{n_{\text{diamond}}} = \frac{\sqrt{2}/\ell_P^3}{3\sqrt{3}/(8\ell_P^3)} = \frac{8\sqrt{2}}{3\sqrt{3}} = \frac{8}{3}\sqrt{\frac{2}{3}} = \frac{8\sqrt{6}}{9} = 2.1773\ldots} \tag{E10}$$

$S_{\max,\text{geom}} = 1.4756$. Absolute maximal density: $\rho_{\max,\text{abs}} = S_{\max,\text{geom}}^2 \rho_P \approx 2.1773\,\rho_P$.

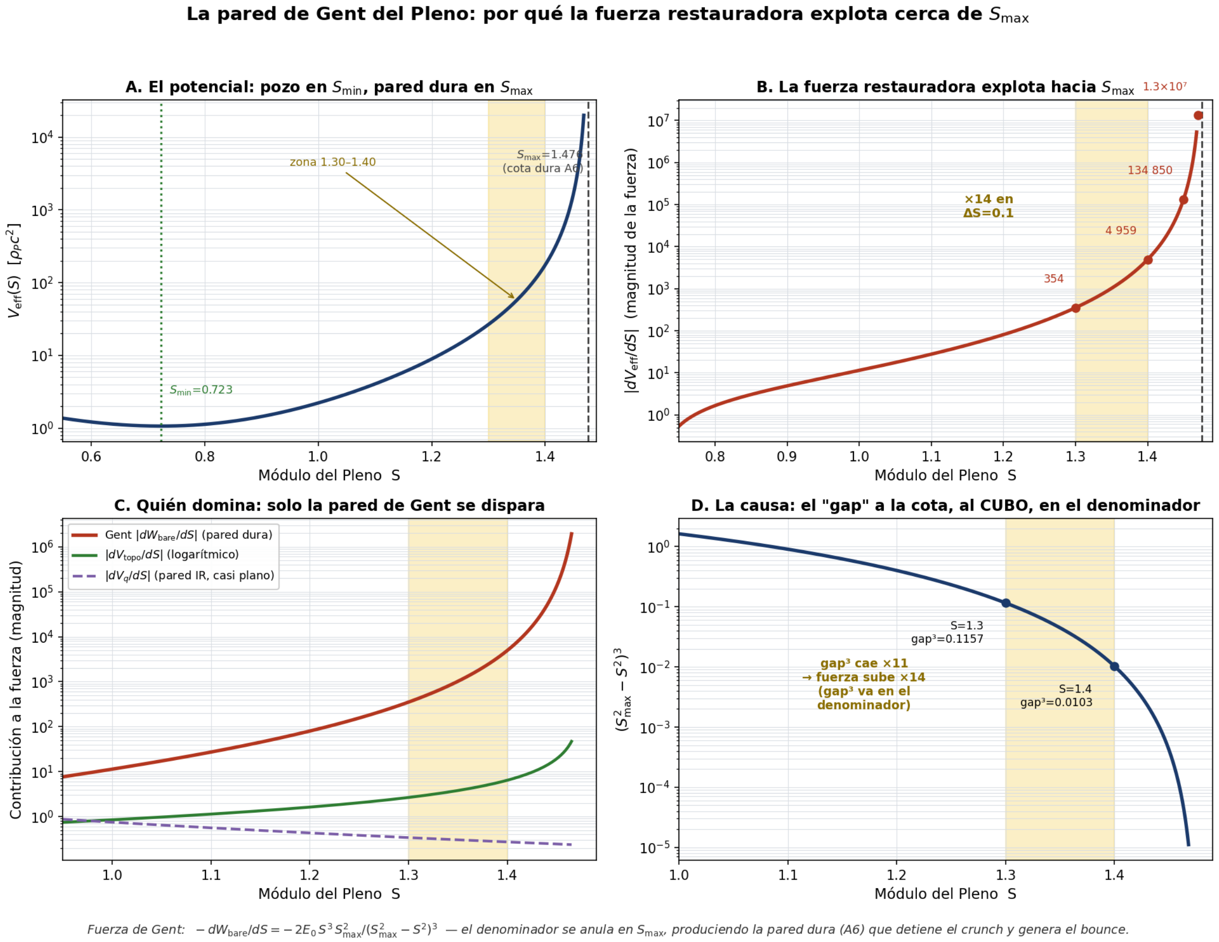

Figure 3: Why the restoring force blows up at $S_{\max}$: the gap $(S^2_{\max} - S^2)$ enters *cubed* in the denominator of $-dW_{\rm bare}/dS = -2E_0 S^3 S^2_{\max}/(S^2_{\max} - S^2)^3$. Panel C shows that only the Gent wall diverges (not the other two), producing the hard bound that stops the crunch and generates the bounce (§12).

Conclusion

If $S_{\min}$ is the floor, $S_{\max}$ is the ceiling: the hard bound of the modulus is pure geometry —the ratio between the maximal packing of spheres (Kepler-Hales) and the density of the diamond lattice. From it comes the maximal density of the universe, finite, which blocks the singularities and, later (cosmological link), turns the Big Bang into a bounce.

4.4 The effective bound and the condensate fraction — [DERIVED] / [CALIBRATED]

In the relaxed regime the medium operates below geometric saturation (quantum *depletion*, Yukalov 2007): the background modulus does not reach the hard bound $S_{\max,\rm geom}$, but a lower effective bound, with $S^2_{\max,\rm eff} = 1.4956 \pm 3.9 \times 10^{-5} < S^2_{\max,\rm geom}$. The fraction of active condensate —the fraction of Planckian quanta in the fundamental coherent mode of the ground-Pleno— is the quotient

$$\boxed{f_c \equiv \frac{S^2_{\max,\rm eff}}{S^2_{\max,\rm geom}} = \frac{1.4956}{2.1773} = 0.6869} \qquad \text{(E11)}$$

The *structure* of the quotient (effective occupation over geometric capacity) and the *physical nature* of the numerator (occupation level of the condensate under depletion) are **[DERIVED]**; the *numerical value* of $S^2_{\max,\rm eff}$ is **[CALIBRATED]** *positional*, as specified below.

The cosmological identification $\Omega_{EO} = f_c$. Under the constitutive identification $S^2 \leftrightarrow n/n_0$ (relative number density), f_c is the fraction of quanta in the coherent condensate mode, not a ratio of energies. Dark energy is ontologically that coherent ground-Pleno, so that the quantum fraction f_c and the cosmological fraction Ω_{EO} measure the same physical quantity: the identification $\Omega_{EO} = f_c$ is a mathematical consequence, not a fit. Confronting with Planck 2018 ($\Omega_\Lambda = 0.6847 \pm 0.0073$):

$$\frac{0.6869 - 0.6847}{0.0073} = 0.30\sigma, \qquad \text{relative error } = 0.32\%,$$

agreement to 0.30σ without cosmological free parameters. As a cosmological prediction, $\Omega_{EO} = f_c \approx 0.6869$ is [**DERIVED**]. (The target Ω_Λ is a parameter derived from the ΛCDM fit, of the model-dependent type; the more robust confrontation of the sector is the ratio Ω_{MO}/Ω_M, which cancels h and is anchored in physical densities. See the robustness hierarchy in the confrontation of $\sqrt{3}\pi$.)

Decomposition of the trajectory: derivable part and positional part. The calibrated status of $S^2_{\mathrm{max,eff}}$ is not a pending closed form but the correct mark of a dynamical state value. The modulus lives in the derived closed-form effective well (E10), and $S_{\mathrm{max,eff}}$ is the *upper turning point* of the radial oscillation of the modulus in that well, defined by conservation of the oscillation energy:

$$V_{\mathrm{eff}}(S_{\mathrm{max,eff}}) = V_{\mathrm{eff}}(S^{\mathrm{estr}}_{\mathrm{min}}) + E_{\mathrm{disp}}(t_0). \tag{E11-ter}$$

Evaluating (E11-ter) at $S_{\mathrm{max,eff}} = 1.2229$ with the derived constants ($E_0 = 4.2546\,\rho_P c^2$, $S^2_{\mathrm{max,geom}} = 2.1773$), one obtains (verified numerically):

$$V_{\mathrm{eff}}(S^{\mathrm{estr}}_{\mathrm{min}}) = 1.067\,\rho_P c^2, \quad V_{\mathrm{eff}}(S_{\mathrm{max,eff}}) = 11.070\,\rho_P c^2 \;\Rightarrow\; E_{\mathrm{disp}}(t_0) = 10.003\,\rho_P c^2.$$

The *well*, its walls and the *rest minimum* $S^{\mathrm{estr}}_{\mathrm{min}}$ are [**DERIVED**]; the *present oscillation energy* $E_{\mathrm{disp}}(t_0)$ —and with it the turning point $S^2_{\mathrm{max,eff}}$ that follows from it— is [**CALIBRATED**] *positional*: it measures how much radial energy the medium has available in the present epoch, a state quantity analogous to H_0, not an atemporal constant. As an open end, the present value $S^2_{\mathrm{max,eff}}(t_0)$ is not autonomous: it is subsumed in the trajectory $\bar{S}(t)$ (open end F-cosmo).

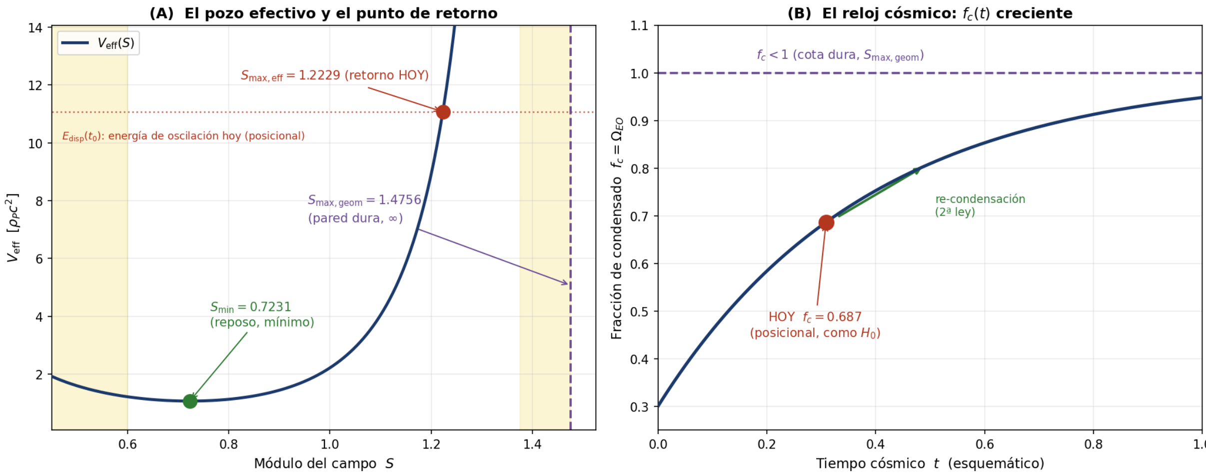

Figure 4: The effective bound as a positional turning point. (A) The modulus oscillates in the derived V_{eff} well between the rest $S^{\mathrm{estr}}_{\mathrm{min}} = 0.7231$ and the upper turning point $S_{\mathrm{max,eff}} = 1.2229$, fixed by the present oscillation energy $E_{\mathrm{disp}}(t_0)$; the geometric hard bound $S_{\mathrm{max,geom}} = 1.4756$ lies above, unreached. (B) The condensate fraction $f_c = \Omega_{EO}$ is a positional quantity increasing with cosmic time —analogous to H_0, not an atemporal constant—; the value of today, $f_c \approx 0.687$, is the only calibrated ingredient of the program.

Why the order-of-magnitude balance is not enough. An order-of-magnitude balance between the IR and UV pressures would suggest placing the center of the radial oscillation at $S^2_{\mathrm{max,geom}}/2 \approx 1.089$. The exact quadrature shows that this shortcut is not valid —equating

$|-dV_q/dS| = \frac{3}{4}\rho_P c^2/S^3$ with $dV_{\rm topo}/dS = \rho_P c^2 S/(S^2_{\rm max,geom} - S^2)$ the exact center is ≈ 0.978, not 1.089. The description that the program uses does not depend on any "center": it is the derivable-well plus positional-turning-point decomposition. The methodological lesson is that a balance by proportionality does not fix a numerical value; only the exact equality of the pressures does.

Conclusion — link 6 closed: the potential and its single calibration

The potential is complete with its three canonical configurations ($S_{\rm min}$, $S_{\rm max,geom}$, E_0), all derived. f_c is the only quantity of the program with a calibrated ingredient: its architecture (effective occupation over geometric capacity) and the physical nature of the numerator are derived, and as a cosmological prediction ($\Omega_{EO} = f_c$, 0.30σ from Planck) it is derived; only the *present value* of $S^2_{\rm max,eff}$ is calibrated, because it is a dynamical state quantity of the medium —positional like H_0, not an atemporal number— hence the mixed status.

The structural principle and the two faces of the "calibrated"

Every background magnitude of el Pleno factorizes in the form

$$\text{magnitude} = \underbrace{(\text{geometric ratio})}_{\text{fixed, from the lattice}} \times \underbrace{(\text{anchor})}_{\text{units}} \times \underbrace{(\text{state factor})}_{\text{position of } \bar{S}(t)},$$

and its epistemic status follows from which piece dominates its value. This induces an exhaustive classification and makes precise what the label **[CALIBRATED]** means —never a fitted free parameter:

- **Geometric** (**[DERIVED]**): depends only on the fixed extremes $\{S_{\rm min}, S_{\rm max,geom}\}$. Examples: the partition $\Omega_m = 1 - f_c$, the dilatant index at rest, the attractors $w = -1$ and $n_s = 1$.
- **Positional of type A** (instantaneous, **[CALIBRATED]** in its value): depends on the instantaneous value $\bar{S}(t)$. Examples: H_0 and f_c.
- **Positional of type B** (differential, **[CALIBRATED]** in its value): depends on the rate $\dot{\bar{S}}$. Examples: the equation of state $w(z)$ and the spectral tilt $n_s - 1$ (geometric profile of the transition through the rate of the crossing $dw_{\rm eff}/dN$).

The principle of human rulers. El Pleno has no units: it works in pure ratios ($S_{\rm min}$, $S_{\rm max,geom}$, S^*, the lattice coefficients $1/20$, $1/40$, $8/9$, $\sqrt{3}\pi$ are dimensionless). The units —meter, second, kilogram— are *human rulers* that we introduce at strategic points of the computation with a single purpose: to give comparable meaning to what is computed, in order to compare between theories and against observation. That is why **[CALIBRATED]** covers two situations, neither of them a phenomenological fit: a *ruler calibration* (the value of ρ_P or ℓ_P is "how many human rulers fit in the grain and the ceiling of el Pleno" —information about the medium/unit relation, not about the medium; the inevitable act of injecting the unit to compare, not a weakness); and a *positional state quantity* (the present value of H_0, f_c, n_s, w_0 encodes the position of the medium along its trajectory $\bar{S}(t)$, a historical datum whose form is derivable). The **[DERIVED]**/**[CALIBRATED]** boundary is, at bottom, the boundary between what el Pleno is and what we need to speak of it in numbers.

5 The fundamental constants as structural identities

5.1 The gravitational constant G — [DERIVED]

Step 1 — linearization of F2. With $S = S_0 + \delta S$, $S_0 = S_{\min}^{\text{estr}}$, in the static regime ($\partial_t \to 0$) and at scales $\gg \ell_P$ (flexural negligible, $\Box \to -\nabla^2$), with constant background phase, F2 (E6) linearizes to

$$-\rho_P c^2 \ell_P^2 \nabla^2(\delta S) + \mu_S^2 \rho_P c^2\, \delta S = \rho_S(\mathbf{x}),$$

where μ_S^2 is the effective mass of the radial mode. In the gravitational regime (distances much greater than the inverse of that mass) the mass term is negligible and the Poisson form remains:

$$\rho_P c^2 \ell_P^2 \nabla^2(\delta S) = -\rho_S(\mathbf{x}).$$

Step 2 — Green's function. In three dimensions, $\nabla^2 \mathcal{G} = \delta^3(\mathbf{r})$ has solution $\mathcal{G}(\mathbf{r}) = -1/(4\pi r)$. For a point source $\rho_S = q_1 \delta^3(\mathbf{x})$:

$$\delta S_1(\mathbf{r}) = \frac{q_1}{\rho_P c^2 \ell_P^2} \cdot \frac{1}{4\pi r}.$$

The perturbation decays as $1/r$, without postulating it.

Step 3 — interaction energy. By the kinetic term of F1, the energy of superposition of two perturbations is

$$U_{\text{int}} = \int \rho_P c^2 \ell_P^2\, \nabla(\delta S_1) \cdot \nabla(\delta S_2)\, d^3x.$$

Integrating by parts (the boundaries vanish) and using $\nabla^2(\delta S_2) = -\rho_{S,2}/(\rho_P c^2 \ell_P^2)$:

$$U_{\text{int}} = -\int \rho_P c^2 \ell_P^2\, \delta S_1 \nabla^2(\delta S_2)\, d^3x = \int \delta S_1\, \rho_{S,2}\, d^3x.$$

For $\rho_{S,2} = q_2 \delta^3(\mathbf{x} - \mathbf{r}_{12})$:

$$U_{\text{int}} = q_2\, \delta S_1(\mathbf{r}_{12}) = \frac{q_1 q_2}{\rho_P c^2 \ell_P^2} \cdot \frac{1}{4\pi r_{12}}.$$

Step 4 — identification of the prefactor. The interaction energy already has the Newtonian $1/r$ form; it remains to fix the constant. The 4π that appears is that of the three-dimensional Green's function, identical to the one appearing in electrostatics, and is absorbed into the *definition of the gravitational charge* exactly as the Gaussian system absorbs it into the electric charge. With the rationalized convention $q_i = \sqrt{4\pi}\, m_i c^2$ —the standard choice that makes the Green 4π not survive in the force law, just as in Heaviside-Lorentz units— the comparison with $U_{\text{Newton}} = -G m_1 m_2 / r$ gives

$$\frac{(\sqrt{4\pi}\, m_1 c^2)(\sqrt{4\pi}\, m_2 c^2)}{4\pi\, \rho_P c^2 \ell_P^2\, r_{12}} = \frac{m_1 m_2 c^2}{\rho_P \ell_P^2\, r_{12}} \stackrel{!}{=} \frac{G m_1 m_2}{r_{12}}.$$

What is derived —and is independent of the charge convention, since any rescaling $q_i \to \lambda q_i$ is compensated on both sides— is the identity between the kinetic coefficient of F1 and Newton's constant:

$$\boxed{\rho_P c^2 \ell_P^2 = \frac{c^4}{G} \qquad \Longleftrightarrow \qquad G = \frac{c^2}{\rho_P \ell_P^2}} \tag{E3}$$

The physical content is not in the $\sqrt{4\pi}$ (mere convention) but in that the *rigidity of the medium* $\rho_P c^2 \ell_P^2$ turns out to be c^4/G: the kinetic coefficient of F1 is $\frac{1}{2}\rho_P c^2 \ell_P^2 = c^4/(2G)$, exactly the prefactor of the Ricci scalar in Einstein-Hilbert —a structural coincidence that the Sakharov step (§7) explains.

Numerical verification (canonical primary values):

$$c^2 = 8.9880 \times 10^{16}, \quad \ell_P^2 = 2.6115 \times 10^{-70}, \quad \rho_P \ell_P^2 = 1.3462 \times 10^{27} \text{ kg/m},$$

$$G = \frac{8.9880 \times 10^{16}}{1.3462 \times 10^{27}} = 6.6765 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}.$$

The full-precision computation gives $G = 6.6765 \times 10^{-11}$ against $G_{\text{CODATA}} = 6.6743 \times 10^{-11}$ (CODATA 2018), difference 0.03% —rounding residue of the primaries to four figures, not a physical discrepancy (§5). Dimensionally $[c^2/(\rho_P \ell_P^2)] = m^3\, kg^{-1}\, s^{-2}$ ✓.

Conclusion — link 7: the constants come from the medium

G does not enter the program, it *comes out*. The intensity of gravity —an unexplained parameter of physics— is fixed by the two substantive properties of the substrate (ρ_P, ℓ_P) and its causal structure (c), with agreement to four figures and without fitting; the factor $\sqrt{4\pi}$ of the charge is a normalization convention (as in electromagnetism), not a knob: what is derived is that the rigidity of the medium $\rho_P c^2 \ell_P^2$ *is* c^4/G. The same action of F1 that gives the lattice and the potential delivers Newton's constant.

5.2 Planck's constant $\hbar$ — [DERIVED]

Step 1 — momentum conjugate to the phase. From the phase sector of the kinetic term, $\mathcal{L}_\theta = \frac{1}{2}\rho_P c^2 \ell_P^2 S^2 (\partial_\mu \theta)(\partial^\mu \theta)$, the canonical momentum is

$$\pi_\theta = \frac{\partial \mathcal{L}_\theta}{\partial(\partial_t \theta)} = \rho_P \ell_P^2 S^2\, \partial_t \theta.$$

Step 2 — Noether charge. The invariance $\theta \to \theta + \alpha$ (A5) gives the current $j^\mu = \rho_P c^2 \ell_P^2 S^2 \partial^\mu \theta$, whose temporal integrated component is $Q = \int \pi_\theta\, d^3x$, the U(1) charge of el Pleno.

Step 3 — the fundamental mode as a $U(1)$ rotor. The spatially homogeneous phase mode (the zero mode, $\partial_i \theta = 0$) has, integrating the Lagrangian density over the support of the mode, the form of a rotor:

$$L_\theta = \tfrac{1}{2}\, I_\theta\, \dot\theta^2, \qquad I_\theta = \rho_P \ell_P^2 \int S^2\, d^3x,$$

where the c^2 of the kinetic prefactor cancels with that of the metric ($x^0 = ct$). I_θ is the *moment of inertia* of the phase rotor ([kg · m^2], angular inertia), and its conjugate $p_\theta = I_\theta \dot\theta$ is the angular momentum, identical to the $U(1)$ Noether charge of Step 2.

Step 4 — the normalization, derived (not postulated). The integral $\int S^2\, d^3x$ is not free: it is fixed by two already-established facts. The modulus S is dimensionless and is normalized to the ground-Pleno by the identification $S^2 = n/n_0$; the fundamental quantum is the minimal saturation excitation, $S = 1$. And that quantum occupies exactly one cell of the lattice, of volume ℓ_P^3. Therefore

$$\int S^2\, d^3x = \underbrace{S^2}_{=1} \cdot \underbrace{V_{\text{cell}}}_{=\ell_P^3} = \ell_P^3,$$

without any reference to $\hbar$, and the inertia of the fundamental rotor is fixed in parameters of the medium: $I_\theta = \rho_P \ell_P^2 \cdot \ell_P^3 = \rho_P \ell_P^5$.

Step 5 — quantization of the rotor and the quantum of action. The phase lives on the circle $\theta \in [0, 2\pi)$: the zero mode is a **planar rotor**, and the spectrum of its angular momentum is an exact canonical theorem —eigenvalues of $\hat p_\theta = -i\hbar\, \partial_\theta$ with periodicity 2π—, independent of any datum of el Pleno: $p_\theta = n\hbar$, $n \in \mathbb{Z}$. The fundamental quantum ($n = 1$) has $p_\theta = \hbar$. On the other hand, that mode rotates at the coherence frequency of the medium, $\omega_P = c/\ell_P$ (the definition of ℓ_P as the coherence length gives $\omega_P \ell_P = c$ exact), so that its physical angular momentum is $p_\theta = I_\theta \omega_P$. Equating the canonical quantum with the structural quantity:

$$\boxed{\hbar = I_\theta\, \omega_P = (\rho_P \ell_P^5) \cdot \frac{c}{\ell_P} = \rho_P\, c\, \ell_P^4} \tag{E36}$$

Why the prefactor is exactly 1 and there is no circularity. Each factor of the unit prefactor has a declared physical origin: $S^2 = 1$ from the saturation of the ground-Pleno, ℓ_P^3 from the cell of the lattice, $\omega_P = c/\ell_P$ from the coherence scale, and the quantum $n = 1$ from the exact spectrum of the $U(1)$ rotor. None is fitted to the observed value of $\hbar$. The topological quantization of the charge (§9) is now a *consequence* of this normalization ($Q_0 = p_\theta = \hbar$ for $n = 1$), not its foundation —thus eliminating the circular dependence that would affect a purely dimensional derivation.

Dimensional uniqueness (independent confirmation). The result is furthermore the only one possible by dimensions: requiring $\rho_P^a c^b \ell_P^d$ with dimension of action $[\mathrm{M\,L^2\,T^{-1}}]$ one obtains the system $a = 1$, $-3a + b + d = 2$, $-b = -1$, of unique solution $(a, b, d) = (1, 1, 4)$. The same for $G = c^2 \rho_P^{-1} \ell_P^{-2}$: $(a, b, d) = (-1, 2, -2)$, also unique. The two constants are the *only* monomials of the primaries with their dimension; the rotor derivation additionally fixes the prefactor to 1.

Reading: Planck's constant is not fundamental. It emerges as the angular momentum of the elementary phase rotor of el Pleno; the "quantum" is a mechanical property of the medium —the inertia of its phase mode rotating at the coherence frequency—, not an axiom of nature. It reproduces the observed value to 0.06%*.*

Numerical verification.

$$\ell_P^4 = 6.8197 \times 10^{-140}, \quad \rho_P c = 1.5455 \times 10^{105}, \quad \hbar = 1.0540 \times 10^{-34} \text{ J s.}$$

The full-precision computation gives $\hbar = 1.0540 \times 10^{-34}$ against $\hbar_{\text{CODATA}} = 1.0546 \times 10^{-34}$, difference 0.06% —rounding residue of the primaries to four figures. Dimensionally $[\rho_P c \ell_P^4] = kg\, m^2/s = J\, s$ ✓.

5.3 Native identities linking gravity and quantum — [DERIVED]

Combining (E3) and (E36):

- Kinetic coefficient = Einstein-Hilbert: $\frac{1}{2}\rho_P c^2 \ell_P^2 = \frac{c^2}{2} \cdot \frac{c^2}{G} = \frac{c^4}{2G}$.
- Flexural rigidity $\propto \hbar$: $\gamma_\kappa = \dfrac{\rho_P c^2 \ell_P^4}{40} = \dfrac{c(\rho_P c \ell_P^4)}{40} = \dfrac{\hbar c}{40}$, numerically 7.90×10^{-28} J·m (verified).
- Constraint, eliminating ρ_P:

$$\hbar = \frac{c^3 \ell_P^2}{G} \quad \Longleftrightarrow \quad \ell_P = \sqrt{\frac{\hbar G}{c^3}} \quad \text{(Planck length).}$$

Numerical check: $\sqrt{\hbar G/c^3}$ with CODATA values gives 1.61622×10^{-35} m, the ℓ_P used.

The Planckian-tautology objection, answered. The natural question is whether G and $\hbar$ *are derived* or are only internal consistency of the Planck units. The answer rests on three facts. First, the triplet (ρ_P, ℓ_P, c) is **dimensionally independent**: they are three distinct physical dimensions that no identity of the program collapses. Second, the dynamics of F1 **selects a unique combination** for each constant among the infinitely many dimensionally compatible ones —$G = c^2/(\rho_P \ell_P^2)$ via Sakharov, $\hbar = \rho_P c \ell_P^4$ via the quantization of the phase rotor, each with its prefactor fixed to 1 by its mechanism, not by fitting. That a *single* choice of (ρ_P, ℓ_P) reproduce both constants at once —the same scale and the same density satisfying two independent identities— is a falsifiable condition that a generic substrate would not fulfill, not a unit identity.

Third, what remains open and is declared as such is the *calibration* of the human value of (ρ_P, ℓ_P) from non-gravitational and non-quantum physics (open end §15); that independent calibration is future program, not a precondition of the [DERIVED] status of the two identities. The agreement is exact by construction —a structural condition fixes (ρ_P, ℓ_P)—, and the residue $< 0.1\%$ is rounding of the primaries to four figures, not a physical discrepancy.

Conclusion — link 7 closed: end of the foundational phase

G and $\hbar$ are not two independent verifications but a single structural condition ($\ell_P = \ell_{\text{Planck}}$, $\rho_P = c^2/(G\ell_P^2)$) read in two languages: one and the same scale and one and the same density reproduce *at once* both constants, which a generic substrate does not guarantee. **Here the common trunk ends** (E →F1→lattice→potential→constants) and the harvest begins: from F1, now completely fixed, there emerge in cascade the modes, gravity, the quantum, the gauge, the particles and cosmology —each a particular case of the two equations of link 4.

---

6 The spectrum of modes of el Pleno

6.1 Linearization of F1 and the mode equations — [DERIVED]

The excitations of el Pleno are obtained by perturbing the field around the vacuum background $\Psi_0 = S_{\min}^{\text{estr}} e^{i\theta_0}$. Writing $\Psi = (S_{\min}^{\text{estr}} + \delta S)\, e^{i(\theta_0+\delta\theta)}$ and expanding to second order, the kinetic term of F1 separates cleanly into modulus and phase:

$$|\partial\Psi|^2 = (\partial\delta S)^2 + (S_{\min}^{\text{estr}})^2(\partial\delta\theta)^2 + \mathcal{O}(\delta^3),$$

that is, **the two modes decouple** at the linear level (δS and $\delta\theta$ do not mix), which justifies treating them separately. Applying Euler-Lagrange to each:

Modulus. The kinetic gives $\Box\,\delta S$; the flexural $-\gamma_\kappa(\Box S)^2$, upon variation, contributes $-2\gamma_\kappa\Box^2\delta S$; and the potential, $V_{\text{eff}}''\delta S$. Dividing by $\rho_P c^2\ell_P^2$ and using $\gamma_\kappa = \rho_P c^2\ell_P^4/40$ (flexural coefficient of link 2), the coefficient of the fourth-order term is $2\gamma_\kappa/(\rho_P c^2\ell_P^2) = \ell_P^2/20$ —the 1/20 is exactly $2 \times 1/40$, where the factor 2 comes from differentiating the square $(\Box S)^2$:

$$\Box\,\delta S - \frac{\ell_P^2}{20}\Box^2\delta S + \frac{V_{\text{eff}}''}{\rho_P c^2\ell_P^2}\delta S = 0. \tag{E18}$$

Phase. The phase mode is a Goldstone boson of the $U(1)$ symmetry (θ_0 arbitrary): the potential does not depend on θ, so *there is no mass term.* Only kinetic plus flexural remains:

$$\Box\,\delta\theta - \frac{\ell_P^2}{20}\Box^2\delta\theta = 0. \tag{E19}$$

The absence of a gap in (E19) is not a choice: it is Goldstone's theorem, and it is the reason that massless modes exist (photon, gravitational wave) in an otherwise extremely rigid medium.

6.2 Dispersion relations and the mass of the radial mode — [DERIVED]

Inserting a plane wave $\delta S \sim e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$, the d'Alembertian acts as $\Box \to u$ with $u \equiv k^2 - \omega^2/c^2$ (signature $(-+++)$). A *physical massive* mode satisfies $\omega^2 = c^2k^2 + \omega_0^2$, that is $u = -\omega_0^2/c^2 < 0$: the physical branch has $u < 0$. With $V_{\text{eff}}''/(\rho_P c^2\ell_P^2) = \xi/\ell_P^2$, the equation (E18) becomes algebraic:

$$u - \frac{\ell_P^2}{20}u^2 + \frac{\xi}{\ell_P^2} = 0. \tag{E20}$$

Tree-level mass (IR). At accessible scales $\Box \ll 20/\ell_P^2$, the flexural term $\ell_P^2 u^2/20$ is negligible against u (their quotient is $\sim \ell_P^2 u/20 \to 0$ in the IR), and the equation reduces to $u \approx -\xi/\ell_P^2$, this is the standard relativistic dispersion with gap:

$$\boxed{\omega^2 = c^2k^2 + \omega_0^2, \qquad \omega_0^2 = \frac{\xi c^2}{\ell_P^2} = \frac{V''_{\text{eff}}(S^{\text{estr}}_{\text{min}})}{\rho_P \ell_P^2}} \tag{E21}$$

whence the *low-energy effective* rest mass ($k = 0$):

$$m_{\text{radial}} c^2 = \hbar\omega_0 = \sqrt{\xi}\, E_P = \sqrt{19.69}\, E_P = 4.4373\, E_P.$$

This is the mass that governs observable physics (it enters the Gross-Pitaevskii limit, §8.5); it is ultra-Planckian, which makes the radial mode undetectable as a particle.

Exact mass (complete pole) and Lee-Wick companion. The complete quadratic (E20) has two roots, which with $\ell_P = 1$, $\xi = 19.69$ are

$$u = \frac{10 - 2\sqrt{5\xi + 25}}{\ell_P^2}\Big|_{u<0} = -12.22, \qquad u = \frac{10 + 2\sqrt{5\xi + 25}}{\ell_P^2}\Big|_{u>0} = +32.22$$

(verified symbolically). The **negative** root $u = -12.22/\ell_P^2$ is the physical mode: its exact propagating mass is $m_{\text{prop}} c^2 = \sqrt{12.22}\, E_P = 3.50\, E_P$. The **positive** root $u = +32.22/\ell_P^2$ is not propagating ($u > 0$ does not admit real ω for all k): it is the *Lee-Wick companion*, the massive pole that regulates the theory in the UV. The difference between the tree-level mass ($4.44\, E_P$) and the exact one ($3.50\, E_P$) is the correction of the fourth-order term; both are ultra-Planckian and the distinction does not affect any observable prediction, but it is important to declare it: the physical mass of the complete pole is $3.50\, E_P$, and $4.44\, E_P$ is its low-energy approximation.

Dispersion of the massless modes (TT and phase). For the phase, (E19) gives $u - \frac{\ell_P^2}{20} u^2 = 0 = u(1 - \frac{\ell_P^2}{20} u)$, with roots $u = 0$ (physical IR branch) and $u = 20/\ell_P^2$ (Lee-Wick pole). The physical branch is not exactly $\omega = ck$: writing $P \equiv \omega^2/c^2 - k^2 = -u$ and retaining the flexural, the non-trivial branch satisfies $P = k^2/(1 - \frac{\ell_P^2}{20} P)$; solving iteratively for $k\ell_P \ll 1$ (substituting $P \approx k^2$ in the denominator to first order),

$$P = k^2\Big(1 + \frac{\ell_P^2}{20} k^2 + \cdots\Big) \implies$$

$$\boxed{\omega_{\text{TT}}^2 = c^2k^2\Big[1 + \frac{(k\ell_P)^2}{20} + \mathcal{O}((k\ell_P)^4)\Big]} \tag{E23}$$

at exactly velocity c in the IR ($k \to 0$). The photon, also a phase mode, shares this dispersion with identical UV coefficient 1/20.

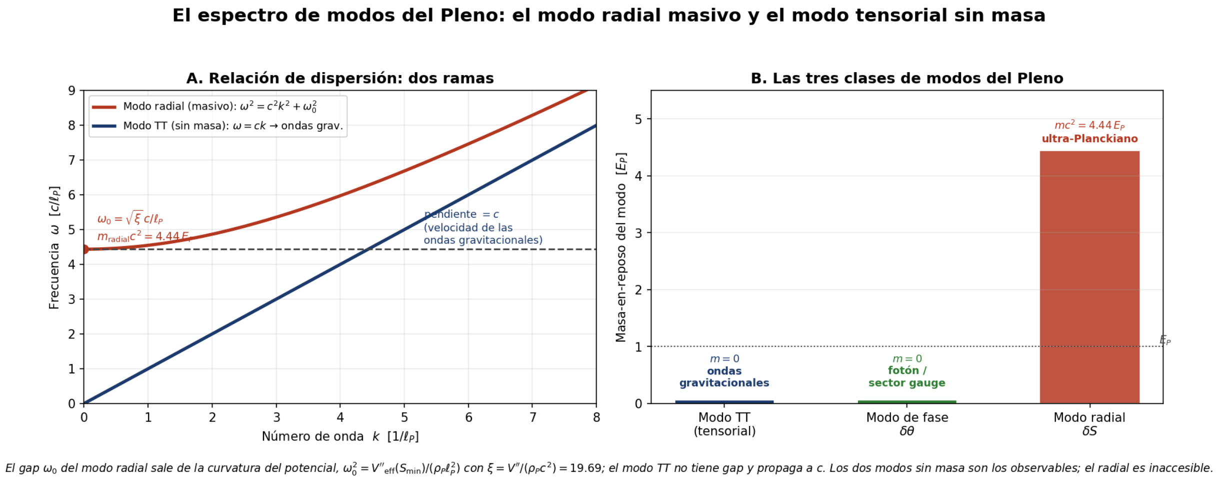

Figure 5: The spectrum of modes: the radial mode (massive, gap $\omega_0 = \sqrt{\xi}\, c/\ell_P$, mass $4.44\,E_P$) against the TT mode (massless, slope c). Panel B shows the three classes: only the radial has mass (ultra-Planckian, inaccessible); the phase and TT modes, massless, are the observables (photon and gravitational wave).

Regime of validity. For accessible frequencies ($k\ell_P \sim 10^{-43}$ in gravitational waves of LIGO/LISA), the correction $(k\ell_P)^2/20 \sim 10^{-86}$ is suppressed ~ 69 orders below the observational bound, so that the program reproduces general relativity in propagation with astronomical precision.

Conclusion — the harvest begins: the modes

First consequence of the now-closed trunk: linearizing F1, three classes of mode appear —the radial (massive, $4.44\,E_P$, ultra-Planckian and undetectable as a particle) and the phase and TT modes (massless, at velocity c), which are the observables: photon and gravitational wave. The non-detection of el Pleno "as a particle" is a consequence of its only massive mode being at the Planck scale. These modes are the raw material of the two following links: gravity (TT mode) and electromagnetism (phase mode).

---

7 Gravity: from Newton's law to the Einstein equations

7.1 The $1/r$ potential and the identity of G — [DERIVED]

Newtonian gravity is the first link, and it emerges without postulating either the force law or its constant. The chain, whose complete steps are in §5.1, is as short as this: in the static regime and at scales $\gg \ell_P$, the modulus equation (F2) reduces to the **Poisson form**

$$\rho_P c^2 \ell_P^2\, \nabla^2(\delta S) = -\rho_S(\mathbf{x}),$$

whose three-dimensional Green's function, $\nabla^2\mathcal{G} = \delta^3$ with $\mathcal{G} = -1/4\pi r$, gives for a point source a modulus perturbation that **decays as** $1/r$, $\delta S(r) \propto 1/r$ —the Newtonian potential, not imposed but a consequence of the dimensionality of space and the form of F1. The interaction energy of two of those perturbations, via the kinetic term of F1 and integrating by parts, results in $U_{\rm int} \propto q_1 q_2/(\rho_P c^2 \ell_P^2\, r)$, of exact Newtonian form. Comparing with $U_{\rm Newton} = -Gm_1m_2/r$ —a comparison independent of the charge convention, since any rescaling is compensated on both sides— fixes the **structural identity**

$$\rho_P c^2 \ell_P^2 = \frac{c^4}{G} \qquad \Longleftrightarrow \qquad G = \frac{c^2}{\rho_P \ell_P^2}, \tag{E3}$$

without residual factor. The logical order is important: G is not measured and inserted, but the kinetic coefficient of F1 —already fixed by ρ_P and ℓ_P— *is* c^4/G. The gravity of Newton remains as the statics of the modulus of el Pleno, and its constant as a mechanical property of the medium. This result feeds everything that follows: the g_{00} of Schwarzschild (§7.3) and the scale of the Einstein coupling rest on this same identity.

7.2 Sakharov induced action and the Einstein equations — [DERIVED] / [SLD]

Step 1 — The effective metric as the Cauchy-Green tensor of the medium. The effective metric $g^{\text{eff}}_{\mu\nu}$ is not primitive geometric structure external to el Pleno: it is the **Cauchy-Green tensor** of el Pleno as a hyperelastic medium —the pull-back of the flat causal structure (axiom A3) by the deformation map $X^a(x)$ of the medium—,

$$g^{\text{eff}}_{\mu\nu} = \eta_{ab}\,\frac{\partial X^a}{\partial x^\mu}\frac{\partial X^b}{\partial x^\nu} = (F^{\mathsf{T}}\eta\,F)_{\mu\nu}, \qquad F^a{}_\mu \equiv \partial_\mu X^a. \tag{E39a}$$

Writing the deformation map as the relaxed configuration plus a displacement, $X^a(x) = x^a + u^a(x)$, the deformation gradient is $F^a{}_\mu = \delta^a_\mu + \partial_\mu u^a$, and the pull-back is expanded order by order:

$$g^{\text{eff}}_{\mu\nu} = \underbrace{\eta_{\mu\nu}}_{\text{order 0}} + \underbrace{\eta_{a\nu}\partial_\mu u^a + \eta_{b\mu}\partial_\nu u^b}_{\text{order 1}\,\equiv\, h_{\mu\nu}} + \underbrace{\eta_{ab}\,\partial_\mu u^a\,\partial_\nu u^b}_{\text{order 2}}. \tag{E39b}$$

Order zero is the Minkowski metric (relaxed regime, A3); order one is the complete linear metric perturbation $h_{\mu\nu}(\Psi) = \eta_{a\nu}\partial_\mu u^a + \eta_{b\mu}\partial_\nu u^b$, whose transverse traceless component —after fixing the gauge and projecting— is the TT mode of el Pleno (the physical gravitational mode; the trace and longitudinal parts are non-propagating, §6); and order two $\eta_{ab}\,\partial_\mu u^a \partial_\nu u^b$ is the first of the non-linear terms, which are *not added by hand* but are fixed by the same map $X(x)$. The symmetry $g^{\text{eff}}_{\mu\nu} = g^{\text{eff}}_{\nu\mu}$ is manifest to all orders in (E39b), so that the object is a **genuine metric, not a truncated perturbation**: the effective metric is not external structure but the reading that the perturbations make of the state of the medium. This resolves the general non-linear covariance: since Γ_{ind} is built with curvature scalars of g^{eff} and the invariant measure $\sqrt{-g}\,d^4x$, it is diffeo-invariant *to all orders* in the deformation, not only at the linear order. The Schwarzschild metric of el Pleno (§7.3), which reproduces perihelion precession and light deflection —impossible with a merely linearized metric—, is precisely this non-linear Cauchy-Green metric in the static spherical regime.

Step 2. The integration of the fluctuations of el Pleno generates an effective Einstein-Hilbert action:

$$\Gamma_{\text{ind}}[g] = -\frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\,R + \mathcal{O}(R^2/E_P^2). \tag{E34}$$

By the heat-kernel expansion (Seeley-DeWitt coefficients), the coefficient of the term $\sqrt{-g}\,R$ —the a_1— is proportional to the square of the ultraviolet cutoff, $1/G_{\text{ind}} \propto \Lambda^2$, with $\Lambda = 1/\ell_P$ (axiom A4). The dimensional structure is transparent: a bosonic loop in 4D contributes its phase-space volume $16\pi^2 = (4\pi)^2$, so that

$$\frac{c^4}{G_{\text{ind}}} = \frac{\hbar c}{\ell_P^2}\cdot\frac{b}{16\pi^2} \overset{\hbar=\rho_P c\ell_P^4}{=} \rho_P c^2\ell_P^2\cdot\frac{b}{16\pi^2},$$

where $\hbar c/\ell_P^2 = \rho_P c^2\ell_P^2$ was used (from $\hbar = \rho_P c\ell_P^4$) and $b = \mathcal{O}(1)$ is the Seeley-DeWitt factor times the density of modes. Comparing with the identity already derived via the Newtonian route, $c^4/G = \rho_P c^2\ell_P^2$ (E3), the two routes to the same G coincide if and only if $b/16\pi^2 = 1/2$, that is $b = 8\pi^2$.

The status of this equality deserves precision, because it is where scrutiny concentrates. $b = 8\pi^2$ is not a Seeley-DeWitt coefficient computed independently and then fitted to G: it is the **self-consistency condition between two independent derivations of** G —the direct Newtonian

(E3, the kinetic coefficient of F1) and the Sakharov-induced (integration of modes)—, and its two factors are known structure, neither adjustable. The $16\pi^2 = (4\pi)^2$ is the four-dimensional phase-space volume of the heat-kernel, universal to the method and not to the lattice. The 1/2 is the kinetic coefficient of F1, the normalization convention of the scalar field $(\frac{1}{2}\rho_P c^2 \ell_P^2\, \partial_\mu \Psi^* \partial^\mu \Psi)$. Trying to "derive the 1/2" from the lattice would be a category error: it is convention, not a lattice number. The only thing with genuine physical freedom —whether the integrated gravitational mode introduces or not a spurious factor— is the *non-contamination of the TT mode*, and that is indeed **[DERIVED]** (Step 3, below). $\gamma_G = 1$ is therefore, before a number, the falsifiable assertion that the substrate has no scalar gravitational mode nor additional polarizations: a generic substrate that had them would give $b \neq 8\pi^2$ even if the convention of the 1/2 were identical. The scale reproduces G exactly with $\ell_P = \ell_{\text{Planck}}$.

Step 3 — the stationary action. $\delta\Gamma_{\text{ind}}/\delta g^{\mu\nu} = 0$ gives

$$\boxed{R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{(\text{Pleno})}} \tag{E35}$$

Reading: the Einstein equations are not postulated; they emerge as the effective elastic action of el Pleno (Sakharov). Curvature is not primitive geometry but the response of the medium to its own configurations, and the four sources of the universe are states of the same substrate.

with $T_{\mu\nu}^{(\text{Pleno})}$ obtained by variation of F1 with respect to $g^{\mu\nu}$, decomposed into four sources (baryonic matter, dark matter, dark energy, gauge sector), all configurations of the same Pleno. Dimensional verification of the coefficient: $[\rho_P c^2 \ell_P^2/2] = [c^4/(2G)] = [N]$ ✓.

The fixed point $\gamma_G = 1$, derived. The quotient $\gamma_G \equiv G_{\text{eff}}/G$ equals 1 exactly when the induced coefficient $K_{\text{ind}} = b\, \rho_P c^2 \ell_P^2/(16\pi^2)$ equals the kinetic one of F1, $K_{F1} = \rho_P c^2 \ell_P^2/2$, that is when $b/16\pi^2 = 1/2$. This equality is not a lattice number to be determined: its two factors are known structure. The $16\pi^2 = (4\pi)^2$ is the four-dimensional phase-space volume of the heat-kernel —universal to the method, not to the lattice—, and the 1/2 is the kinetic coefficient of F1, the *convention* of normalization of the scalar field $(\frac{1}{2}\rho_P c^2 \ell_P^2\, \partial_\mu \Psi^* \partial^\mu \Psi)$. Trying to "derive the 1/2" from the diamond lattice would be a category error: it is convention, not a lattice number. What does have physical and falsifiable content —that the TT mode, integrated, reproduce the rigidity $1/G = \rho_P \ell_P^2$ *without a spurious factor*— is **[DERIVED]** by three pieces:

1. The gravitational mode is transverse traceless spin-2, with two physical polarizations and representation of dimension $5 = 2\ell+1$ ($\ell = 2$), obtained by *two convergent routes*: the spectral lineage of el Pleno (Casimir $\ell(\ell+1) = 6$, $\ell = 2$) and the standard counting of the graviton in $D = 4$, $(D+1)(D-2)/2 = 5$. There is no scalar mode nor additional polarization that contaminates a_1.

2. The residue of the physical pole is $+1$ (E35-ter, below): standard coupling of a massless mode to curvature.

3. The Lee-Wick fakeon contributes to a_0 ($\sim \Lambda^4$, cosmological constant), *not* to a_1 ($\sim \Lambda^2$, the induced G): it does not contaminate the coefficient that fixes G.

A substrate with a scalar gravitational mode or with additional polarizations would give $b \neq 8\pi^2$ even if the convention of the 1/2 were identical: the value $b = 8\pi^2$ is entirely structure of the method ($16\pi^2$) times convention of F1 (1/2), and the condition with physical content —the non-contamination of the TT mode— is derived. **[DERIVED]** $\gamma_G = 1$; with it the coefficient 1/4 of the Bekenstein-Hawking entropy is fixed, which depends on the same $c^4/(16\pi G)$.

The mechanism by which the lattice does not contaminate $\gamma_G = 1$. The fluctuation operator of the TT mode is of fourth order due to the flexural of F1 (E18–E19):

$$\mathcal{O}_{\text{TT}} = -\Box - \frac{\ell_P^2}{20}\Box^2 = -\Box\Big(1 + \frac{\ell_P^2}{20}\Box\Big). \tag{E35-bis}$$

In momenta ($\Box \to -k^2$, Euclidean signature) the propagator decomposes into partial fractions in a *forced* way (exact algebraic identity):

$$\frac{1}{k^2(1-\frac{\ell_P^2 k^2}{20})} = \frac{1}{k^2} - \frac{1}{k^2 - \frac{20}{\ell_P^2}}, \tag{E35-ter}$$

with two poles of forced residue (verified symbolically): a *physical pole* at $k^2 = 0$ (massless TT mode) with residue $+1$, and a massive *Lee-Wick pole* at $k^2 = 20/\ell_P^2$ with residue -1. The induced Einstein-Hilbert coefficient —the a_1 of the heat-kernel, the one that fixes G— is determined by the residue of the physical pole, $+1$. The k^4 flexural tail of the lattice dispersion belongs *entirely* to the Lee-Wick pole, not to the physical gravitational mode: the lattice correction affects a_0 (induced cosmological constant $\sim \Lambda^4$), *not* a_1 ($\sim \Lambda^2$). Therefore $b/16\pi^2 = 1/2$ is preserved in the sector that fixes G, and $\gamma_G = 1$ is robust against the diamond discretization.

$$\frac{b}{16\pi^2} = \frac{1}{2} \quad \text{is preserved in the sector that fixes } G. \tag{E35-quater}$$

Classification: the residue of the physical pole $= +1$ is **[DERIVED]** (exact partial fractions, E35-ter); the localization of the lattice correction in a_0 and not in a_1 is **[DERIVED]** (the mode counting $2\ell + 1 = 5$ by two routes excludes scalar contamination); consequently $\gamma_G = 1$ is **[DERIVED]**. The objection that the discrete lattice could give $b \neq 8\pi^2$ is answered: the lattice correction lives in a_0, not in the a_1 that fixes G, and the TT form factor of the Brillouin zone (≈ 1.36) belongs to the Lee-Wick pole, not to the physical gravitational mode.

Emergent Bianchi identities and covariant conservation. The exact covariance of (E39a)–(E39b) is not the end of the chain: from it one deduces, without additional postulate, the covariant conservation of the energy-momentum tensor and the contracted Bianchi identities, applying the *second Noether theorem* to the induced action. Let $\Gamma_{\text{ind}}[g]$ be invariant under diffeomorphisms. An infinitesimal diffeomorphism $x^\mu \to x^\mu + \xi^\mu(x)$ acts on the metric as the Lie derivative $\delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$. Defining the induced Einstein tensor as the functional derivative of the action, $G^{\mu\nu} \equiv (2/\sqrt{-g})\, \delta\Gamma_{\text{ind}}/\delta g_{\mu\nu}$, the variation of the action under the diffeomorphism is

$$\delta\Gamma_{\text{ind}} = \int d^4x\, \sqrt{-g}\, \tfrac{1}{2} G^{\mu\nu} (\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu) = \int d^4x\, \sqrt{-g}\, G^{\mu\nu}\, \nabla_\mu \xi_\nu,$$

where the second equality uses the symmetry of $G^{\mu\nu}$ (the two terms contribute equally). Integrating by parts with the covariant derivative (boundary term null for ξ of compact support), $\delta\Gamma_{\text{ind}} = -\int d^4x\, \sqrt{-g}\, (\nabla_\mu G^{\mu\nu})\, \xi_\nu$. The diffeomorphism invariance requires $\delta\Gamma_{\text{ind}} = 0$ for every arbitrary ξ_ν, whence

$$\nabla_\mu G^{\mu\nu} = 0 \tag{E39c}$$

—the *contracted Bianchi identity*, emergent from covariance, not postulated. The induced field equations $G^{\mu\nu} = (8\pi G/c^4)\, T^{\mu\nu}$ (E35) then imply, taking the covariant divergence,

$$\nabla_\mu T^{\mu\nu} = \frac{c^4}{8\pi G}\, \nabla_\mu G^{\mu\nu} = 0, \tag{E39d}$$

the *covariant conservation* of the energy-momentum tensor. Both are a consequence of the second Noether theorem applied to the diffeo-invariance of Γ_{ind} —which in turn rests on the action being built with curvature scalars of the Cauchy-Green metric. **[DERIVED]**. The formal chain of the Einstein dynamics is closed without presupposing Bianchi nor conservation: both emerge from the exact covariance, which emerges from g^{eff} being a genuine metric of the medium (E39a–E39b).

Conclusion — the Einstein structure, closed

The Einstein equations are not postulated: they emerge as the elastic action of el Pleno. The piece that closes it is that the effective metric $g^{\text{eff}} = F^{\mathsf{T}}\eta F$ is a **genuine metric**, the Cauchy-Green tensor of the deformed medium, not a truncated linearized perturbation $\eta + h$. From this follows a complete chain without adding hypotheses: the general covariance is exact to all orders (not only to linear); the coupling is fixed, $\gamma_G = 1$ **[DERIVED]**, and with it the coefficient 1/4 of the Bekenstein-Hawking entropy; and the Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ and the conservation $\nabla_\mu T^{\mu\nu} = 0$ *are deduced* from the second Noether theorem, instead of being imposed. General Relativity remains as the macroscopic limit of the medium, with its structure —not only its Newtonian form— derived. The open end that remains is bounded and declared: the explicit interior metric in the saturation zone $S \to S_{\max}$, where the geometric description ceases to apply and the medium is described by its regular core.

7.3 The Schwarzschild metric as the effective geometry of the static Pleno — [DERIVED] / [SLD]

Under the ontological shield of the program, the metric is not a primitive object to be determined by imposing $R_{\mu\nu} = 0$ on an *ansatz*: it is the reading that the perturbations make of the state of the medium (E-geo). The correct route is inverse: one first solves the static configuration of the modulus $\delta S(r)$ and *reads* from it the effective geometry $g^{\text{eff}}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(\Psi)$.

Step 1 — configuration of the modulus and potential. For a point source of mass M at rest, the static linearized equation of the modulus (§7, with $\rho_P c^2 \ell_P^2 = c^4/G$ and gravitational charge $q = \sqrt{4\pi}\, Mc^2$) gives

$$\rho_P c^2 \ell_P^2\, \nabla^2(\delta S) = -\rho_S(\mathbf{x}), \qquad \delta S(r) = \frac{q}{\rho_P c^2 \ell_P^2} \cdot \frac{1}{4\pi r}, \qquad \Phi(r) = -\frac{GM}{r}. \tag{E-Sch-1}$$

The normalization $q = \sqrt{4\pi}\, Mc^2$ is not ad hoc: it is fixed uniquely by the cancellation of the 4π of the Green's function ($\mathcal{G} = -1/4\pi r$, §5.1), so that $\Phi = -GM/r$ remains without residual factor; any rescaling of q is compensated on both sides of the comparison with Newton. It is the only dynamical input; the rest is geometric reading of Φ.

Step 2 — temporal component g_{00}. The temporal component of the geodesic of a slow particle gives $\mathbf{a} = -\frac{1}{2}c^2\nabla h_{00}$; equating it with $\mathbf{a} = -\nabla\Phi$ (Newton's law already derived) fixes $h_{00} = 2\Phi/c^2$,

$$g_{00} = -\Big(1 + \frac{2\Phi}{c^2}\Big) = -\Big(1 - \frac{2GM}{c^2 r}\Big). \tag{E-Sch-2}$$

The factor 2 is not free: it is fixed by the reproduction of Newton, without a new parameter.

Step 3 — radial component g_{rr} by exterior $R_{\mu\nu} = 0$. Outside the source el Pleno is relaxed and without propagating excitations, $T_{\mu\nu} = 0$, and E35 reduces to $R_{\mu\nu} = 0$. On the static spherically symmetric form $ds^2 = -A(r)c^2dt^2 + B(r)dr^2 + r^2 d\Omega^2$ —where A, B parametrize the components of $h_{\mu\nu}(\Psi)$ allowed by the symmetry, not freely postulated functions— the Ricci components are the standard ones, and the combination $R_{tt}/A + R_{rr}/B = 0$ cancels the terms in A'':

$$\frac{1}{rB}\Big(\frac{A'}{A} + \frac{B'}{B}\Big) = 0 \implies A(r)B(r) = \text{const}. \tag{E-Sch-4}$$

The asymptotic matching ($A, B \to 1$ when $r \to \infty$) fixes the constant at 1, so that $B = 1/A$. With $B = 1/A$, the component $R_{tt} = 0$ reduces —by the standard Ricci computation for this metric (verified symbolically)— to $rA'' + 2A' = 0$. This combination is the second derivative of rA: indeed $(rA)' = A + rA'$ and $(rA)'' = 2A' + rA''$, so that $rA'' + 2A' = (rA)'' = 0$. Integrating once, $(rA)' = \text{const} = 1$ (the constant is fixed to 1 by the limit $A \to 1$, $A' \to 0$ when $r \to \infty$);

integrating again, $rA = r - r_s$ with r_s an integration constant:

$$A(r) = 1 - \frac{r_s}{r}, \qquad B(r) = \left(1 - \frac{r_s}{r}\right)^{-1}. \tag{E-Sch-5}$$

The identification $A \simeq 1 - 2GM/c^2 r$ at large distance (E-Sch-2) fixes $r_s = 2GM/c^2$.

Schwarzschild metric as the effective geometry of the static Pleno

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)c^2dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1}dr^2 + r^2 d\Omega^2, \qquad \boxed{r_s = \frac{2GM}{c^2}}. \tag{E-Sch-7}$$

Geometric reading of the static configuration $\delta S(r) \propto 1/r$, forced in its form by the emergent Einstein equations (E35), with the single freedom r_s fixed by the Newtonian matching.

Dimensional verification. $[r_s] = [G][M]/[c^2] = m$ ✓; for $M = M_\odot$, $r_s \approx 2.95$ km. The expansion of (E-Sch-7) in r_s/r reproduces the three classical tests by being identical to the solution of General Relativity: perihelion precession $\Delta\phi = 6\pi GM/c^2 a(1-e^2)$, deflection $\Delta\theta = 4GM/c^2 b$, redshift $1 + z = (1 - r_s/r)^{-1/2}$. The program inherits the tests, it does not recompute them.

The two pathologies and their cure. *(i)* The horizon $r = r_s$ ($g_{00} \to 0$, escape velocity $= c$) is the sonic surface of the medium: since the velocity of the modes of el Pleno is c (§6), there $\mathrm{Ma} = v/c = 1$. It is the analog of Unruh, exact and not analogical. *(ii)* The central singularity does not exist: the Kretschmann invariant $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 12\, r_s^2/r^6$ would diverge as $r \to 0$, but the hard bound $S \le S_{\max}^{\text{geom}}$ (A6) bounds the density. The interior is a saturated core $S \to S_{\max}$, not a singularity; the black hole resolves the singularity by the same A6 mechanism that the bounce resolves that of the Big Bang. The radius of the core connects with the minimal mass $M_{\min} = m_P\sqrt{3/(32\pi S_{\max}^2)} \approx 0.117\, m_P$: below $M_{\min}$ there is no stable saturated core nor horizon.

Epistemic classification. The exterior form (E-Sch-7) is **[DERIVED]** as a spherical corollary of E35, with g_{00} **[DERIVED]** by consistency with Newton and g_{rr} **[DERIVED]** conditional on E35; the sonic horizon is **[SLD]**; the regular core by A6 is **[SLD]** with individually derived pieces. The explicit interior metric of the transition zone $S \to S_{\max}$ is a declared derivation open end. The exact coefficient of r_s is fixed by two concordant routes: consistency with Newton in the weak field (E-Sch-2) and the induced action with $\gamma_G = 1$ **[DERIVED]**. The program does not claim derivational novelty over the metric: it asserts it as a proof of consistency and a confrontation bridge, with the two cures as distinctive content inherited from already-closed pieces.

Conclusion — what it means physically

A black hole does not have a singularity at the center: it has a core of vacuum compressed to its maximal density. The curvature of space around a mass is the elastic deformation of el Pleno, exactly as a weight sinks a taut membrane; far away, that deformation reproduces the Schwarzschild metric and Newton's gravity. But the medium cannot be compressed without limit —the same hard bound that stops the Big Bang in a bounce—, so where general relativity predicts infinite density, el Pleno saturates and forms a finite core at the Planck density. The horizon, for its part, is the surface where the escape velocity equals the sound velocity of the medium: nothing escapes not because of a magical barrier, but because the flow becomes supersonic inward.

7.4 The structural rigidities of el Pleno — [DERIVED] / [SLD]

If gravity is the macroscopic response of el Pleno, the medium must possess a set of mechanical constants, all derivable from the primaries (ρ_P, c, ℓ_P) and from the pure ratios already obtained. Each one measures the resistance of the medium to a *distinct type* of deformation, and together they draw the character of el Pleno as a solid.

(1) Kinetic rigidity $K_{\mathbf{cin}}$ —effective Young's modulus. It is the coefficient of the kinetic term of F1, the resistance to deforming the *modulus* S:

$$K_{\text{cin}} = \frac{\rho_P c^2 \ell_P^2}{2} = \frac{c^4}{2G} \approx 6.05 \times 10^{43}\ \text{N}. \tag{E38}$$

It has units of force ([J/m]): force per relative deformation of the modulus. It coincides with the Einstein-Hilbert prefactor. Its Planckian magnitude ($\sim c^4/G$) is the ontological key: deforming el Pleno costs an energy of gravitational scale.

Reading: el Pleno is a medium of extreme rigidity. Inverting $K_{cin} = c^4/2G$, the constant G is the ***softness*** *(compliance) of the medium, not an intensity of force. Gravity is the weakest of the interactions because el Pleno is extremely dense and therefore extremely rigid —the gravitational hierarchy ceases to be a mystery.*

(2) Shear modulus μ —resistance to shear. It admits two readings that coincide in scale $\sim \rho_P c^2$ but differ by the packing ratio, and it is important not to confuse them. The *dynamic* reading comes from the velocity of the shear waves, $v_T = \sqrt{\mu/\rho_P} = c$ (the transverse modes travel at c), and gives $\mu_{\text{din}} = \rho_P c^2 \approx 4.63 \times 10^{113}$ Pa. The *static* reading comes from the quadratic response of the Gent saturation potential ($V_{\text{topo}} \approx \frac{1}{2}\mu_{\text{Gent}} S^2$ at small deformation), and gives

$$\mu_{\text{Gent}} = \frac{\rho_P c^2}{S^2_{\text{max,geom}}} \approx 2.13 \times 10^{113}\ \text{Pa},$$

smaller by the factor $S^2_{\text{max,geom}} \approx 2.18$ (E10). The difference is not an error: μ_{din} measures shear propagation, μ_{Gent} the rigidity of the potential against the hard bound. Both confirm that el Pleno is, mechanically, a solid of maximal admissible rigidity.

(3) Relaxation time τ_Π —not free, derived. The dissipation of the medium proceeds from the flexural term, whose UV regulation mode is the Lee-Wick pole at $\Box\delta S = 20/\ell_P^2$, at the scale $M_{\text{LW}} = \sqrt{20}/\ell_P$. The relaxation time is the inverse of the frequency of that mode:

$$\tau_\Pi = \frac{1}{\omega_{\text{LW}}} = \frac{\ell_P}{\sqrt{20}\,c} = \frac{\sqrt{5}\,\ell_P}{10\,c} \approx 1.21 \times 10^{-44}\ \text{s}. \tag{E-relax}$$

The factor $\sqrt{20}$ is exact. The same quantity comes, independently, from the fluctuation-dissipation relation of the medium ($\tau_\Pi k_B T_P/\hbar = 1/\sqrt{20}$ with $k_B T_P = E_P$): two routes, a single τ_Π, without fitting.

(4) Shear viscosity η —internal dissipation. By the Maxwell-Kubo relation $\eta = \mu \cdot \tau$, with the Gent shear and the derived time:

$$\eta_{\text{Pleno}} = \mu_{\text{Gent}}\,\tau_\Pi = \frac{\sqrt{5}\,\rho_P c\,\ell_P}{10\,S^2_{\text{max,geom}}} \approx 2.56 \times 10^{69}\ \text{Pa}\cdot\text{s}.$$

It measures how much energy the medium dissipates upon responding to a deformation in its scale of relaxation. Divided by the entropy density it fixes the ratio η/s, the hydrodynamic signature of el Pleno comparable with the KSS bound (E51).

(5) Characteristic tension $\sigma_{\mathbf{Pleno}}$. From the flexural rigidity $\gamma_\kappa = \hbar c/40$ and the maximal curvature $\kappa_{\max} = 1/\ell_P$ (A4):

$$\sigma_{\text{Pleno}} = \frac{\gamma_\kappa}{\ell_P^2} = \frac{1}{40} \cdot \frac{\hbar c}{\ell_P^2} = \frac{F_{\text{Planck}}}{40} \approx 3.03 \times 10^{42}\ \text{N},$$

the Planck force reduced by the flexural factor 40. It is the internal tension that the medium sustains before the flexural penalizes the curvature.

Combined reading: the elastic balance. Treating el Pleno as an isotropic solid (Hooke, E40), the bulk modulus $K = \frac{1}{4}\xi \rho_P c^2 S^{2,\text{estr}}_{\min} \approx 1.19 \times 10^{114}$ Pa —from the amplitude mode (Higgs), with $\xi \equiv V''_{\rm eff}(S^{\rm estr}_{\rm min})/(\rho_P c^2) \approx 19.69$ already fixed in the spectrum of modes (gap $\sqrt{\xi}\,E_P$, E21), without a new parameter— and the shear μ (from the phase mode, Goldstone, without gap) give the Poisson coefficient

$$\nu = \frac{3K - 2\mu}{2(3K+\mu)} \approx 0.328, \qquad E = \frac{9K\mu}{3K+\mu} \approx 1.23 \times 10^{114}\ \text{Pa}. \tag{E43}$$

Reading: $\nu \approx 0.33$ (comparable to steel) places el Pleno in the range of thermodynamic stability. That its rigidities fall where they fall is not a fit —it is a consequence of compression (Higgs) and shear (Goldstone) being the two modes of the same field, and their balance ν is a geometric property, not calibrated.

Three structural invariants. The rigidities reveal three pure numbers that run through the whole program: $\sqrt{2}$ (packing of the diamond lattice), $\sqrt{20}$ (relaxation, UV regulation) and 40 (flexural tension, in $\gamma_\kappa = \hbar c/40$ and $\sigma_{\rm Pleno} = F_{\rm Planck}/40$). That a handful of pure ratios fixes all the mechanical constants is the mark of a medium without free parameters.

Conclusion — the harvest: gravity

The TT mode, taken to the macroscopic regime, *is* gravity: the Einstein equations emerge by integration of fluctuations of F1, with G already derived in link 7 and without singularities (blocked by the hard bound of link 6). The factor $b = 8\pi^2$ of the coupling is **[DERIVED]**: it is the phase-space volume of the method ($16\pi^2$) times the kinetic convention of F1 (1/2), with the non-contamination of the TT mode derived (residue +1 of the physical pole E35-ter, mode counting $2\ell+1=5$ by two routes, fakeon in a_0); hence $\gamma_G = 1$ and, with it, the coefficient 1/4 of Bekenstein-Hawking. General relativity remains as an effective limit of el Pleno.

---

8 Quantum mechanics as the dynamics of el Pleno

8.1 The uncertainty relation (derived GUP) — [DERIVED]

The scale factor 20. The whole correction of this section is controlled by a single number, the 20 of the UV regulation pole of F1 (the Lee-Wick pole of the flexural term, $\Box\delta S = 20/\ell_P^2$). From it come the two quantities we will use: the minimal resolvable width $\sigma_{\rm min} = \ell_P/\sqrt{20}$ and, by the fluctuation-dissipation relation of the medium, $\tau_\Pi k_B T_P/\hbar = 1/\sqrt{20}$ (with $k_B T_P = E_P$). It is not an adjustable parameter: the 20 is fixed by the flexural coefficient 1/40 of link 2. (Separately, the mass^2 of the radial mode $\xi \approx 19.69$ falls numerically close to 20, but they are distinct quantities —mass of the mode *vs* regulation pole— and that closeness is not used in what follows.)

The starting point: the Gaussian saturation. The standard uncertainty relation, $\Delta x\,\Delta p \geq \hbar/2$, is *saturated* (becomes an equality) for a Gaussian packet, which is the minimum-uncertainty state. The strategy is therefore to take the state that would saturate the ordinary limit and compute what the rigidity of the medium does to it. Let a normalized Gaussian packet of width σ in position,

$$\psi(x) = (2\pi\sigma^2)^{-1/4}\, e^{-x^2/4\sigma^2}, \qquad |\psi|^2 = (2\pi\sigma^2)^{-1/2} e^{-x^2/2\sigma^2},$$

for which $\langle x\rangle = 0$ and $\langle x^2\rangle = \sigma^2$, so that $\Delta x = \sigma$ (verified symbolically). Its Fourier transform is also Gaussian, $\phi(k) \propto e^{-\sigma^2 k^2}$, with moments

$$\langle k\rangle = 0, \qquad \langle k^2\rangle = \frac{1}{4\sigma^2}, \qquad \langle k^4\rangle = \frac{3}{16\sigma^4} = 3\langle k^2\rangle^2, \tag{E-inc-1}$$

all computed by integrating over the Gaussian (verified symbolically; the factor 3 is the fourth Gaussian moment). In the ordinary case $p = \hbar k$ this gives $\Delta p = \hbar/(2\sigma)$ and $\Delta x\,\Delta p = \hbar/2$ exact: the Heisenberg limit, recovered as the base case.

Why this is an uncertainty relation (not a mere product). Before computing, one must justify that $\Delta x \cdot \Delta p_{\text{medium}}$ *is* a legitimate uncertainty relation and not a coincidence of dispersions. The key is that the momentum of the medium is a *function* of the canonical momentum, $p_{\text{medium}} = f(p)$ with $p = -i\hbar\partial_x$ and $f(p) = p\sqrt{1 + (\ell_P p/\hbar)^2/20}$. The Robertson-Schrödinger theorem applied to x and $p_{\text{medium}} = f(p)$ gives

$$\Delta x\, \Delta p_{\text{medium}} \geq \frac{1}{2}|\langle [x, f(p)]\rangle| = \frac{\hbar}{2}|\langle f'(p)\rangle|,$$

using $[x, f(p)] = i\hbar f'(p)$. This is the exact structure of a **generalized uncertainty principle** (GUP): the right side is no longer the constant $\hbar/2$ but depends on the state through $\langle f'(p)\rangle$. Since $f'(0) = 1$, in the limit of long waves ($p \to 0$) the commutator recovers $[x, p_{\text{medium}}] \to i\hbar$ and the floor is $\hbar/2$ exact. That the bound be *generalized* —not the fixed canonical relation— is precisely the physical signature of the medium. The Gaussian computation that follows evaluates this bound over the minimum-uncertainty state and gives its closed form.

The correction of the medium. In el Pleno the momentum is not the canonical one. From the dispersion relation (E23), the flexural rigidity makes the transported momentum be

$$p_{\text{medium}}(k) = \hbar k\sqrt{1 + \frac{(k\ell_P)^2}{20}}.$$

Since $p_{\text{medium}}(k)$ is odd in k and the packet is symmetric ($\langle k\rangle = 0$), it follows $\langle p_{\text{medium}}\rangle = 0$, so that the dispersion is directly the second moment:

$$\Delta p^2_{\text{medium}} = \langle p^2_{\text{medium}}\rangle = \hbar^2\Big\langle k^2\Big(1 + \frac{(k\ell_P)^2}{20}\Big)\Big\rangle = \hbar^2\Big[\langle k^2\rangle + \frac{\ell_P^2}{20}\langle k^4\rangle\Big].$$

Substituting the moments (E-inc-1),

$$\Delta p^2_{\text{medium}} = \hbar^2\Big[\frac{1}{4\sigma^2} + \frac{\ell_P^2}{20}\cdot\frac{3}{16\sigma^4}\Big] = \frac{\hbar^2}{4\sigma^2}\Big[1 + \frac{3}{80}\frac{\ell_P^2}{\sigma^2}\Big].$$

Multiplying by $\Delta x^2 = \sigma^2$ and taking the root, the product is obtained in closed form:

$$\boxed{\Delta x \cdot \Delta p = \frac{\hbar}{2}\sqrt{1 + \frac{3}{80}\Big(\frac{\ell_P}{\sigma}\Big)^2}} \tag{E-inc-3}$$

The coefficient $3/80 = \frac{1}{20}\cdot 3\cdot\frac{1}{4}$ combines the flexural rigidity ($1/20$), the fourth Gaussian moment (3) and the normalization of the second moment ($1/4$). **Symbolic verification**: the difference between $\sigma^2\Delta p^2_{\text{medium}}$ and the closed form $(\hbar/2)^2[1 + \frac{3}{80}(\ell_P/\sigma)^2]$ is exactly zero.

Recovery of Heisenberg. The result has the two correct limits. *(i) Ordinary regime*: for $\sigma \gg \ell_P$ the correction term vanishes and

$$\Delta x\, \Delta p \to \frac{\hbar}{2} \quad \text{exact},$$

independent of ξ and of any parameter of the medium: the Heisenberg limit is not an input of the model but the floor that the Gaussian saturation produces when the flexural rigidity is negligible. *(ii) Extreme regime*: at the minimal width $\sigma = \sigma_{\min} = \ell_P/\sqrt{20}$ (the Lee-Wick pole), substituting $(\ell_P/\sigma_{\min})^2 = 20$,

$$\Delta x\, \Delta p = \frac{\hbar}{2}\sqrt{1 + \frac{3}{80}\cdot 20} = \frac{\hbar}{2}\sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{4}\,\hbar \approx 0.6614\,\hbar,$$

an excess of 32.3% over the floor: the maximum possible deviation, at the limit of resolution of the medium.

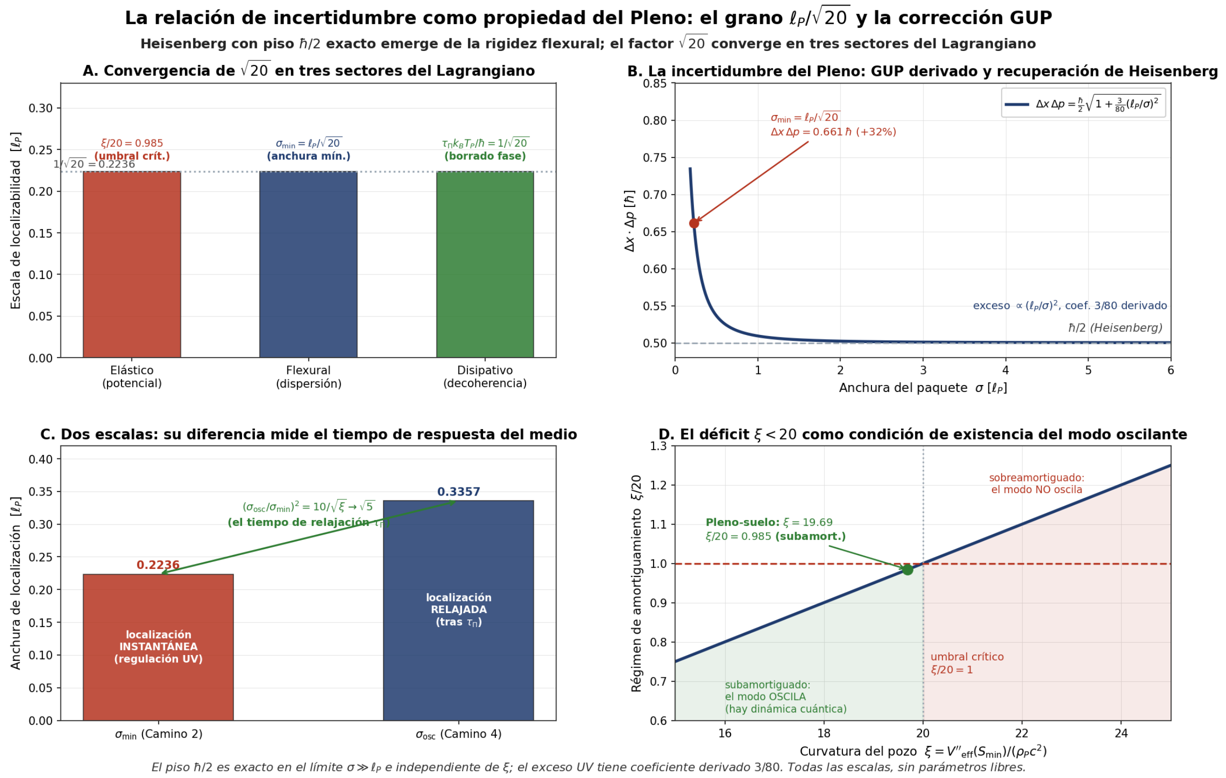

Figure 6: The uncertainty relation as a property of the medium. Panel B: the product $\Delta x\,\Delta p$ falls to the exact floor $\hbar/2$ for $\sigma \gg \ell_P$ and rises to $+32\%$ at $\sigma_{\min} = \ell_P/\sqrt{20}$, with the UV excess of derived coefficient $3/80$. Panel A: the factor $\sqrt{20}$ converges from three independent sectors of the Lagrangian (elastic, flexural, dissipative).

The sign $\geq$. Since $1 + \frac{3}{80}(\ell_P/\sigma)^2 \geq 1$ by the strictly positive flexural rigidity (A4), the Heisenberg inequality *is* the positivity of the curvature rigidity of el Pleno. The closed form and the recovery of $\hbar/2$ are [**DERIVED**]; the reading of the sign is [**SLD**]; the GUP as a prediction is suppressed $\sim 10^{-50}$ at atomic scales (not tested).

Conclusion — the harvest: the quantum

If gravity is the medium in the large, the quantum is the medium in the small: the uncertainty relation is the phenomenology of a medium with positive curvature rigidity (the flexural of F1). The floor $\hbar/2$ is exact (Gaussian saturation of long waves) and independent of ξ; the ultraviolet excess $(3/80)(\ell_P/\sigma)^2$ is a prediction with a derived coefficient, not parametrized. The same rigidity that regulates F1 in the UV produces the quantum limit —gravity and the quantum, a single substrate.

8.2 The Chandrasekhar limit as an internal balance — [DERIVED]

The chain, internal to el Pleno. (1) The fermionic statistics is not imported: it is Finkelstein-Rubinstein over the solitons ($(-1)^B$, §10). (2) The degeneracy pressure is the uncertainty (flexural rigidity). (3) Gravity is the macroscopic response of el Pleno (§7). (4) The equilibrium of a self-gravitating degenerate relativistic gas is the polytrope of index $n = 3$ (Lane-Emden), which fixes the coefficient.

The Lane-Emden equation $\theta'' + \frac{2}{\xi}\theta' + \theta^3 = 0$, with $\theta(0) = 1$, $\theta'(0) = 0$, integrated numerically up to the first root, gives (verified): first root $\xi_1 = 6.8968$ and $(-\xi^2\theta')_{n=3} = 2.0182$. The balance produces a limiting mass independent of the radius:

$$\boxed{M_{\text{Ch}} = \frac{\sqrt{3\pi}}{2}\left(-\xi^2\theta'\right)_{n=3}\frac{(\hbar c/G)^{3/2}}{(\mu_e m_u)^2} \approx 1.456\,M_\odot} \tag{E-Ch-1}$$

with $\mu_e = 2$ (C-O matter). Factored with $m_P = \sqrt{\hbar c/G}$:

$$M_{\rm Ch} = m_P \cdot \Big(\frac{m_P}{\mu_e m_u}\Big)^2 \cdot \underbrace{\frac{\sqrt{3\pi}}{2}\big(-\xi^2\theta'\big)_{n=3}}_{=3.098}, \tag{8.1}$$

where m_P is the scale of gravity-uncertainty equalization, $(m_P/\mu_e m_u)^2$ the Planck-hadronic hierarchy (the same as the mass of the fundamental soliton), and 3.098 the pure coefficient of the $n = 3$ polytrope. Three mass limits in one framework: $M_{\rm min} \approx 0.117\, m_P$ (black hole), $M_{\rm Ch} \approx 1.456\, M_\odot$, $M_Q = M_P/S_{\rm max,eff}$.

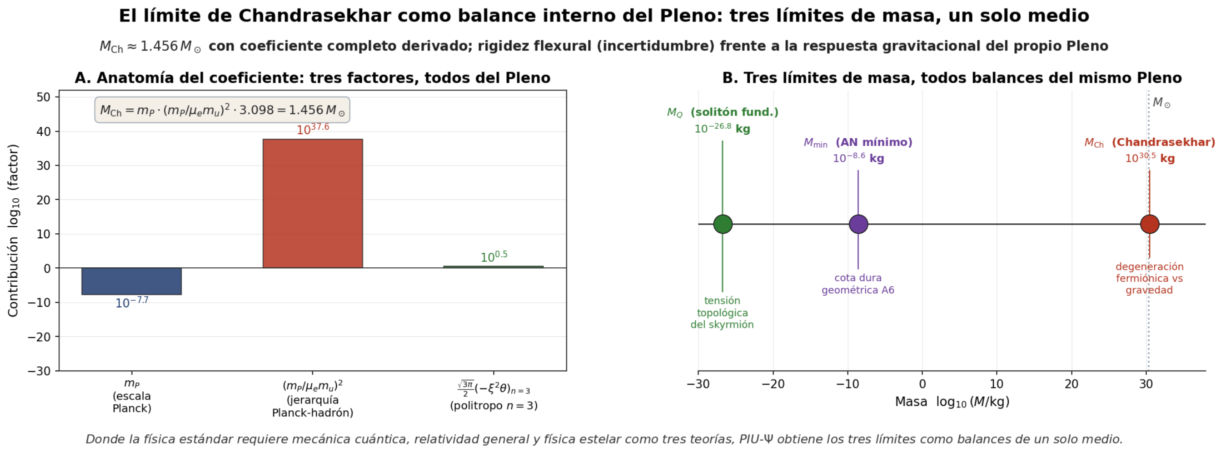

Figure 7: The Chandrasekhar limit as an internal balance. Panel A: anatomy of the coefficient in three factors —the Planck scale m_P, the hierarchy $(m_P/\mu_e m_u)^2$ and the pure coefficient of the $n = 3$ polytrope, $\frac{\sqrt{3\pi}}{2}(-\xi^2\theta') = 3.098$. Panel B: the three mass limits (M_Q, $M_{\rm min}$, $M_{\rm Ch}$) as balances of the same medium, spanning 57 orders of magnitude.

Conclusion

The Chandrasekhar limit is not borrowed from astrophysics: it is the maximum mass that the flexural rigidity of el Pleno sustains against its own gravitational response —gravity and the quantum, the two sectors of the same medium, together fixing a critical stellar mass. The polytropic framework ($n = 3$, Lane-Emden) is imported, and is declared as such; the coefficient 2.0182 is a pure ratio of the polytropic profile, and the scales that enter (m_P, the mass hierarchy) are those of el Pleno itself. The result, $1.456\, M_\odot$, comes out without a fitting parameter.

8.3 Born, Bell and the Tsirelson bound — [DERIVED] / [SLD]

Three features that make quantum mechanics "strange" —the Born rule, the Bell non-locality and the quantum Tsirelson limit— emerge here from the same medium, without being postulated. The wave function ψ is not an abstract probability object: it *is* the perturbation $\delta S\, e^{i\theta}$ of el Pleno.

The Born rule as accounting of conserved charge. The localization probability $|\psi|^2$ is not a separate law: it is the fraction of the U(1) Noether charge (axiom A5) of the quantum that is in each region. For a quantum $\psi = \delta S\, e^{i\theta}$, the Noether current of F1 (E7c) and its charge density are

$$j^\mu = \rho_P c^2 \ell_P^2\, S^2 \partial^\mu \theta, \qquad \rho_Q = \rho_P \ell_P^2\, \omega\, S^2 \propto |\psi|^2,$$

so that the probability of finding the quantum in a region R is the fraction of charge localized there:

$$P(R) = \frac{Q_R}{Q_{\rm total}} = \frac{\int_R S^2\, d^3x}{\int S^2\, d^3x} = \int_R |\psi_{\rm norm}(\mathbf{x})|^2\, d^3x. \tag{E-Born}$$

The normalization $\int |\psi|^2 = 1$ is not imposed: it is closed by the topological quantization of the total charge (§9). The Born rule is thus *accounting of something that is conserved*, not an independent probabilistic axiom. [**DERIVED**].

Reading: the quantum probability ceases to be a mysterious postulate. $|\psi|^2$ measures what fraction of a physically conserved quantity —the phase charge of the medium— is in each region; the "randomness" of the measurement is the macroscopic reading of the internal noise of el Pleno, not an external collapse.

Bell correlations, step by step. A pair of subsystems of el Pleno in antisymmetric configuration realizes the analog of the singlet state; from it the correlation function is *derived*, not postulated.

- **Step 1 — singlet state.** Two complementary phase modes form $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$, the total-zero-angular-momentum configuration of the pair.

- **Step 2 — measurement operators.** To measure in the direction $\mathbf{a}$ is the operator $\boldsymbol{\sigma} \cdot \mathbf{a}$; the correlation is the expectation value of the product:

$$E(\mathbf{a}, \mathbf{b}) = \langle \Psi^- | (\boldsymbol{\sigma}_1 \cdot \mathbf{a})(\boldsymbol{\sigma}_2 \cdot \mathbf{b}) | \Psi^- \rangle.$$

- **Step 3 — evaluation.** The Pauli-algebra identity over the singlet gives $E = -\mathbf{a} \cdot \mathbf{b} = -\cos\theta_{ab}$. Parametrizing the directions by measurement angles, $\theta_{ab} = 2(a - b)$ —the factor 2 is the signature that the phase mode is a spinor (a physical rotation α rotates the spinor by $\alpha/2$):

$$E(a, b) = -\cos\big(2(a - b)\big). \tag{E-Bell}$$

- **Step 4 — Tsirelson bound.** With the CHSH combination $\mathcal{S} = E(a, b) - E(a, b') + E(a', b) + E(a', b')$, optimizing over the angles (optimum at separations of 22.5°) gives

$$\boxed{|\mathcal{S}|_{\max} = 2\sqrt{2}} \qquad \text{(Tsirelson bound)}, \tag{8.2}$$

  verified numerically. It violates the classical Bell limit ($|\mathcal{S}| \leq 2$) but respects the quantum one, exactly as observed.

- **Step 5 — no-signaling.** The reduced state of a subsystem is $\rho_B = \mathrm{Tr}_A |\Psi^-\rangle\langle\Psi^-| = \frac{1}{2}\mathbb{I}$, independent of the direction measured at A: no measurement at A alters the local statistics of B. There is no superluminal communication; no-signaling is verified exactly.

Why $2\sqrt{2}$: the tetrahedral connection. The bound is not an arbitrary number: it is anchored in the geometry of the lattice. The tetrahedral angle $\theta_{\text{tet}} = \arccos(-\frac{1}{3}) = 109.47°$ satisfies exactly

$$\sin\theta_{\text{tet}} = \frac{2\sqrt{2}}{3}, \qquad \tan\frac{\theta_{\text{tet}}}{2} = \sqrt{2},$$

and the binary tetrahedral group $2T$ (order 24) is the symmetry of the diamond lattice. The $\sqrt{2}$ of Tsirelson is the same $\sqrt{2}$ of the packing (structural invariant of el Pleno): the quantum limit of the correlations is the imprint of the geometry of the medium. [**SLD**].

Reading: the quantum "non-locality" is not action at a distance. It is a single object —el Pleno— whose configuration is global by construction, prior to the metric that defines the distances. EPR is resolved by the pre-metric uniqueness of the medium: there are not two systems that communicate, there is one whose state manifests in two regions.

Conclusion — what it means physically

Quantum mechanics, at its core, is the phenomenology of a physical medium. Its three puzzling features have here a mechanical origin and a clear meaning: the **uncertainty** is the positive curvature rigidity of el Pleno (localizing costs energy, floor $\hbar/2$ exact); the **Born rule** is accounting of conserved charge ($|\psi|^2$ = fraction of localized phase charge); the **Bell correlations** up to $2\sqrt{2}$ are the tetrahedral geometry of the lattice read in the measurement space. There is no collapse, no fundamental randomness, no superluminal signal: there is a single substrate whose deterministic dynamics is perceived as probabilistic at the level of its perturbations. Three features, three distinct origins within F1, a single medium.

8.4 The non-relativistic limit: F1 → Gross-Pitaevskii — [DERIVED]

A structural correspondence closes the quantum link: in the **non-relativistic** limit of F1 —modes of frequency much lower than the Planckian $\omega_P = c/\ell_P$ and slow dynamics against the microscopic scale— F1 reduces formally to the **Gross-Pitaevskii equation** (Gross 1961, Pitaevskii 1961), the best-validated macroscopic quantum field equation of physics (Bose-Einstein condensates, superfluids, superconductors). The same decomposition $\Psi = Se^{i\theta}$ of el Pleno is that of the order parameter of those systems, so that the reduction is not an analogy but a limit.

Step 1 — Regime. $S = S_{\min}^{\text{estr}} + \delta S$ with $\delta S \ll S_{\min}^{\text{estr}}$ and slow dynamics against ω_P.

Step 2 — Field substitution. The fast phase of the rest mass of the radial mode is factored out and the modulus is normalized to its reading as *number density* of the medium. By the constitutive identification $S^2 \leftrightarrow n_{\text{quanta}}/n_0$ with $n_0 = 1/\ell_P^3$ (E-dens, §4), the perturbation δS has the number-density reading $\delta S^2/\ell_P^3$, which fixes the prefactor of the effective wave function:

$$\psi(\mathbf{x}, t) = \frac{\delta S(\mathbf{x}, t)}{\ell_P^{3/2}}\, e^{-i\omega_0 t}, \qquad \omega_0 = \frac{m_{\text{radial}} c^2}{\hbar} = \sqrt{\xi}\, \frac{c}{\ell_P},$$

so that $|\psi|^2 = \delta S^2/\ell_P^3$ is **number density** ($[\,|\psi|^2\,] = \mathrm{m}^{-3}$), the canonical Gross-Pitaevskii convention in which $|\psi|^2$ counts quanta per unit volume —coherent with the reading of the modulus of el Pleno established in E-dens (§4), not with a normalization to energy density. The phase $e^{-i\omega_0 t}$ separates the fast dynamics ($\omega_0 = \sqrt{\xi}\,\omega_P \approx 4.44\,\omega_P$, ultra-Planckian) from the non-relativistic one ($\omega_{\delta S} \ll \omega_0$).

Step 3 — Expansion of the potential to fourth order around $S_{\min}^{\text{estr}}$, retaining the cubic term:

$$V_{\text{eff}}(S_{\min} + \delta S) = V_{\text{eff}}(S_{\min}) + \tfrac{1}{2} V''_{\text{eff}}\, \delta S^2 + \tfrac{1}{6} V'''_{\text{eff}}\, \delta S^3 + \tfrac{1}{24} V^{(4)}_{\text{eff}}\, \delta S^4 + \mathcal{O}(\delta S^5).$$

The cubic term has a nonzero coefficient at the minimum. The three derivatives of the potential at $S_{\min}^{\text{estr}}$ are obtained from the same $V = W_{\text{bare}} + V_q + V_{\text{topo}}$ (§4, first subsection, where they are computed and verified symbolically from the closed form of the potential):

$$V''_{\text{eff}} = 19.69\,\rho_P c^2, \qquad V'''_{\text{eff}} = 27.92\,\rho_P c^2, \qquad V^{(4)}_{\text{eff}} = 855.6\,\rho_P c^2. \tag{E24-bis}$$

With $\delta S = \frac{1}{2}(A\,e^{-i\omega_0 t} + A^* e^{+i\omega_0 t})$, the square $\delta S^2 = \frac{1}{4}A^2 e^{-2i\omega_0 t} + \frac{1}{4}A^{*2} e^{+2i\omega_0 t} + \frac{1}{2}|A|^2$ has two components at $2\omega_0$ and one DC component. The reason the cubic does not contribute *at first order* is not that it averages to zero, but that none of those components is in resonance with ω_0, the carrier frequency of ψ. *At second order*, in contrast, those components fed back by the cubic generate a resonant term at ω_0 that **renormalizes** the effective quartic: the coupling g_{eff} receives a cubic correction that is computed explicitly in Step 6.

Step 4 — the reduction to Schrödinger (rotating average). Here is the heart of the limit, and it is worth being explicit about what "non-relativistic" means. The perturbation δS

obeys a Klein-Gordon-type equation with mass, that of the radial mode (E18). Substituting $\delta S \propto \mathrm{Re}[\psi\, e^{-i\omega_0 t}]$ with ψ slow and $\omega_0 = m_{\rm eff}c^2/\hbar$, the second time derivative gives

$$e^{i\omega_0 t}\,\partial_t^2(\psi\, e^{-i\omega_0 t}) = \ddot{\psi} - 2i\omega_0\dot{\psi} - \omega_0^2\psi.$$

The term $-\omega_0^2\psi$ cancels *exactly* with the mass one $(m_{\rm eff}c/\hbar)^2\psi = \omega_0^2\psi/c^2$ (that is the factored-out rest mass). The *non-relativistic* condition is $|\ddot{\psi}| \ll \omega_0|\dot{\psi}|$ —the residual dynamics of ψ is slow against ω_0, not that $m_{\rm eff}$ be small; on the contrary, it is precisely the enormity of ω_0 (ultra-Planckian) that guarantees the scale separation. Neglecting $\ddot{\psi}$ remains

$$-\frac{2i\omega_0}{c^2}\dot{\psi} = -\nabla^2\psi + (\text{non-linear term}) \implies i\hbar\,\dot{\psi} = -\frac{\hbar^2}{2m_{\rm eff}}\nabla^2\psi + \ldots,$$

using $\omega_0 = m_{\rm eff}c^2/\hbar$. The corresponding effective Lagrangian, in which the effective quartic $g_{\rm eff}$ already incorporates the renormalization of the cubic at second order (Step 6), is

$$\mathcal{L}_{\rm eff} = \frac{i\hbar}{2}(\psi^*\partial_t\psi - \psi\partial_t\psi^*) - \frac{\hbar^2}{2m_{\rm eff}}|\nabla\psi|^2 - \frac{g_{\rm eff}}{2}|\psi|^4.$$

Step 5 — Effective mass. Equating the rest-mass term $\hbar\omega_0 = m_{\rm eff}c^2$, with $\omega_0 = \sqrt{V''_{\rm eff}(S_{\rm min})/(\rho_P\ell_P^2)}$ from (E21):

$$\boxed{m_{\rm eff}\,c^2 = \hbar\sqrt{\frac{V''_{\rm eff}(S_{\rm min}^{\rm estr})}{\rho_P\,\ell_P^2}} = \sqrt{\xi}\,\frac{\hbar c}{\ell_P} = \sqrt{\xi}\,E_P} \tag{E24}$$

With $\xi \approx 19.69$: $\sqrt{\xi} = 4.4373$, so that $m_{\rm eff}c^2 = \sqrt{\xi}\,E_P \approx 4.44\,E_P \approx 5.42\times10^{19}$ GeV (with $E_P \approx 1.22\times10^{19}$ GeV), an ultra-Planckian excitation that justifies that the observable regime be that of the massless phase mode, not the radial one. Dimensional verification: $[\sqrt{V''_{\rm eff}/(\rho_P\ell_P^2)}] = [\sqrt{(J/m^3)/((kg/m^3)\,m^2)}] = [1/\mathrm{s}]$, whence $[(\hbar/c^2)\cdot(1/\mathrm{s})] = [\mathrm{kg}]$ ✓.

Step 6 — Coupling constant $g_{\rm eff}$ at second order. The direct quartic contributes $V^{(4)}_{\rm eff}/12$, but the second-order cubic contribution renormalizes it (Step 3). The derivation proceeds by adiabatic elimination (multiscale) of the equation of motion of the radial mode around the minimum. Writing it in units $\rho_P\ell_P^2 = 1$ with $a \equiv V''_{\rm eff}$, $b \equiv V'''_{\rm eff}$, $c \equiv V^{(4)}_{\rm eff}$:

$$\ddot{\delta} + a\,\delta = -\tfrac{1}{2}b\,\delta^2 - \tfrac{1}{6}c\,\delta^3, \tag{E25a}$$

with the multiscale expansion $\delta = \varepsilon\delta_1 + \varepsilon^2\delta_2 + \varepsilon^3\delta_3$ and slow time $T_2 = \varepsilon^2 t$ where the amplitude $A(T_2)$ lives.

Order ε: $\delta_1 = A\,e^{i\omega_0 t} + \text{c.c.}$, with $\omega_0^2 = a$.

Order ε^2 (modes generated by the cubic). The source $-\frac{1}{2}b\,\delta_1^2$ has a DC component and one at $2\omega_0$, which excite respectively

$$\text{DC mode:}\quad d = -\frac{b\,|A|^2}{a}, \qquad \text{mode } 2\omega_0:\quad p = \frac{b\,A^2}{6a}, \tag{E25b}$$

where p comes from $(a - 4\omega_0^2)\,p = -\frac{1}{2}bA^2$ with $\omega_0^2 = a$, so that $a - 4a = -3a$.

Order ε^3 (non-secularity condition). The resonant terms at ω_0 that determine the amplitude equation are: from the quartic, $-\frac{1}{6}c\cdot 3|A|^2A = -\frac{1}{2}c\,|A|^2A$; and from the crossed cubic $-b\,\delta_1\delta_2$, whose component at ω_0 is $-b(A\,d + A^*p)$, with two channels,

$$\text{DC channel:}\quad -b\,A\,d = +\frac{b^2|A|^2A}{a}\quad(+6/6), \qquad \text{channel } 2\omega_0:\quad -b\,A^*p = -\frac{b^2|A|^2A}{6a}\quad(-1/6), \tag{E25c}$$

whose sum is $+\frac{5}{6}\,b^2|A|^2A/a$ (the $2\omega_0$ channel *subtracts*, being out of phase). The net coefficient of $|A|^2A$ combines quartic and cubic:

$$\text{coef} = -\tfrac{1}{2}c + \tfrac{5}{6}\frac{b^2}{a} = -\tfrac{1}{2}c\left[1 - \frac{5}{3}\frac{b^2}{a\,c}\right]. \tag{E25d}$$

Restoring the normalization to number density ($n_0 = 1/\ell_P^3$, factor $1/(12\,n_0^2)$ of the quartic combinatorics), the complete coupling constant is:

$$\boxed{g_{\text{eff}} = \frac{V_{\text{eff}}^{(4)}\,\ell_P^6}{12}\left[1 - \frac{5}{3}\frac{(V_{\text{eff}}''')^2}{V_{\text{eff}}''\,V_{\text{eff}}^{(4)}}\right]} \tag{E25}$$

The coefficient 5/3 is exact and invariant under the normalization of the amplitude (a rescaling $A \to \lambda A$ multiplies numerator and denominator of the quotient $g_{\text{corr}}/g_{\text{naive}}$ by $|\lambda|^2\lambda$ and cancels), and coincides with the canonical result of the anharmonic oscillator (Landau-Lifshitz, *Mechanics* §28). The static minimization of the potential would give instead a coefficient 3, distinct, because it does not capture the dynamic channel $2\omega_0$; that GP requires 5/3 confirms that that channel is real.

Dimensional verification. $[V_{\text{eff}}^{(4)}\,\ell_P^6] = [J/m^3]\cdot[m^6] = [J\,m^3]$, the contact dimension in 3D; the factor in brackets is dimensionless ✓. The factor $1/(12\,n_0^2) = \ell_P^6/12$ combines the combinatorics of the quartic term ($\delta S^4/24$ differentiated twice) with the conversion of δS^2 to number density $|\psi|^2$ of Step 2.

Numerical verification. With the three derivatives (E24-bis), $V_{\text{eff}}'' = 19.69\,\rho_P c^2$, $V_{\text{eff}}''' = 27.92\,\rho_P c^2$, $V_{\text{eff}}^{(4)} = 855.6\,\rho_P c^2$, the cubic factor is

$$1 - \frac{5}{3}\frac{(27.92)^2}{19.69 \times 855.6} = 1 - \frac{5}{3}(0.04627) = 0.9229,$$

whence

$$g_{\text{eff}} = \frac{855.6\,\rho_P c^2\,\ell_P^6}{12} \times 0.9229 = 65.8\,\rho_P c^2\,\ell_P^6 \approx 5.43 \times 10^{-94}\ \text{J}\cdot\text{m}^3,$$

a correction of -7.7% with respect to the direct quartic ($71.3\,\rho_P c^2\ell_P^6 \approx 5.88 \times 10^{-94}$ J·m^3). The positive sign is preserved ($g_{\text{eff}} > 0$): the self-interaction $|\psi|^4$ remains repulsive and the ground-Pleno condensate remains stable. [**DERIVED-ALGEBRA**]: g_{eff} is a pure ratio of the potential derivatives, zero free parameters.

Step 7 — Euler-Lagrange. Varying $\mathcal{L}_{\text{eff}}$:

$$\boxed{i\hbar\,\partial_t\psi = -\frac{\hbar^2}{2m_{\text{eff}}}\nabla^2\psi + g_{\text{eff}}|\psi|^2\psi} \tag{E26}$$

Closure dimensional verification (the three terms). With $[\psi] = \text{m}^{-3/2}$, $[m_{\text{eff}}] = \text{kg}$ and $[g_{\text{eff}}] = \text{J m}^3$, the three terms of (E26) share units $[\hbar\,\partial_t\psi] = [\hbar^2\nabla^2\psi/m_{\text{eff}}] = [g_{\text{eff}}|\psi|^2\psi] = \text{kg m}^{1/2}\text{s}^{-2}$ (verified symbolically): the equation is dimensionally consistent term by term, which fixes the prefactor of ψ of Step 2 uniquely to $\ell_P^{-3/2}$. The effective mass $m_{\text{eff}} = \sqrt{19.69}\,m_P = 4.4373\,m_P$ coincides with the radial mode of (E21), obtained by an independent route (linearized spectrum): the frequency ω_0 that is factored out in Step 4 is the same as that of the massive mode, which confirms the internal coherence of the reduction.

Conclusion — what this limit means

The implication is ontological, not only formal. That F1 contain Gross-Pitaevskii as an *exact* non-relativistic limit —not as an analogy— means that the same equation that governs Bose-Einstein condensates, superfluids and superconductors in the laboratory governs el Pleno: the universe *is*, in its slow regime, a condensate. The decomposition $\Psi = Se^{i\theta}$ of el Pleno is literally the order parameter of those systems. Three consequences: *(i)* the normalization of the effective field is not free —the reading of the modulus as number density (E-dens) fixes the prefactor $\psi = \delta S/\ell_P^{3/2}$ and closes the three terms of (E26) without fitting; *(ii)* the effective mass of the radial mode coincides with the linearized spectrum by two independent routes, a proof of internal consistency; *(iii)* all laboratory-validated GP phenomenology —Bogoliubov-de Gennes spectra, bright solitons, collapse-rebound in *Bosenovas*— is then **indirect experimental validation** of el Pleno: tabletop experiments that probe the structure of the cosmic vacuum. Status **[DERIVED]** (dimensionally consistent computation, E24–E26), with the complete coupling g_{eff} at second order —the direct quartic plus the cubic renormalization (factor 5/3, E25a–E25d) leave it as a pure ratio of the potential derivatives, without free parameters, **[DERIVED-ALGEBRA]**; as a parametric mapping, conditional on the GP phenomenology, experimentally established.

8.5 The Dirac structure as an effective theory of the phase modes — [DERIVED] / [SLD]

The *structure* of the Dirac equation —first-order operator, Clifford algebra, relativistic energy-momentum relation, spin-$\frac{1}{2}$— is derived as an effective low-energy theory of the phase modes θ_- over the bipartite diamond lattice. The structure is **[DERIVED]**: two routes converge on it. The *band route* (developed here) derives each link —the doublet A/B, the bands and the nodal line, the $SU(2)$ phase, the covariance by $v_F = c$— from the geometry of the medium. The *covariant route* (corpus §V.7.2) establishes it independently of the value of λ_{SO}: the two binary structures of the medium (bipartition A/B and modulus-phase duality) fix a unique representation of the algebra by the Pauli theorem, so that the Dirac form does not depend on any fine parameter. What remains as an open end is the *value* of two constants (λ_{SO} and the masses m), tied to the open ends of fermionic masses and chirality (§15).

Step 1 — doublet and bipartite Bloch Hamiltonian. The bipartition A/B (§9) provides the doublet $\Phi = (\varphi_A, \varphi_B)^T$. Since an excitation on an A site only hops to its four B neighbors (tetrahedral geometry), the Bloch Hamiltonian is strictly off-diagonal:

$$H(\mathbf{k}) = t\begin{pmatrix} 0 & f(\mathbf{k}) \\ f^*(\mathbf{k}) & 0 \end{pmatrix}, \qquad f(\mathbf{k}) = \sum_{j=1}^{4} e^{i\,\mathbf{k}\cdot\mathbf{d}_j}, \qquad E(\mathbf{k}) = \pm\, t\,|f(\mathbf{k})|. \tag{E-Dirac-1}$$

They are the two bands that the bipartition already produces (§3, form factor of $S^{\text{estr}}_{\text{min}}$).

Step 2 — nodal lines. The set where the bands touch is $f(\mathbf{k}) = 0$, with

$$|f(\mathbf{k})|^2 = 4 + 4\cos\tfrac{ak_x}{2}\cos\tfrac{ak_y}{2} + 4\cos\tfrac{ak_x}{2}\cos\tfrac{ak_z}{2} + 4\cos\tfrac{ak_y}{2}\cos\tfrac{ak_z}{2}, \tag{E-Dirac-2}$$

whose zero set is *one-dimensional*: nodal lines, not points. The pure lattice gives a nodal-line semimetal, not the 3D Dirac cone; the ingredient that opens the line into isolated points is missing.

Step 3 — the spin-orbit is forced $\neq 0$. The intrinsic second-neighbor spin-orbit coupling, the only one allowed by $Fd\bar{3}m$ (Fu-Kane),

$$H_{\text{SO}} = i\,\lambda_{\text{SO}} \sum_{\langle\langle ij\rangle\rangle} (\mathbf{d}_{il} \times \mathbf{d}_{lj}) \cdot \boldsymbol{\sigma}, \tag{E-Dirac-3}$$

is forced to be nonzero by the $SU(2)$ character of the mode θ_- (§3). The transport over a plaquette of two non-collinear bonds, with $A_1 = \theta\sigma_x$ and $A_2 = \theta\sigma_y$, gives the holonomy

$$W = e^{iA_1}e^{iA_2}e^{-iA_1}e^{-iA_2} = \mathbb{1} - \theta^2[\sigma_x, \sigma_y] + \mathcal{O}(\theta^3) = \mathbb{1} - 2i\theta^2\sigma_z + \mathcal{O}(\theta^3), \tag{E-Dirac-4}$$

using $[\sigma_x, \sigma_y] = 2i\sigma_z$: proportional to $\boldsymbol{\sigma}$ and distinct from the identity. Hence the equivalence

$$\lambda_{\mathrm{SO}} = 0 \iff \text{holonomy } SU(2) \text{ trivial} \iff \theta_- \text{ lives in } U(1). \tag{E-Dirac-5}$$

Since θ_- lives in $SU(2)$ by derivation —the same property from which the fermions hang—, $\lambda_{\mathrm{SO}} \neq 0$ is forced. It is the *same* holonomy $SU(2)$ that produces the non-abelian structure of Yang-Mills (§9): they are not independent facts, but a single property of the medium read in two sectors.

Step 4 — opening of the node and the Weyl operator. With $\lambda_{\mathrm{SO}} \neq 0$ the nodal line opens into 3D Dirac points. Near a node,

$$H(\mathbf{q}) = v_F\,(q_x\Gamma_1 + q_y\Gamma_2 + q_z\Gamma_3), \qquad \Gamma_1 = \sigma_x \otimes \mathbb{1},\ \Gamma_2 = \sigma_y \otimes \mathbb{1},\ \Gamma_3 = \sigma_z \otimes \sigma_x, \tag{E-Dirac-6}$$

with 4×4 matrices (two sublattices $\times$ two spin components) that satisfy the Euclidean Clifford algebra, verified symbolically,

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}\mathbb{1}_4, \qquad i, j \in \{1, 2, 3\}, \tag{E-Dirac-7}$$

whence $H^2 = v_F^2\mathbf{q}^2\mathbb{1}_4$ and the isotropic linear cone $E = \pm v_F|\mathbf{q}|$.

Step 5 — mass and complete equation. A fourth matrix $\Gamma_0 = \sigma_z \otimes \sigma_z$ ($\Gamma_0^2 = \mathbb{1}$, anticommutes with the three Γ_i), originating in the A–B imbalance (Peierls dimerization), opens a gap. The complete Hamiltonian and its spectrum are

$$H = v_F\,\boldsymbol{\Gamma}\cdot\mathbf{q} + mc^2\,\Gamma_0, \tag{E-Dirac-8}$$

$$H^2 = (v_F^2\,\mathbf{q}^2 + m^2c^4)\mathbb{1}_4 \implies E = \pm\sqrt{v_F^2\,\mathbf{q}^2 + m^2c^4}. \tag{E-Dirac-9}$$

Since the phase modes propagate at velocity c (§6), with $v_F = c$ the relation is exactly $E^2 = c^2\mathbf{q}^2 + m^2c^4$ and the operator is the Dirac equation, with Clifford-Lorentz algebra $\{\Gamma_\mu, \Gamma_\nu\} = 2\eta_{\mu\nu}$. The covariance is not imposed: it is inherited from $v_F = c$ (axiom A3).

The Dirac equation as an effective theory of el Pleno

$$\boxed{(i\hbar\Gamma^\mu\partial_\mu - mc)\Psi_D = 0} \qquad \iff \qquad H = c\,\boldsymbol{\Gamma}\cdot\mathbf{q} + mc^2\,\Gamma_0, \qquad \{\Gamma_\mu, \Gamma_\nu\} = 2\eta_{\mu\nu}. \tag{E-Dirac-10}$$

The doublet comes from the bipartition A/B; the Clifford matrices, from the generators $SU(2)$ of θ_-; the first order, from the spin-orbit opening of the nodal line; the covariance, from $v_F = c$.

Reading: the spin-$\frac{1}{2}$ and the relativistic structure of the electron are not postulated —they emerge from the topology of the lattice. The fermion is a mode of el Pleno, not a fundamental entity; what remains as an open end is the value of the masses, not the form of the equation.

Dimensional verification. In (E-Dirac-9), $[v_F q\,\hbar] = [J]$ and $[mc^2] = [J]$: energy ✓. The covariant form has $[\hbar\Gamma^\mu\partial_\mu] = [mc] = kg\cdot m/s$: momentum ✓.

Epistemic classification. The *structure* of Dirac is **[DERIVED]**: the covariant route (corpus §V.7.2) establishes it from the two binary structures of the medium with a unique representation by the Pauli theorem, without depending on the value of λ_{SO}; the band route (here) confirms it with each link **[DERIVED]** (doublet A/B, bands and nodal line, phase $SU(2)$, $v_F = c$) or **[SLD]**

($\lambda_{SO} \neq 0$ by holonomy, opening of the node). The emergent spin-$\frac{1}{2}$ is **[DERIVED]** as a property of the doublet. The *value* of λ_{SO} and of the masses m is a declared open end; the chirality ($SU(2)_L$ vs $SU(2)_R$) and the number of generations remain open ends. The program affirms the Dirac structure as a derived result and does not claim to have derived masses, chirality or generations.

Conclusion — what it means physically

Matter with spin —the electron, the quarks— is a torsion vibration of the medium. The diamond lattice has two interlaced sublattices (A and B), and a phase mode that oscillates out of phase between them cannot be described with a single number: it needs two coupled components, exactly the Dirac spinor. The first-order structure of the equation, the Clifford algebra and the spin-$\frac{1}{2}$ are not imposed: they are the obligatory form of propagation of an excitation that lives over two sublattices at once. Spin is thus a real geometric property —the way in which the vibration is distributed between the two faces of the crystal of the vacuum—, not an abstract quantum label. What the medium does not yet fix by itself alone (the value of the masses, the chirality, how many generations) remains explicitly as the pending part; what it does deliver is the physical reason that matter be spinorial.

---

9 The gauge sector: Maxwell, Yang-Mills and the charge

9.1 The Maxwell equations from phase circulation — [DERIVED]

The two energies of the vector sector. A perturbation of the phase $\delta\theta$ over the lattice decomposes by tensor rank into scalar (Goldstone), transverse vector (electromagnetism) and transverse tensor (TT mode). The vector sector a_μ admits two orthogonal energies:

(a) Bond energy (magnitude of the stretching $\sum_i |\Psi_n - \Psi_m|^2$): produces a *symmetric* tensor, with the cross term $\partial_i a_j\, \partial_j a_i$ of coefficient $+2$. It is a vector phonon, not a photon.

(b) Oriented circulation of the phase over the plaquettes: produces an *antisymmetric* tensor $F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$.

The coefficient $8/9$ (sum of plaquettes, verified symbolically). The circulation energy is the sum of the square of the circulation $\delta_i^a \delta_j^b F_{ab}$ over the six pairs $i<j$ of tetrahedral bonds, with the normalized vectors $\boldsymbol{\delta}_i = \frac{1}{\sqrt{3}}(\pm1, \pm1, \pm1)$ (each component therefore carries the factor $1/\sqrt{3}$, and the product $\delta_i^a \delta_j^b$ the factor $1/3 \cdot 1/3 = 1/9$):

$$E_{\text{circ}} = \sum_{i<j} \left(\delta_i^a\, \delta_j^b\, F_{ab}\right)^2.$$

Expanding the quadratic form with F_{ab} antisymmetric (sympy), the contributions over the six pairs sum, for each independent component,

$$E_{\text{circ}} = \frac{16}{9}\left(F_{01}^2 + F_{02}^2 + F_{12}^2\right) = \frac{16}{9} \cdot \tfrac{1}{2}\, F_{\mu\nu}F^{\mu\nu} = \frac{8}{9}\, F_{\mu\nu}F^{\mu\nu},$$

where $F_{\mu\nu}F^{\mu\nu} = 2(F_{01}^2 + F_{02}^2 + F_{12}^2)$ was used for an antisymmetric tensor. The $16/9$ already includes the $1/9$ of the bond normalization; the crude 16 is $Z^2 = 4^2$ (square of the coordination). In terms of derivatives, each $F_{12}^2 = (\partial_1 a_2 - \partial_2 a_1)^2$ contributes the diagonal $(\partial_1 a_2)^2$ with coefficient $+16/9$ and the cross term $(\partial_1 a_2)(\partial_2 a_1)$ with $-32/9$, exactly in the ratio -2 that characterizes $F_{\mu\nu}F^{\mu\nu}$:

$$\boxed{E_{\text{mag}} = \frac{8}{9}\, F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu} \tag{E-Maxwell-1}$$

The cross term $-32/9$, opposite to the $+2$ of the symmetric bond sector, is the signature that the object is antisymmetric, genuinely $F_{\mu\nu}$.

Verification by discrete Stokes. That the object is the field tensor and not an artefact is confirmed independently: the oriented circulation of the phase over each tetrahedral plaquette equals the flux of the curl through it —the discrete Stokes theorem— over the six bond pairs of the lattice (verified symbolically). The sum of the six circulations reproduces $E_{\mathrm{circ}} = 8\, F_{\mu\nu}F^{\mu\nu}$ with cross term -32 and diagonal $+16$ (ratio -2), against the bond sector with cross term $+16$ and diagonal $+8$ (ratio $+2$): the two sectors are orthogonal, and only the circulation one carries the antisymmetry of $F_{\mu\nu}$. The decomposition $8 = 2^3$ (spectral quotient of the coordination $Z = 4$) and $9 = 3^2$ (the three dimensions of the average) is a reading of the result, not its origin; the origin is the sum of plaquettes.

The four Maxwell equations. With the Lagrangian $\mathcal{L}_{\mathrm{EM}} = -\frac{8}{9}F_{\mu\nu}F^{\mu\nu}$ (E-Maxwell-1) the four equations come out, they are not postulated:

- **Homogeneous** (Faraday $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$ and $\nabla \cdot \mathbf{B} = 0$). Since $F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is a curl, it identically satisfies the Bianchi identity

$$\partial_{[\lambda}F_{\mu\nu]} = \partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0,$$

  verifiable by substituting $F = \partial a - \partial a$: each term cancels by the commutativity of the partial derivatives. It is not a dynamical law but a geometric identity —that is why there are no magnetic monopoles in the U(1) sector.

- **Inhomogeneous in vacuum** ($\nabla \cdot \mathbf{E} = 0$, $\nabla \times \mathbf{B} = \partial_t \mathbf{E}$). The Euler-Lagrange equation of $\mathcal{L}_{\mathrm{EM}}$ with respect to a_ν,

$$\partial_\mu \frac{\partial \mathcal{L}_{\mathrm{EM}}}{\partial(\partial_\mu a_\nu)} = 0 \implies \partial_\mu F^{\mu\nu} = 0,$$

  where the global coefficient 8/9 cancels upon equating to zero, so that the form of the equation is independent of it (the 8/9 fixes the *normalization* of the field, not the dynamics).

- **With source** ($\partial_\mu F^{\mu\nu} = J^\nu$). The current $J^\nu = \rho_P c^2 \ell_P^2 S^2 \partial^\nu \theta$ is the same U(1) Noether current (E7c) of the charged solitons; coupled to the vector sector, it appears as a source term. The coefficient remains tied to the hypercharge embedding (**[SLD]**, open end §15).

Why electromagnetism is complex: the origin of i. This is the conceptual piece of the result. The phase lives in $U(1)$: $\Psi = Se^{i\theta}$, and the imaginary unit i is the generator of rotations of the phase circle, that is, a rotation of 90° in the plane $(\mathrm{Re}\,\Psi, \mathrm{Im}\,\Psi)$. When the phase circulates over a plaquette, that 90° turn is what converts the *circulation* of phase (an oriented gradient) into a *perpendicular field* —exactly the relation $\mathbf{E} \perp \mathbf{B}$, both perpendicular to the propagation. Electromagnetism is not complex by mathematical convention: it is so because it *is* the dynamics of a $U(1)$ phase field, and the complex structure is intrinsic to the circle on which the phase lives. Light is, literally, a wave of phase circulation of el Pleno propagating at c.

Conclusion — electromagnetism, derived

The four Maxwell equations come out of el Pleno, they are not postulated. The field tensor $F_{\mu\nu}$ is the oriented circulation of the phase over the plaquettes of the lattice, with coefficient 8/9 *computed* —sum of the six tetrahedral circulations, verified by the discrete Stokes theorem, without a free parameter. The homogeneous ones (Faraday, $\nabla \cdot \mathbf{B} = 0$) are the Bianchi identity that $F = \mathrm{d}a$ satisfies identically —that is why there are no monopoles in the U(1) sector; the inhomogeneous ones come out of Euler-Lagrange; the source is the U(1) Noether current of the charged solitons. And the complex structure of electromagnetism has an origin: the i of $\Psi = Se^{i\theta}$ is the 90° turn that converts phase circulation into a perpendicular field, the relation $\mathbf{E} \perp \mathbf{B}$. The only open coefficient is the coupling of the source, tied to the hypercharge embedding (declared, §15).

9.2 Consistency theorem: light propagates at the change ceiling — [DERIVED]

Once Maxwell is derived, a question arises about the architecture of primaries: if Maxwell is derived and c appears in Maxwell, could c be derived from Maxwell and cease to be primary? The answer is negative by circularity; the positive content is the theorem that follows.

Theorem — speed of light = change ceiling

The massless transverse vector mode of the gauge sector, with Lagrangian $\mathcal{L}_{\rm EM} = -\frac{8}{9}F_{\mu\nu}F^{\mu\nu}$, $F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ (E-Maxwell-1), has phase and group velocity exactly equal to c, the primary change ceiling, at every infrared scale. **[DERIVED]**

Proof. (i) The wave equation. The Euler-Lagrange equation of $\mathcal{L}_{\rm EM}$ is $\frac{8}{9}\partial_\mu F^{\mu\nu} = 0 \Rightarrow \partial_\mu F^{\mu\nu} = 0$: the global coefficient 8/9 factors out and cancels —it does not affect the propagation dynamics, only the normalization of the energy. In the Lorenz gauge ($\partial_\mu a^\mu = 0$),

$$\Box\, a^\nu = 0, \qquad \Box = \frac{1}{c^2}\partial_t^2 - \nabla^2,$$

with the d'Alembertian of the medium, built with the same c of the kinetic term of el Pleno.

(ii) Dispersion relation. For a plane wave $a^\nu \sim \exp[i(kx - \omega t)]$:

$$\Box\, a^\nu = 0 \implies -\frac{\omega^2}{c^2} + k^2 = 0 \implies \omega = ck,$$

whence $v_{\rm phase} = \omega/k = c$ and $v_{\rm group} = d\omega/dk = c$, both exactly c, without a factor. The coefficient 8/9, which does fix the normalization of the magnetic energy (E-Maxwell-1), does not appear in the velocity.

(iii) Absence of dispersion in the light branch. The complete phase equation (E19) includes the flexural correction:

$$\Box\,\delta\theta - \frac{\ell_P^2}{20}\Box^2\delta\theta = 0 \implies \Box\Big(1 - \frac{\ell_P^2}{20}\Box\Big)\delta\theta = 0,$$

which factors into two branches: (a) $\Box\,\delta\theta = 0$, the massless infrared mode (light), with $\omega = ck$ exact and without dispersion; (b) $(1 - \frac{\ell_P^2}{20}\Box)\delta\theta = 0$, a massive mode at the ultraviolet scale $\sim 1/\ell_P$ (Lee-Wick pole), which is not light. The electromagnetic branch is (a): $v = c$ at every infrared scale. ■

Non-triviality. The theorem is not a tautology. (i) A massive mode does not saturate the ceiling: the modulus mode (matter, E18) obeys $\Box\,\delta S + \mu_S^2\,\delta S = 0$, with $\omega^2/c^2 = k^2 + \mu_S^2$ and $v_g = ck/\sqrt{k^2 + \mu_S^2} < c$; only the massless mode propagates at c, and that light does so depends on $F_{\mu\nu}$ being a curl and the U(1) symmetry protecting the zero mass of the photon. (ii) A sector with its own rigidity $c_{\rm EM} \neq c$ would give two limiting velocities, in violation of A3. (iii) The velocity is c because the vector sector a_μ is a perturbation of the *same* phase θ whose kinetic term $\frac{1}{2}\rho_P c^2 \ell_P^2 S^2 (\partial\theta)^2$ defines c: light has no medium of its own, it inherits c from the term that defines it.

Corollary: c cannot be derived from Maxwell. For two independent reasons. *Circularity*: the standard derivation $c^2 = 1/(\varepsilon_0\mu_0)$ presupposes ε_0, μ_0 prior to c; in PIU the vacuum constants are properties of el Pleno that already contain c (the Lagrangian is built over $F_{\mu\nu}[\theta]$, and the dynamics of θ carries c in its kinetic term), so that deriving c from Maxwell would return c to itself —a true but empty identity, like measuring a ruler with itself. *Ontological inversion*: c is not the speed of light but the change ceiling of the energy, prior to space, to time and to light (from c emerge time as accumulated change and causality A3); light is the consequence —the massless mode that saturates the ceiling—, and deriving c from Maxwell would put the consequence as the origin. Moreover c is the standard of commensurability ($E = Mc^2$, spacetime, sectors); a measurement standard does not measure itself. The theorem is the correct exploitation of the question: c is not derived, it is verified that light —derived— respects the ceiling that c embodies. The constancy of the speed of light, a postulate in relativity, is here a result.

9.3 Yang-Mills from the non-abelian circulation — [DERIVED]

Applying the same mechanism to the matrix phase $\theta_- \in SU(2)$, the bond factors are $U_i = \exp(iA^a_\mu T^a \delta^\mu_i)$ with non-commuting generators $[T^a, T^b] = i\varepsilon^{abc}T^c$. The difference from $U(1)$ is that the transport over a closed plaquette no longer cancels: the holonomy $W = U_1 U_2 U_1^{-1} U_2^{-1}$, expanded by Baker-Campbell-Hausdorff, leaves a residue proportional to the commutator,

$$\ln W = -[A_1, A_2] + \mathcal{O}(\varepsilon^3),$$

which does not vanish because the A_μ do not commute. That residue is the extra term of the field tensor:

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]} \tag{E-YM-1}$$

The term $i[A_\mu, A_\nu] = -f^{abc}A^a_\mu A^b_\nu T^c$ is born from the non-commutativity; the abelian limit ($[A, A] = 0$) recovers Maxwell exactly —Yang-Mills is Maxwell when the phase ceases to commute. The three gauge structures of the Standard Model are the same phase-circulation mechanism: $\theta_+ \in U(1)$ (electromagnetism), $\theta_- \in SU(2)$ (weak), $\mathbb{Z}_3$ vortices (color $SU(3)$).

9.4 Quantization of the charge and beta function — [DERIVED]

The phase lives on a circle ($\theta \in [0, 2\pi)$); the finite-energy configurations tend to a constant value at infinity, and the integer winding (number of turns) is a topological invariant. The conserved U(1) charge is proportional to it:

$$Q = n\,Q_0, \qquad Q_0 = \hbar, \qquad n \in \mathbb{Z}.$$

The difference of beta-function coefficients of $U(1)_Y$ and $SU(2)$ is $b_Y - b_2 = 29/4$ **[DERIVED]**. The mixing angle $\sin^2\theta_W$ is **[SLD]** conditional on the hypercharge embedding $Y(\Phi) = 1/2$ (open end, §15).

Conclusion — the harvest: the gauge sector

The conserved charge of the previous link unfolds: Maxwell and Yang-Mills are the same word of the medium —phase circulation over plaquettes— distinguished only by whether the phase commutes or not. The quantization of the charge is topological (integer winding), which explains why the electric charge comes in integer multiples, a fact that the Standard Model incorporates without explaining.
What it means physically: the fundamental forces are not separate entities added to the universe. Electromagnetism is the way in which el Pleno propagates the twists of its own phase; light is a wave of that circulation. The factor 8/9, the complex structure of electrodynamics, the quantization of the charge and the unification of the three gauge forces in a single mechanism are not coincidences of the formalism: they are consequences of there being a single phase field over a single lattice. What remain open are the value of the coupling g and the hypercharge embedding, but the ***form*** *of the gauge sector —why there are forces of the Maxwell-Yang-Mills type— is derived.*

---

10 Solitons as particles

10.1 Topological classification and statistics — [DERIVED]

A material particle is, in this program, a *localized and stable configuration of el Pleno*: a region where the field Ψ departs from the background $\Psi_0 = S^{\text{estr}}_{\text{min}} e^{i\theta_0}$ and returns to it at infinity. That such configurations exist, be stable and have a definite statistics is not postulated: it follows from the topology of the field and the form of F1.

Two topological classes. According to the topology of the internal configuration space of the soliton, there are two families, and the distinction *fixes the statistics*:

- **Topological class (skyrmions).** Over the target $SU(2) \cong S^3$ (antisymmetric mode θ_-), the finite-energy configurations with $U \to \mathbb{1}$ at infinity are classified by $\pi_3(SU(2)) = \pi_3(S^3) = \mathbb{Z}$. The integer is the **baryon number** B, a conserved topological charge: it cannot change continuously, which gives stability without need of an external confining force.

- **Non-topological class (Q-balls).** Over the contractible sub-target $[S_{\min}, S_{\max}]$ ($\pi_n = 0$), there is no topological charge; the stability is sustained dynamically by the U(1) Noether charge (Coleman conditions).

The hedgehog ansatz ($B = 1$). The fundamental configuration with baryon number unity is the "hedgehog", where the internal orientation of the field points radially:

$$U(\mathbf{x}) = \cos f(r) + i\,\boldsymbol{\sigma}\cdot\hat{\mathbf{x}}\,\sin f(r), \qquad f(0) = \pi,\ f(\infty) = 0,$$

with $f(r)$ the radial profile that interpolates between the north pole ($f = \pi$ at the center) and the south pole ($f = 0$ at infinity) of S^3, covering it *exactly once*: that unique covering *is* $B = 1$. The profile $f(r)$ is obtained by minimizing the energy over the non-linear field equations (F2).

The baryon number as a topological integral. B is not a label but the degree of the map $\mathbb{R}^3 \cup \{\infty\} \cong S^3 \to SU(2) \cong S^3$, given by the integral of the baryon density with $L_i = U^{-1}\partial_i U$:

$$B = -\frac{1}{24\pi^2}\int \varepsilon^{ijk}\,\mathrm{Tr}(L_i L_j L_k)\,d^3x.$$

For the hedgehog ansatz this integral reduces to a single dimension,

$$B = -\frac{2}{\pi}\int_0^\infty \sin^2 f\,\frac{df}{dr}\,dr = \frac{2}{\pi}\int_0^\pi \sin^2 f\,df = \frac{2}{\pi}\cdot\frac{\pi}{2} = 1,$$

where the change of variable uses $f(0) = \pi$, $f(\infty) = 0$ (verified symbolically). The result is an *integer* independent of the concrete profile $f(r)$: any continuous deformation of the field leaves it at 1 —that invariance is the topological stability.

Stability: the Derrick balance. Derrick's theorem (1964) warns that a pure scalar soliton in 3D is unstable —either it collapses to zero size or it dilutes. F1 evades it because it combines *three* terms with opposite tendencies that balance at a finite size:

- the **kinetic** $\rho_P c^2 \ell_P^2 (\partial\Psi)^2$ favors *expanding* (its energy decreases as the size grows);

- the **potential** $V(S)$, with its Gent hard wall at $S_{\max}$, favors *contracting* (a compact profile avoids saturation);

- the **flexural** $-\gamma_\kappa(\Box S)^2$ prevents the *collapse* to zero size (it penalizes infinite curvature).

The balance of the three fixes a characteristic scale of the order of ℓ_P by dimensionless factors of F1: the soliton has its own size, neither pointlike nor diffuse. This is what makes matter have finite extension.

The profile equation and the mass. The profile $f(r)$ minimizes the energy functional of the hedgehog. With the dimensionless variable $\tilde{r} = r/\ell_P$ and the structure of F1 (kinetic + Skyrme-type flexural + potential), the radial energy density has the form

$$E[f] = 4\pi\,\rho_P c^2 \ell_P^3 \int_0^\infty \Big[\underbrace{\tfrac{1}{2}\tilde{r}^2 f'^2 + \sin^2 f}_{\text{kinetic}} + \underbrace{\sin^2 f\Big(f'^2 + \tfrac{\sin^2 f}{2\tilde{r}^2}\Big)}_{\text{flexural (stabilizes)}} + \underbrace{\tilde{r}^2\,\mathcal{V}(f)}_{\text{potential}}\Big]\,d\tilde{r},$$

and the condition $\delta E/\delta f = 0$ gives the profile equation, a non-linear second-order ODE:

$$(\tilde{r}^2 + 2\sin^2 f)f'' + 2\tilde{r}f' + \sin 2f \left(f'^2 - 1 - \tfrac{\sin^2 f}{\tilde{r}^2}\right) - \tilde{r}^2\,\mathcal{V}'(f) = 0,$$

with $f(0) = \pi$, $f(\infty) = 0$. Its numerical solution, substituted into $E[f]$, gives a pure number $\mathcal{E}_0 = \mathcal{O}(1)$ of order unity, so that the mass is $M_Q = \mathcal{E}_0\, M_P/S_{\mathrm{max,eff}}$. With the normalization of the program ($\mathcal{E}_0 \to 1$ by the reading of $S_{\mathrm{max,eff}}$):

$$M_Q = \frac{M_P}{S_{\mathrm{max,eff}}} \approx \frac{2.18 \times 10^{-8}\,\mathrm{kg}}{\sqrt{1.4956}} \approx 9.98 \times 10^{18}\ \mathrm{GeV}/c^2, \tag{E27}$$

with $M_P = \sqrt{\hbar c/G}$ the Planck mass. The fundamental soliton is ultra-heavy ($\sim M_P$): it is the *simplest* configuration, not the one identified with the proton —that is a composite cluster, developed below, whose *structure* (three vortices, fractional charge, confinement) emerges from the topology. Its absolute hadronic mass, in contrast, is not derived: $m_p c^2 \approx 938.3$ MeV is the **only calibrated energy scale** of the hadronic sector, analogous to Λ_{QCD} in QCD (status [**CALIBRATED**]). What the program derives is the *structure* and the *ratios* (split Δ-N, radius, charges), not the absolute scale.

Spin-statistics, derived (Finkelstein-Rubinstein). The spin-statistics theorem is not postulated: it *emerges* from the topology. The idea of Finkelstein-Rubinstein is that rotating a soliton by 2π is not the identity if its configuration space has non-contractible loops. For skyrmions of charge B, the configuration space $\mathcal{Q}_B$ (maps of degree B with boundary condition) has fundamental group

$$\pi_1(\mathcal{Q}_B) = \mathbb{Z}_2 \quad (B \neq 0),$$

so that the closed loop that describes the physical rotation of 2π of the soliton is the non-trivial generator of that $\mathbb{Z}_2$: it cannot be contracted to a point. The quantization over a space with $\pi_1 = \mathbb{Z}_2$ admits two sectors —even or odd wave function under the loop— and the physical sector is fixed by the degree: the rotation of 2π produces on the wave function of the soliton the factor

$$\chi_{\mathrm{FR}} = (-1)^B: \qquad B \text{ odd} \to -1 \text{ (fermion)}, \quad B \text{ even} \to +1 \text{ (boson)}.$$

For the fundamental baryon $B = 1$ the factor is -1: **mathematically obligatory fermionic quantization** (Witten 1983; explicit computation by Krusch 2003). A Q-ball, with contractible configuration space ($\pi_1 = 0$), only admits the trivial sector, factor $+1$: boson. The statistics is not a choice of the model nor an added axiom —it is a consequence of the distinct topology of each class.

Sub-target	**Topology**	χ_{FR}	**Statistics**
$SU(2) \cong S^3$	$\pi_3 = \mathbb{Z}$	$(-1)^B = -1$	Fermionic
$[S_{\min}, S_{\max}]$	$\pi_n = 0$	$+1$	Bosonic

Reading: that ordinary matter be fermionic —the Pauli exclusion principle, on which all of chemistry rests— ceases to be a postulate. It is the imprint of the matter particles being topological knots (B odd) in the field of el Pleno, and the topology of the knot forces the statistics.

10.2 The Borromean baryon and the $\mathbb{Z}_3$ charge — [DERIVED] / [SLD]

The observed baryon (proton, neutron) is not the fundamental monolithic soliton, but a *richer sub-structure* of the same $B = 1$. By the Gudnason-Nitta theorem (2020), the topological degree $B \in \pi_3(SU(2))$ is equivalent to the linking number (linking number) of the preimages under the Hopf map, and admits a decomposition into vortex winding:

$$B = \mathrm{Link}(L_1, L_2, L_3) = \sum_{\mathrm{clusters}} \prod_{i=1}^{3} w_i. \tag{E28}$$

Applied to the baryon of el Pleno, the integer winding $B = 1$ splits into **three vortices of fractional winding** $1/3$, which may have distinct profiles without altering the total:

$$B = 1 = w_1 + w_2 + w_3 = \tfrac{1}{3} + \tfrac{1}{3} + \tfrac{1}{3}.$$

Those three vortices of $1/3$ *are* the three quarks: neither their number nor their fractional charge is postulated, they come from the fact that the unit linking number factorizes into three thirds.

The $\mathbb{Z}_3$ arithmetic and the $2+1$ asymmetry. By the Borromean supercounterfluidity theorem (Blomquist-Syrwid-Babaev 2021), the *stable* configurations of the cluster satisfy a congruence modulo $N = 3$ over the **integer numerators** of the charges (in units of $1/3$), not over the fractions:

$$\sum_{j=1}^{3} q_j \equiv 0 \pmod 3. \tag{E29}$$

- **Proton** $(+\frac{2}{3}, +\frac{2}{3}, -\frac{1}{3}) \to q = (2, 2, -1)$: $\sum = 3 \equiv 0$ ✓stable; total charge $\sum q_j/3 = +1$.
- **Neutron** $(+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}) \to q = (2, -1, -1)$: $\sum = 0 \equiv 0$ ✓stable; total charge 0.

The complete structure of the nucleon emerges without postulates: the fractional charges $\pm\frac{2}{3}, \pm\frac{1}{3}$, their number (three), the $2+1$ asymmetry between quarks, and the confinement as a $\mathbb{Z}_3$ center obstruction —why the quarks do not separate is the same topological reason that the Borromean rings do not unlink pairwise.

Reading: the proton is not a bag of three point particles put in by hand. It is a single topological knot of el Pleno whose unit linking is realized, in the most stable way, as three sub-vortices knotted à la Borromeo. The fractional charge of the quark and its confinement —two of the strangest facts of the Standard Model— are consequences of that linking geometry, not axioms.

10.3 Rotational tower and the Δ-N split — [DERIVED]

Collective-coordinate quantization. The classical skyrmion is invariant under internal rotations; upon quantizing that freedom (collective coordinate $A(t) \in SU(2)$), the rotational Lagrangian is that of a rigid rotor,

$$L_{\mathrm{rot}} = \tfrac{1}{2}\Theta_p\,\omega^2, \qquad H_{\mathrm{rot}} = \frac{\mathbf{J}^2}{2\Theta_p},$$

with Θ_p the moment of inertia of the soliton and $\mathbf{J}$ the spin. The eigenvalues of $\mathbf{J}^2$ are $\hbar^2 J(J+1)$, so that each rotational level has energy $\hbar^2 J(J+1)/(2\Theta_p)$. Taking the nucleon ($J = \frac{1}{2}$) as reference and subtracting its value $J(J{+}1) = \frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4}$:

$$m(J) = m_N + \frac{\hbar^2}{2\Theta_p}\big[J(J+1) - \tfrac{3}{4}\big]. \tag{E30}$$

The half-integer spin is *forced* by the fermionic quantization of Finkelstein-Rubinstein ($B = 1$): the skyrmion only admits $J = \frac{1}{2}, \frac{3}{2}, \ldots$, not integer J. The tower is, therefore, a consequence of the same topology that gives the statistics.

Moment of inertia. For an approximately uniform baryon-charge distribution in the radius R_p, the moment of inertia is $\Theta_p = m_p R_p^2/3$ (imported Skyrme framework). The split between $J = \frac{3}{2}$ (Δ) and $J = \frac{1}{2}$ (N):

$$\Delta m = \frac{\hbar^2}{2\Theta_p}\Big[\tfrac{3}{2}\cdot\tfrac{5}{2} - \tfrac{1}{2}\cdot\tfrac{3}{2}\Big] = \frac{\hbar^2}{2\Theta_p}\cdot 3,$$

and substituting $\Theta_p = m_p R_p^2/3$:

$$\boxed{\Delta m = \frac{9\hbar^2}{2m_p R_p^2} = \frac{9(\hbar c)^2}{2m_p c^2 R_p^2}} \tag{E31}$$

Numerical verification. With $\hbar c = 197.327$ MeV·fm, $m_p c^2 = 938.272$ MeV, $R_p = 0.8409$ fm:

$$\Delta m = \frac{9 \times (197.327)^2}{2 \times 938.272 \times (0.8409)^2} = 264.1 \text{ MeV}.$$

Observed $m_\Delta - m_N \approx 293.08$ MeV: agreement to 90%, with m_p as the only calibrated input. The residual 10% is not a structural error: it is the expected correction of the non-rigid-rotor model —centrifugal deformation and vibrational/Coriolis modes, standard physics of the Skyrme model— over the rigid-rotor approximation that gives the dominant term. The proton radius is an independent prediction $R_p^{\text{pred}} = \xi_{\text{PIU}}/2 = 0.851$ fm against 0.8409 fm observed (98.8%). The complete tower (base factor 88.0 MeV, verified):

J	$J(J+1) - 3/4$	$m(J)$ predicted	Observed	Agreement
1/2	0	938.3	$N(939)$	calibr.
3/2	3	1202.4	$\Delta(1232)$	97.6%
5/2	8	1642.6	$N^*(1680)$	97.8%
7/2	15	2258.8	$\Delta^*(1950)$	84.1%

Conclusion — what it means physically

Matter is not an ingredient added to the universe: it is the knotted form of el Pleno itself. A particle is a localized and stable configuration of the field, and everything that characterizes it emerges from its topology and the form of F1, without postulates: its **existence and stability** (Derrick balance between kinetic, potential and flexural, which gives it finite size); its **statistics** (Finkelstein-Rubinstein: B odd $\to$ fermion, whence the Pauli principle and, with it, the structure of chemistry); its **charge** (topological winding, quantized in integers); and, for the baryon, its **three-quark sub-structure** with fractional charges, 2+1 asymmetry and confinement, all coming from the Borromean $\mathbb{Z}_3$ linking. The hadronic masses and the rotational tower are predicted with a single calibrated scale (m_p), to 90–98%. The spin-$\frac{1}{2}$, the fractional charge and the confinement —three of the facts that the Standard Model incorporates without explaining— are here geometric consequences. What remains as an open end is the *value* of the individual masses and the rotational framework, imported from Skyrme; the **form** of matter is derived.

---

11 Quantitative cosmology

11.1 The dark matter / baryonic ratio $\sqrt{3}\pi$ — [DERIVED] / [SLD]

$$\boxed{\frac{\Omega_{MO}}{\Omega_M} = \sqrt{3}\,\pi \approx 5.4414} \tag{E37}$$

The ratio is computed as the quotient of partition functions $\mathcal{Z}_{MO}/\mathcal{Z}_M$ at the freeze-out temperature, integrating over the topological configurations of each sector. Three pillars fix it; the important thing is that none of the three is a free assumption —each follows from already-established structure.

Pillar 1 — the weight is proportional to the area (derived, not postulated). The key claim —that $\mathcal{Z}_i \propto A_i$, linear in the area— is not postulated: it follows from the Gent wall. The potential $V''_{\rm topo}(S) \propto (S_{\rm max,geom} - S)^{-2}$ diverges as the hard bound is approached (§4.3), which *confines* the modulus fluctuations to a surface shell of thickness $\xi \to 0$ around the defect. In consequence the log-determinant of the fluctuations (the contribution to $\ln \mathcal{Z}$) scales with the *area* of that shell, not with the volume: $\mathcal{Z}_i \propto A_i \phi$. The dominance of surface modes is, then, a consequence of the Gent divergence of link 6, not a hypothesis. The prefactor ϕ is the vibrational density of states of the ground-Pleno ($v_s = c$, cutoff $1/\ell_P$), *identical* for M and MO because they are solitons of the *same* ground; what distinguishes them (topology, moduli) is accounted for separately, so that $\phi_{MO}/\phi_M = 1$ exact and ϕ cancels in the quotient. **[SLD]**.

Pillar 2 — spinor factor 1/2. The quotient of invariant volumes $\mathrm{Vol}(SO(3))/\mathrm{Vol}(SU(2)) = \pi^2/(2\pi^2) = 1/2$ (Peter-Weyl, $SO(3) \cong SU(2)/\mathbb{Z}_2$): ordinary matter (fermionic, lives in $SU(2)$) undergoes the reduction 1/2 upon projecting to $SO(3)$ observables; the dark one (scalar, in $SO(3)$) does not. **[DERIVED]**.

Pillar 3 — geometric assignment. The defects with conserved topological charge (M, $B = 1$) anchor to the local Voronoi cell of the diamond lattice (tetrahedron, $A_T = (8\sqrt{3}/3)R^2$); the chargeless ones (MO, Q-balls) expand isotropically (sphere, $A_S = 4\pi R^2$). The two forms *emanate from* $\mathcal{L}_{F1}$, they are not postulated: the chargeless Q-ball ($B = 0$, global Noether charge, unwound phase) has a free boundary and minimizes the isotropic gradient term $\frac{1}{2}\rho_P c^2 \ell_P^2 |\nabla S|^2$ at fixed volume → sphere (isoperimetric theorem); the charged soliton ($B = 1$, wound phase, winding = holonomy) has its boundary *pinned* to the nodes of the substrate —it cannot be rounded off without unwinding the charge, forbidden by topological conservation— and remains confined to the Voronoi cell of the $Z = 4$ node → tetrahedron. The Kibble-Zurek transition fixes *how many* defects are nucleated, not *what shape* each takes; the shape is selected by the topology of F1. **[DERIVED]**.

Assembly (verified symbolically):

$$\frac{\Omega_{MO}}{\Omega_M} = \frac{A_S \cdot 1}{A_T \cdot (1/2)} = \frac{2A_S}{A_T} = \frac{2 \cdot 4\pi R^2}{(8\sqrt{3}/3)R^2} = \frac{8\pi}{8\sqrt{3}/3} = \frac{3\pi}{\sqrt{3}} = \sqrt{3}\,\pi.$$

Confrontation. Planck 2018: $\Omega_{\rm dm}/\Omega_b = 5.375 \pm 0.077$; difference 1.23%, equivalent to 0.86σ, without free parameters.

Against what it is confronted: robustness hierarchy of the "Planck values"

Caution that governs the cosmological confrontation (continues the principle of §11.9). The "Planck values" are not of the same type: the acoustic angle θ_* and the *physical densities* $\omega_b = \Omega_b h^2$, $\omega_{\rm dm} = \Omega_{\rm dm} h^2$ are read in a nearly model-independent way from the peak heights; in contrast H_0, Ω_m, Ω_Λ, σ_8 are *outputs* of the six-parameter ΛCDM fit. The reading has two sides. The **strength, which must not be minimized:** the ratio $\Omega_{MO}/\Omega_M = \omega_{\rm dm}/\omega_b$ is the *most robust* confrontation of the sector —the factor h^2 cancels and it is compared against physical densities— and that a pure-geometry number fall to 0.86σ of it *without a free parameter and without looking at the datum* is an improbable coincidence under the null hypothesis (Bayes factor, §15), genuine cross-validation, not an artefact; that the target come out of a fit does not make it arbitrary, since the peaks fix it with little freedom. The **caution**, at the other extreme: $\Omega_{EO} = f_c$ and $\Omega_m = 1 - f_c$ are of the derived type (they share the model dependence of Ω_Λ); their agreement is real and counts in favor, but their target is a fit parameter, not a raw datum. The rule is not to distrust coincidences —they are the evidence— but to grade their weight by the robustness of the target. One avoids *calibrating* against an interpretation; one preserves the success of *coinciding* with it without seeking it.

The objection: is the geometry forced? The strongest criticism against (E37) is not that the computation fails —the algebra is exact— but that the *input geometry* could be chosen: the formula combines the area of a sphere and that of the Voronoi of a tetrahedron; is the sphere/tetrahedron choice forced, or is it one of several that coincidentally reproduces the datum? The sphere is inevitable (isotropy ⇒ sphere, by definition). All the force falls on whether the lattice of the substrate *must* be the diamond $Z = 4$. The distinction D1-a/D1-b (link 5) situates the weight of the argument: the **tetrahedral coordination** $Z = 4$ **is a theorem**, not a conjecture —the only arrangement of four nearest neighbors compatible with the isotropy of A1 (D1-a, via spherical 2-design)— and the **cubic stacking is derived** (D1-b, minimum of the isotropic flexural functional: fourth-order uniaxial anisotropy 0 cubic vs 4/81 wurtzite). $\sqrt{3}\pi$ depends on the tetrahedral coordination and the A/B bipartition —both fixed— and the geometry of the lattice is derived in its two pieces. The precise statement is, then, "$\sqrt{3}\pi$ rests on a derived tetrahedral coordination (D1-a), a derived cubic stacking (D1-b) and an axiomatic bipartition". The status of (E37) is **[DERIVED]** in its algebra and **[SLD]** in its input geometry: the weight of the argument falls on an entirely derived lattice and an axiomatic bipartition, without a pending geometry open end.

Conclusion — the harvest: cosmology

F1 taken to the whole universe: the proportion of dark to baryonic matter —in ΛCDM, two free parameters fitted separately— is here a pure number of geometry, two areas (sphere vs tetrahedron) and a spinor factor 1/2, with the surface weight derived from the Gent wall. The agreement at 0.86σ with zero fitting is one of the two firm cosmological predictions of the program, both inherited from the diamond lattice of link 5; with the coordination $Z = 4$ already closed (D1-a), its only residual limitation is the polytypy of the stacking (D1-b), of low severity.

11.2 Effective Friedmann equations and $w(z)$ — [DERIVED] / [SLD]

On FLRW, the background $\bar{S}(t)$ obeys (restriction of F2 to the homogeneous background):

$$\ddot{\bar{S}} + 3H\dot{\bar{S}} + \frac{1}{\rho_P \ell_P^2} V'_{\text{eff}}(\bar{S}) = 0,$$

with $\rho_{\text{Pleno}} = \frac{1}{2}\rho_P \ell_P^2 \dot{\bar{S}}^2 + V_{\text{eff}}$, $p_{\text{Pleno}} = \frac{1}{2}\rho_P \ell_P^2 \dot{\bar{S}}^2 - V_{\text{eff}}$. Substituting into the emergent Einstein equations (E35) with G from (E3):

$$H^2 = \frac{8\pi G}{3}(\rho_{\text{Pleno}} + \rho_M + \rho_{MO}), \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i(\rho_i + 3p_i),$$

identical in form to the standard ones (the "components" are configurations of the same medium, G derived). The equation of state is the quotient $w = p_{\text{Pleno}}/\rho_{\text{Pleno}}$; writing the kinetic term as $K \equiv \frac{1}{2}\rho_P \ell_P^2 \dot{\bar{S}}^2$ and defining $\delta(t) \equiv K/V_{\text{eff}}(\bar{S}) \geq 0$,

$$w = \frac{p_{\text{Pleno}}}{\rho_{\text{Pleno}}} = \frac{K - V_{\text{eff}}}{K + V_{\text{eff}}} = \frac{\delta V_{\text{eff}} - V_{\text{eff}}}{\delta V_{\text{eff}} + V_{\text{eff}}},$$

whence, cancelling V_{eff},

$$\boxed{w(t) = \frac{\delta(t) - 1}{\delta(t) + 1}} \tag{11.1}$$

and the gravitational coupling $\kappa = c^4/(2G)$. In the relaxed limit ($\dot{\bar{S}} \to 0$, $\delta \to 0$), $w \to -1$ exact. The canonical kinetics guarantees $w \geq -1$, since $w+1 = 2K/(K+V_{\text{eff}}) = \dot{\bar{S}}^2/(\frac{1}{2}\dot{\bar{S}}^2 + V_{\text{eff}}/(\rho_P \ell_P^2)) \geq 0$: the phantom region ($w < -1$) is structurally vetoed, without need to impose it.

Conclusion — what it means physically

The entire history of the universe —its expansion, its content, its acceleration— is the history of a single medium changing configuration. What cosmology calls "matter", "radiation" and "dark energy" are not separate ingredients thrown into an empty container, but distinct states of the same Pleno: how much its modulus moves ($\dot{S}$) against how much tension it stores ($V_{\rm eff}$) fixes the equation of state at each epoch. From this comes, without fitting, that the universe can never behave like phantom energy ($w < -1$): that region is physically forbidden because it would require negative kinetic energy, something a real medium does not have. The Friedmann expansion is, literally, the medium breathing between motion and tension.

11.3 Dark energy $w = -1$ — [DERIVED] / [SLD]

The equation of state of the ground-Pleno is not $w = -1$ by postulate, but the result of the derived relation $w(t) = (\delta - 1)/(\delta + 1)$ (§11.2), with $\delta = K/V_{\rm eff} \geq 0$. The late cosmological sector is the phase condensate at $S_{\rm min}$ with $\dot{\theta}_0 = 0$ and the radial modulus frozen (§11.9): $\delta = 0$ *structurally*, so that $w = -1$ *exact* is the present value, not only an asymptotic attractor. The positivity of the canonical kinetics also vetoes the phantom region ($w + 1 \geq 0$), and no internal mechanism displaces w from -1 (the two candidates are excluded by algebra, below). The value $w = -1$ is geometric —the symmetry $T_{\mu\nu} \propto g_{\mu\nu}$ of the ground state—, not a calibrated number.

In the relaxed limit, with $p = -\rho$, the energy-momentum tensor of the fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\, g_{\mu\nu}$ reduces to

$$T^{(w=-1)}_{\mu\nu} = (\rho - \rho)u_\mu u_\nu + \rho\, g_{\mu\nu} = \rho\, g_{\mu\nu} :$$

the four-velocity disappears, the dark energy has no dynamical rest frame —it is a homogeneous structural property (DE-Curtain). The acceleration:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho(1 + 3w) = -\frac{4\pi G}{3}\rho(1 - 3) = +\frac{8\pi G}{3}\rho > 0.$$

Density: $\rho^{(0)}_{\rm DE} = V(S^{\rm estr}_{\rm min}) = \rho_P (S^{\rm estr}_{\rm min})^2 = 0.5229\,\rho_P$. Matter is the excess $\rho_{\rm matter} = \rho_P[S^2 - (S^{\rm estr}_{\rm min})^2]$. Thermodynamically, $\rho + p = 0$ implies entropy $s = 0$: the relaxed vacuum is an elastic solid, not a fluid.

The w_0–w_a signal of DESI as a parametrization artefact. The late cosmological sector is the phase condensate at $S_{\rm min}$ with $\dot{\theta}_0 = 0$: $T_{\mu\nu} \propto g_{\mu\nu}$ gives $w = -1$ *exact*, without a kinetic contribution to displace it (the radial modulus is ultra-Planckian and frozen, §11.9). The two mechanisms that could produce a dynamic deviation are vetoed by algebra: a condensate that gained physical density would give $w < -1$ phantom (forbidden by the positivity of the canonical kinetics, $w + 1 \geq 0$), and a phase mode that *rolled* ($\dot{\theta}_0 \neq 0$) does not give $w = -1$ constant but dilutes like matter —from the conserved Noether charge $Q = S^2\dot{\theta}\, L^3 = \text{const}$ and the continuity $S^2 L^3 = \text{const}$ it follows $\dot{\theta} = \text{const}$ and $\rho_{\rm kin} = \frac{1}{2}\rho_P \ell_P^2 S^2 \dot{\theta}^2 \propto S^2 \propto L^{-3}$, dust scaling incompatible with a $w = -1$ component. The ground state has $\dot{\theta}_0 = 0$ exact (minimum of the phase potential), so that only $w = -1$ remains. The DESI observations (2024–2025), combined with BAO/SNe/CMB, prefer at 2.5–3.9σ dynamical dark energy with $w_0 \approx -0.75$, $w_a \approx -1$ when fitted with the w_0–w_a parametrization. A medium with $w = -1$ exact whose $H(z)$ coincides with ΛCDM at linear order, but differs in the non-linear regime of the walls, produces exactly that type of signal when forced within the parametric mold: the deviation is a property of the parametrization, not of the equation of state of the medium. The falsifiable prediction is that the dynamical signal attenuates or disappears when measured with model-independent observables (raw D_H/r_d, the shape of $E(z)$), and that its residual —if any— tracks the non-linear correction of the walls, not a dynamical $w_0 > -1$. **[SLD]**.

Conclusion — what it means physically

Dark energy is not a substance added to the universe: it is the tension of the relaxed vacuum itself. When el Pleno rests in its configuration of minimal rigidity ($S_{\rm min}$), its phase is frozen in coherence and its pressure equals in magnitude its density with opposite sign ($\rho + p = 0$): it is an elastic solid under uniform tension, not a gas. That tension does not dilute as space expands —they are not particles that separate, but a property of the medium at each point—, and that is why it dominates more and more as matter rarefies, accelerating the expansion. Its equation of state is $w = -1$ exact: the signal of dynamical dark energy that DESI hints at when fitting with the w_0–w_a parametrization is not a physical deviation of the vacuum, but the artefact of forcing a medium with $w = -1$ exact —whose $H(z)$ differs from ΛCDM only in the non-linear regime of the walls— within that mold. The program predicts that the signal dissolves in raw observables.

11.4 The ponderomotive drag of matter by the dark-energy wave — [DERIVED] / [SLD]

The DE-Curtain is the homogeneous component θ_0 of the phase, with $w = -1$ and without a rest frame ($T_{\mu\nu} = \rho\, g_{\mu\nu}$): being homogeneous, it exerts no net local force. On that background, the phase perturbations $\delta\theta$ —the gapless Goldstone mode— ripple it, and their coupling to matter emerges from the second order of the kinetic term of F1.

The two modulations. On the relaxed vacuum at $S_{\min}$, the phase kinetic term gives, at linear order, $|\partial\Psi|^2 = (\partial\delta S)^2 + S_{\min}^2(\partial\delta\theta)^2 + \mathcal{O}(\delta^3)$: the two modulations decouple. The wave $\delta\theta$ (Goldstone, massless, $\Box\delta\theta = 0$, at c) and the matter δS (massive by V''_{eff}, localized in defects) do not mix at linear order: the drag is necessarily non-linear.

The coupling from F1. The phase kinetic term is $\frac{1}{2}\rho_P c^2 \ell_P^2 S^2(\partial\theta)^2$. With $S = S_{\min} + \delta S$, at second order:

$$S^2(\partial\delta\theta)^2 = \underbrace{S_{\min}^2(\partial\delta\theta)^2}_{\text{free wave}} + \underbrace{2\,S_{\min}\,\delta S\,(\partial\delta\theta)^2}_{\text{coupling}} + \mathcal{O}(\delta^3).$$

The cross term $\mathcal{L}_{\text{couple}} = \rho_P c^2 \ell_P^2\, S_{\min}\,\delta S\,(\partial\delta\theta)^2$ couples the matter to the energy of the wave. It is not an added ingredient: it is the second order of the same term that at first order gives the free equations E6 and E7. **[DERIVED]**.

The matter equation with drag. Varying with respect to δS:

$$\boxed{\Box\,\delta S - \frac{\ell_P^2}{20}\Box^2\delta S + \mu_S^2\,\delta S = \frac{S_{\min}}{\ell_P^2}\,(\partial\delta\theta)^2} \tag{11.2}$$

with $\mu_S^2 = V''_{\text{eff}}/(\rho_P c^2 \ell_P^2)$. The right side $(\partial\delta\theta)^2$ is the local energy density of the wave: where it concentrates energy, it forces the matter field —the structure of a ponderomotive force.

The drag force. For a traveling wave $\delta\theta = A\cos(kx - \omega t)$ with $\omega = ck$, the time average $\langle(\partial_x\delta\theta)^2\rangle = \frac{1}{2}A^2k^2$. The matter feels the gradient of this energy:

$$\boxed{\mathbf{F}_{\text{drag}} = -\rho_P c^2 \ell_P^2\, S_{\min}\,\nabla\langle(\partial\delta\theta)^2\rangle} \tag{11.3}$$

A uniform wave does not drag ($\nabla\langle\cdots\rangle = 0$); the wave (massless, at c) goes faster than the matter (massive, $v < c$), which follows it partially; and the sign $\mathbf{F} = -\nabla\langle\cdots\rangle$ attracts the matter toward the minima of phase agitation. **[DERIVED]** (coupling and ponderomotive form, second order of F1).

Falsifiable consequences. The drag is a dark-energy–matter coupling with two local consequences of definite sign, **[SLD]**: (i) modified structure growth —the matter feels, besides the emergent curvature, $\mathbf{F}_{\text{drag}}$ (effective fifth force mediated by the wave), deviating σ_8 and $f\sigma_8$ from ΛCDM—; (ii) local phase-density anti-correlation —the matter accumulates in the minima of phase energy, distinguishable from the structure by curvature alone. These feed the open end F-cosmo (structure growth, S_8) with a derived mechanism, without closing it: the magnitude depends on the amplitude of the phase perturbations, not fixed without additional input, and remains [**CALIBRATED**]. The global deviation of $w = -1$ that DESI observes does not come from this mechanism (it is the non-relaxed background, zeroth order), but the drag is a separate perturbative effect on the growth of structure.

Conclusion — what it means physically

Matter is dragged by dark energy like sand by a wave. When the phase of the vacuum ripples, those waves transport energy, and the concentrated energy pushes the matter toward where the wave is calmest —the same effect by which an intense light beam pushes particles (radiation pressure), here exerted by the waves of the vacuum itself. It is not gravity: it is a real fifth force, mediated by dark energy, that adds to the ordinary attraction and leaves two observable signatures —it alters the speed at which galaxies grow and clusters matter where the phase is calm. The decisive thing is that this force is not postulated: it appears by itself upon expanding el Pleno to second order, which reveals that matter and dark energy are not two distinct things that interact, but two ripples of the same medium that share substance.

11.5 Phase freezing: origin of dark energy and matter (Kibble-Zurek) — [SLD] / [CALIBRATED]

Angular ontology of $S_{\max}$. The saturation bound $S^2_{\max,\text{geom}} = 8\sqrt{6}/9$ admits an angular reading: the factor $8/9$ is $\sin^2\theta_{\text{tet}}$, with $\theta_{\text{tet}} = \arccos(-1/3) = 109.47^\circ$ the tetrahedral bond angle,

$$\sin^2\theta_{\text{tet}} = 1 - \cos^2\theta_{\text{tet}} = 1 - \tfrac{1}{9} = \tfrac{8}{9}.$$

At $S_{\max}$ el Pleno is continuous mass without vacuum, compressed to saturate the geometry of the bond —not packed spheres with gaps; the Kepler-Hales packing is the contact anchor, the angular reading is the ontology of the continuous medium.

The path of the continuum. The post-bounce expansion traverses the modulus from $S_{\max}$ to $S_{\min}$ (densities $\rho \propto S^2$, in ρ_P): from $S_{\max}$ ($\rho = 2.177$, amorphous continuous mass, U(1) restored) to S^* ($\rho = 1.067$, crystallization to diamond lattice and phase separation) to $S_{\min}$ ($\rho = 0.523$, relaxed diamond lattice, the present sea). The total compression is $S^2_{\max}/S^2_{\min} \approx 4.16$.

The freezing. At $S_{\max}$ the phase θ is thermally disordered (U(1) restored, without condensate). Upon crossing the transition at S^*, U(1) breaks (A5): causally disconnected regions choose independent phases, forming domains of coherent phase (Kibble-Zurek). Upon the completion of crystallization, the phase remains frozen in two complementary structures:

- Within each domain: uniform coherent phase, $\Box\theta = 0$, propagating mode. It is the dark energy (the DE-Curtain, ground-Pleno in its coherent mode).
- At the boundaries: trapped phase circulation, topological defects of *winding*. They are the particles (the depleted matter, the vortices of the hadronic sector).

The volume fraction in coherent domains is f_c; the fraction in defects is $1 - f_c$.

Derived properties of dark energy. As intra-domain coherent phase over the sea $S_{\min}$: it has $w = -1$ ($\Box\theta = 0$, $S = S_{\min}$ uniform, $\rho + p = 0$, [**DERIVED**]); it does not dilute (the coherent phase is not countable, does not go as a^{-3}, [**DERIVED**]); it propagates over the medium (satisfies E7, $\partial_\mu(S^2\partial^\mu\theta) = 0$, with coefficient S^2: the phase does not exist without the modulus). The matter (defects) is trapped circulation, with mass, countable, $\rho \propto a^{-3}$. The IR wall of Klauder guarantees $S \geq S_{\min}$: the sea never empties.

Status of the partition f_c. The partition between coherent dark energy ($f_c \approx 0.6869$) and depleted matter ($1 - f_c \approx 0.3131 = \Omega_m$) is not derived geometrically: although its *present value* is of type A —it is read from the instantaneous value of the numerator $S^2_{\max,\text{eff}}(t_0)$, as H_0 is read from Θ_0 (§4.4)—, that value was *imprinted* by the rate of the last phase transition: the fraction of defects in Kibble-Zurek depends on the *rate* of the transition (the velocity of the bounce). This situates f_c as a measure of the freezing velocity of the last phase transition of the universe: physical but trajectory-dependent, on equal footing with H_0 and with the numerator $S^2_{\mathrm{max,eff}}(t_0)$ of f_c itself. The mechanism (freezing at S^*) is **[SLD]**; the value remains **[CALIBRATED]**, the same positional character that governs the tilt n_s —the three quantities are imprinted in the same bounce transition, and all are state readings of the single trajectory $\bar{S}(t)$.

Conclusion — a single substance, three modulations

One and the same continuous medium, modulated in three ways over $S_{\min}$, gives dark energy (coherent phase wave), matter (trapped vortices) and space (the diamond lattice of the sea). One and the same event —the crystallization at S^* during the bounce— gives the spectral tilt, the matter/dark-energy separation and the origin of coherence. That the wave-matter coupling be derived and not postulated —that it come out of the same kinetic term that gives the free waves— is the signature that matter and dark energy share a single substance: el Pleno, modulated.

11.6 The perfect blackbody and the primordial spectrum — [DERIVED]

Blackbody. The photon is the phase mode θ_- of el Pleno (§9), an excitation of the medium that can be created and destroyed freely: there is no conserved charge nor photon number. In statistical mechanics, the chemical potential μ is the Lagrange multiplier of the conservation of number; if the number is not conserved, the minimization of the free energy with respect to N ($\partial F/\partial N = \mu = 0$) imposes

$$\mu = 0 \quad \Longrightarrow \quad n(\nu) = \frac{1}{e^{h\nu/k_BT} - 1}, \qquad \text{(E-cosmo-4)}$$

the Bose-Einstein distribution without chemical potential, that is the exact Planck one, $u(\nu)d\nu = \frac{8\pi h\nu^3}{c^3}\frac{d\nu}{e^{h\nu/k_BT}-1}$. The observed perfection ($|\mu| \lesssim 9\times10^{-5}$, FIRAS) is not a fine-tuning: it is structural, a consequence of the photon being a mode of the medium without its own number.

Spectral index without inflation. In conformal time $d\eta = dt/a$, with the Mukhanov-Sasaki variable $z = aS$ and $v_k = z\mathcal{R}_k$, the mode equation is

$$v_k'' + \Big(c^2k^2 - \frac{z''}{z}\Big)v_k = 0.$$

With $z \propto (-\eta)^s$, the symbolic computation gives $z''/z = s(s-1)/\eta^2$, so that the equation reduces to Bessel's with Hankel solutions $v_k = \sqrt{-\eta}\,[AH_\nu^{(1)}(-ck\eta) + BH_\nu^{(2)}]$, $\nu = |s - \frac{1}{2}|$. In the super-horizon limit ($H_\nu^{(1)} \sim x^{-\nu}$), $\mathcal{P}_\mathcal{R} \propto k^{3-2\nu}$, whence the master formula:

$$\boxed{n_s - 1 = 3 - 2\nu = 3 - 2\big|s - \tfrac{1}{2}\big|} \qquad \text{(E-cosmo-1)}$$

The exponent s. The depleted matter dilutes $S^2 \propto a^{-3}$ ($w = 0$), hence $S \propto a^{-3/2}$ and $z = aS \propto a^{-1/2}$. With $w = 0$, Friedmann in conformal time gives $a \propto (-\eta)^2$, whence $z \propto (-\eta)^{-1}$, that is $s = -1$ and $\nu = 3/2$. Substituting:

$$n_s - 1 = 3 - 2\cdot\tfrac{3}{2} = 0 \quad \Longrightarrow \quad n_s = 1 \text{ (zeroth order)}.$$

The degeneracy of the pure components. The observed red tilt ($n_s \approx 0.965$, $n_s - 1 \approx -0.035$) does not come from an individual component of the background. For a component of equation of state w, conformal Friedmann gives $a \propto (-\eta)^p$ with $p = 2/(1+3w)$, and the Mukhanov variable inherits

$$s_z(w) = p\cdot\frac{-1-3w}{2} = \frac{-1-3w}{1+3w}.$$

Evaluating the two components that dominate the crunch:

$$s_z(0) = -1 \text{ (depleted matter)}, \qquad s_z(-2) = \frac{-1+6}{1-6} = -1 \text{ (Klauder, } w_q = -2).$$

Both give the same exponent $s_z = -1$, hence $\nu = 3/2$ and $n_s = 1$: neither the depleted matter ($w = 0$) nor the IR wall of Klauder ($w_q = -2$) produces, separately, a tilt. The red sign requires the *crossing* between regimes, not the regime.

The mechanism: the packing transition in the bounce. The bounce occurs from the finite maximal density $\rho_{\max} = S^2_{\max,\text{geom}}\,\rho_P = \frac{8\sqrt{6}}{9}\rho_P$, which the program derives as the ratio between the maximal FCC packing (Kepler-Hales, $Z = 12$) and the density of the diamond lattice ($Z = 4$):

$$S^2_{\max,\text{geom}} = \frac{n^{\max}_{\text{FCC}}}{n_{\text{diamante}}} = \frac{\sqrt{2}/\ell_P^3}{3\sqrt{3}/(8\ell_P^3)} = \frac{8\sqrt{6}}{9}.$$

The two structures of this identity —compact $Z = 12$ and diamond $Z = 4$— are the phases between which el Pleno transits in the bounce ($Z = 12 \to Z = 4$). The imprinting of the primordial spectrum occurs during that transition, over the persistent background $f_c = S^2_{\max,\text{eff}}/S^2_{\max,\text{geom}} = 1.4956/2.1773 = 0.6869 = \Omega_{EO}$, with $w_{EO} = -1$.

Red sign [SLD]. The Klauder component that enters the transition has $w_q = -2 < -\frac{1}{3}$: it accelerates the background. An accelerated phase in the imprinting of the modes adds power at large scale relative to small, running the spectrum to the red; the persistent background f_c with $w_{EO} = -1$ reinforces the same sign. Therefore $n_s - 1 < 0$ structural, not fitted.

Smallness [SLD]. In the early crunch, when the large-scale modes cross the horizon, the Klauder fraction $x = \rho_q/\rho_m \propto a^6$ is small: the regime is dominated by the depleted matter ($n_s = 1$), and Klauder enters as a first-order correction in $x \ll 1$. Therefore $|n_s - 1| \ll 1$. The band $n_s = 0.965 \pm 0.004$ is reached with $|w_{\text{eff}}| \approx 0.006$, the order that Klauder introduces in that epoch.

Exact value, positional [CALIBRATED]. The value $n_s = 0.965$ depends on the Klauder fraction at the precise moment of horizon crossing, that is, on where over the trajectory $\bar{S}(t)$ the modes were imprinted. There is no exact scaling solution that cancels that dependence: for $V_q \propto S^{-2}$, the ansatz $S \propto t^\beta$, $a \propto t^\gamma$ imposes $\beta = 1/2$ by balance of the EOM, but then Friedmann is inconsistent in the exponents of t ($H^2 \sim t^{-2}$ against $\rho \sim t^{-1}$). The present value of the tilt inherits the positional condition of $S^2_{\max,\text{eff}}(t_0)$, on equal footing with f_c and H_0. By Rule 17 it is a state variable, not a derivable number; the datum itself is a band (± 0.0042).

11.7 The matter density $\Omega_m = 1 - f_c$ — [DERIVED]

By conservation of the condensate population, $f_c + \Omega_{\text{depleted}} = 1$:

$$\boxed{\Omega_m = 1 - f_c = 1 - 0.6869 = 0.3131} \qquad \text{(E-cosmo-2)}$$

Against Planck ($\Omega_m = 0.315 \pm 0.007$): 0.27σ, without fitting. Decomposing with $\Omega_{MO}/\Omega_M = \sqrt{3}\pi$:

$$\Omega_M = \frac{0.3131}{1+\sqrt{3}\pi} = 0.0486 \ (1.2\sigma), \qquad \Omega_{MO} = 0.2645 \ (0.07\sigma).$$

The dilution $\rho_m \propto a^{-3}$ is topological (conserved countable objects): $n = N/V \propto a^{-3}$, $\rho_m = n\, m_{\text{sol}} c^2 \propto a^{-3}$, the exponent emerges from the conservation plus the geometry.

11.8 The expansion history $H(z)$ — [DERIVED] / [CALIBRATED]

Nondimensionalizing $V_{\text{eff}} = \rho_P c^2 v(S)$ and using $G = c^2/(\rho_P \ell_P^2)$, the Friedmann prefactor of the field is $\frac{8\pi G}{3}\rho_P c^2 = \frac{8\pi}{3}\frac{c^4}{\ell_P^2} \equiv \mathcal{C}$. From $H^2 = \frac{8\pi G}{3}(\rho_m + \rho_S)$ with $\rho_m = \rho_{m,0}(1+z)^3$ (diluted matter) and $\rho_S(z)$ the density of the field, upon forming $E(z) \equiv H(z)/H_0$ the common prefactor $\mathcal{C}$ appears in numerator and denominator and cancels along with every dimensional scale; normalizing $E(0) = 1$:

$$\boxed{E^2(z) = \frac{\Omega_m(1+z)^3 + \rho_S(z)/\rho_{c,0}}{\Omega_m + \rho_S(0)/\rho_{c,0}}} \tag{E-cosmo-3}$$

scale-invariant form, forced by the derived potential, with δ_0 (initial condition $\leftrightarrow w_0$) as the only observable freedom. Confrontation with BAO DESI DR2 (5 points):

Model	χ^2	χ^2/dof
ΛCDM ($\Omega_m = 0.31$)	6.53	1.63
PIU ($\delta_0 \to 0$, $w_0 \approx -1$)	6.54	1.63
PIU ($\delta_0 = 0.02$, $w_0 = -0.96$)	11.24	2.81

The program fits BAO DESI DR2 as well as ΛCDM with less freedom. The scale invariance of the form is [**DERIVED**]; w_0 (or δ_0) is [**CALIBRATED**], a state variable.

Equivalence with ΛCDM and distinctive advantage

PIU-Ψ reproduces the central cosmological observables with the same statistical precision as ΛCDM and with the same degrees of freedom —it is parity, not superiority of fit. On that basis, the program contributes three features that ΛCDM does not possess. *(i) Derived, not fitted, parameters:* Ω_{EO} and Ω_{MO}/Ω_M come out of the geometry of the substrate (at 0.30σ and 0.86σ, zero free parameters), where ΛCDM fits them to the datum. *(ii) Unique and falsifiable predictions:* the scalar/tensor asymmetry with red→blue transition (CMB-S4/LiteBIRD), the baryonic rotational tower, the vortex separation $d \approx R_p$, and the signatures of the Lee-Wick pole in gravitational waves (LISA/ET/CE) —ΛCDM formulates none; they are the points where the program exposes itself to refutation. *(iii) Ontological reduction:* a single substrate read in distinct configurations, instead of an inventory of independent components each with its postulated equation of state. The ontological economy is what allows *deriving* the fractions instead of postulating them.

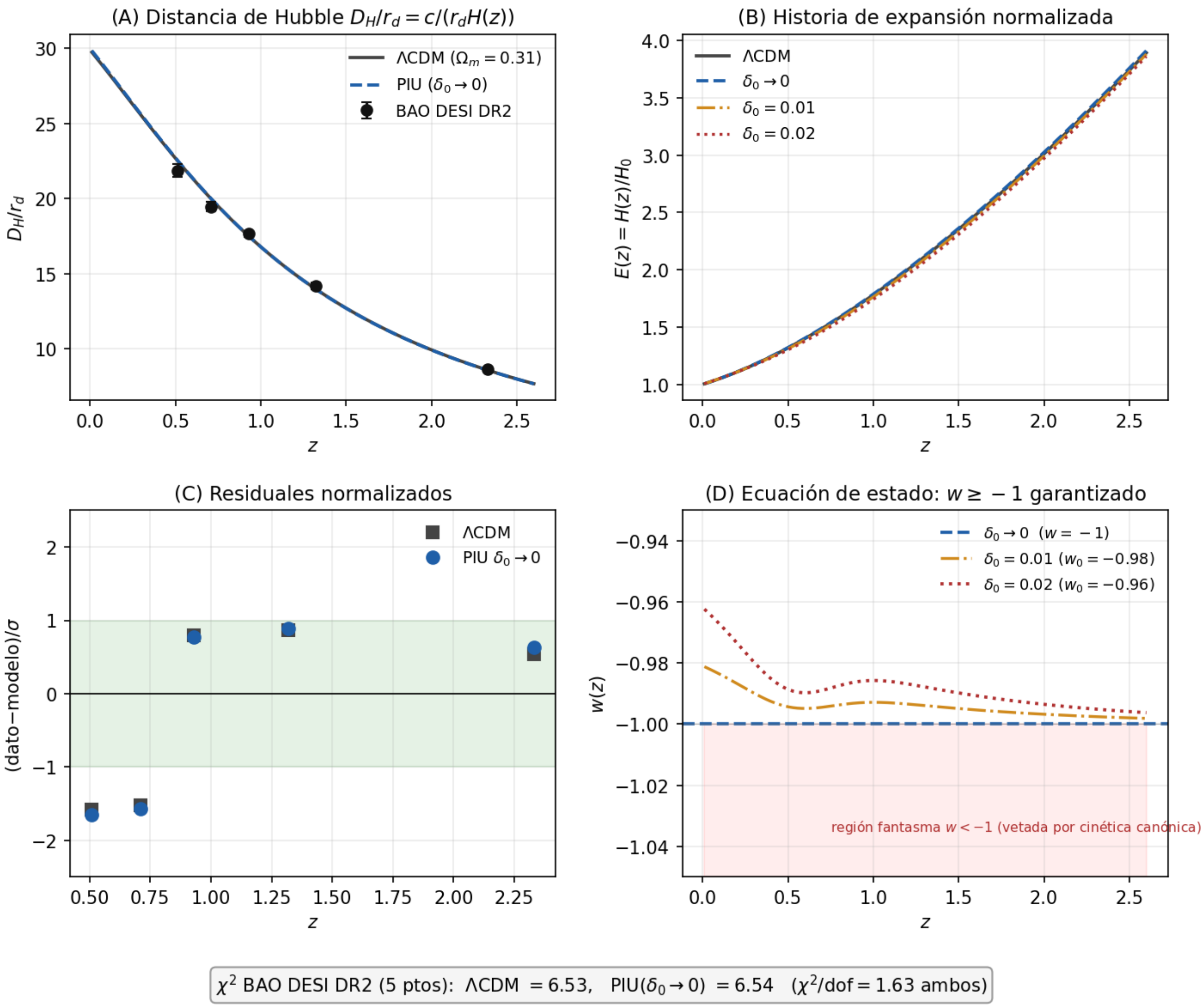

Figure 8: Confrontation with BAO DESI DR2. Panel A: the Hubble distance D_H/r_d of PIU ($\delta_0 \to 0$) and of ΛCDM are indistinguishable over the five points. Panel C: identical normalized residuals. Panel D: the canonical kinetics guarantees $w \geq -1$ (the phantom region is vetoed). The $\chi^2 = 6.54$ of PIU equals the 6.53 of ΛCDM —same fit, less freedom.

Conclusion — what it means physically

Cosmology ceases to be a catalog of fitted ingredients and becomes the reading of the same Pleno at the scale of the universe. What in ΛCDM are free parameters, here are consequences: the dark/baryonic matter ratio $\sqrt{3}\pi$ is pure geometry (two areas and a spinor factor 1/2); $\Omega_m = 1 - f_c$ is the depleted fraction of the condensate; the **dark energy is the floor of the potential** —the same $0.5229\,\rho_P$ of the ground-Pleno of link 6—, so that $w = -1$ is not an exotic fluid but the elastic rigidity of the relaxed vacuum ($\rho + p = 0$, zero entropy: a solid, not a fluid). The perfect blackbody and $n_s \approx 1$ without inflation fall out as structure, not as a fit. And all of it holds with *less* freedom than ΛCDM: two firm predictions at 0.27σ and 0.86σ without a single parameter, inherited from the diamond lattice of link 5. The universe does not contain dark matter and dark energy as separate substances: it exhibits them as two readings —excited and background— of a single medium.

11.9 The Hubble constant as a positional quantity: factorization, condensate occupation and tension — [DERIVED] / [SLD]

The scale-invariant form $E(z) = H(z)/H_0$ cancels the Friedmann prefactor: the theoretical content lives in the *shape* of the curve, not in the absolute value H_0. It remains to classify what kind of object H_0 is. Starting from the effective Friedmann equation of the background today, $H_0^2 = \frac{8\pi G}{3}\rho_{\rm tot,0}$, the density is nondimensionalized by its anchor, $\rho_{\rm tot,0} \equiv \Theta(t_0)\,\rho_P$, and G is eliminated with the native identity $G = c^2/(\rho_P \ell_P^2)$ (§5.1). The anchor density ρ_P cancels exactly:

$$H_0^2 = \frac{8\pi G}{3}\,\rho_{\rm tot,0} = \frac{8\pi}{3}\,\frac{c^2}{\rho_P \ell_P^2}\,\Theta\,\rho_P = \frac{8\pi}{3}\,\Theta\,\frac{c^2}{\ell_P^2} = \frac{8\pi}{3}\,\Theta\left(\frac{c}{\ell_P}\right)^2. \tag{11.4}$$

Taking the root and using the definitional identity $\tau_P = \ell_P/c$ ($c/\ell_P = 1/\tau_P$):

$$\boxed{H_0 = \underbrace{\sqrt{\tfrac{8\pi}{3}}}_{\text{geom. ratio, [DERIVED]}} \times \underbrace{\frac{c}{\ell_P}}_{\text{anchor } = 1/\tau_P} \times \underbrace{\sqrt{\Theta(t)}}_{\text{state, [CALIBRATED]}}} \tag{E-cosmo-5}$$

with $\sqrt{8\pi/3} = 2.8944$ and $c/\ell_P = 1.8552 \times 10^{43}\,\mathrm{s}^{-1}$.

What the factorization asserts and what it does not assert. The factor $\sqrt{8\pi/3}$ is the geometric Friedmann coefficient —common to all FLRW cosmology, fixed by the density-curvature relation in 3+1 dimensions—, not an exotic invariant of the lattice. What the program contributes is the *identification of the anchor*: the dimensional scale of H_0 is exactly $c/\ell_P = 1/\tau_P$, the frequency of the Planck tick (the same $\dot\gamma_{\rm max}$ of §12.2), once G is eliminated by its native identity. The non-trivial claim is threefold: (i) the factorization introduces no parameter —the first two pieces are fixed and the third is a state quantity, not a free constant; (ii) the dimensional content of H_0 resides entirely in ℓ_P (with c reabsorbable causal structure), which passes the purity test of the Language of el Pleno —in units $c = \ell_P = 1$ the anchor dissolves to unity; (iii) the pure number of H_0 in units of el Pleno, $\sim 10^{-61}$, alien to the $O(1)$ ratios that the program derives, is the diagnosis that H_0 is positional. The numerical verification with $H_0 = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}} = 2.184\times10^{-18}\,\mathrm{s}^{-1}$ gives $\Theta_0 = 1.655 \times 10^{-123}$ and reconstructs H_0 exactly.

Today's density is the age in ticks, squared. Defining the number of ticks of the Hubble time $N_{\rm tic} \equiv 1/(H_0\tau_P) = 8.493 \times 10^{60}$ (with the Planck tick $\tau_P = \ell_P/c = 5.391 \times 10^{-44}$ s), from $H_0 = \sqrt{8\pi/3}\,\sqrt{\Theta}/\tau_P$ one solves $\sqrt{\Theta} = \sqrt{3/8\pi}\,(H_0\tau_P) = \sqrt{3/8\pi}/N_{\rm tic}$, and squared:

$$\boxed{\Theta(t) = \frac{3}{8\pi\,N_{\rm tic}^2(t)}} \quad\Longleftrightarrow\quad N_{\rm tic} = \sqrt{\frac{3}{8\pi\,\Theta}}. \tag{E-cosmo-6}$$

Verification: $3/(8\pi\cdot(8.493\times10^{60})^2) = 1.655\times10^{-123} = \Theta_0$. The number of ticks admits two independent readings of the same clock that agree: from the rate, $N_{\rm tic} = 1/(H_0\tau_P) = 8.49\times10^{60}$; from the observational age, $N = t_0/\tau_P = (13.787\,\mathrm{Gyr})/\tau_P = 8.07\times10^{60}$. The agreement to $\sim 5\%$ without fitting confirms that $N_{\rm tic}$ is the age of the universe in Planck ticks, and the small difference is the factor $O(1) \approx 0.95$ between $1/H_0$ and the true age (the same as the magnitude open end).

Reformulation of the cosmological constant problem

The number $\Theta_0 \approx 10^{-123}$ *is* the crude formulation of the cosmological constant problem: the vacuum energy density of today is $\sim 10^{-123}$ times that of Planck, the worst fine-tuning of the standard model. The identity (E-cosmo-6) rewrites it: the "10^{-123}" is not a number to explain, but $1/N_{\rm tic}^2$ up to the geometric ratio $3/8\pi$. Any universe that has lived $N_{\rm tic}$ ticks automatically has $\Theta \sim 1/N_{\rm tic}^2$ by arithmetic. There is no need to explain the exponent 123; one has to explain how many ticks the clock has run ($N_{\rm tic} \approx 8.5\times10^{60}$), and that is an age, not a tuned parameter. This extends to the absolute *scale* what $\Omega_m = 1 - f_c$ did with the *partition*: the "120 orders" of the matter/vacuum discrepancy were replaced by the order-one fraction $f_c \approx 0.69$. The two sides of the problem —why Λ is small (scale) and why it is comparable to matter today (partition)— are reformulated as clock readings.

The Hubble tension as a method artefact. If H_0 is the reading of a clock —the frequency of the tick modulated by the age in ticks—, then the "primordial H_0" and the "local H_0" are not two measurements of the same constant in disagreement, but two distinct objects: the first is the value that ΛCDM *needs* to reproduce the acoustic angle of the CMB under its own template, not a raw datum. Analyzing a medium with $w = -1$ exact and elastic $H(z)$ —which coincides with ΛCDM at linear order and differs in the non-linear regime of the walls— under the ΛCDM template biases the primordial inference. The structure of the bias is seen in the propagation to the inference of the primordial method, which anchors H_0 to the acoustic angle $\theta_* = r_s/D_M(z_*)$ of the CMB:

$$D_M(z_*) = \int_0^{z_*} \frac{c\,dz}{H(z)}, \qquad H(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)\,g(z)},$$
$$g(z) = \exp\Big[3\int_0^z \frac{1+w(z')}{1+z'}\,dz'\Big]. \tag{11.5}$$

When the real $H(z)$ is that of the elastic medium but the inference is made under the rigid ΛCDM fit, the H_0 that reproduces the same θ_* at fixed r_s falls below the locally measured one: the geometric degeneracy in the distance to the last scattering produces a bias of definite sign. The number ~ 5.8 of the "tension" is a property of the ΛCDM interpretation, not a raw datum. The falsifiable prediction of the program is that the tension *dissolves* in model-independent observables (raw D_H/r_d of BAO, raw θ_*, the shape of $E(z)$): there are not two true values of H_0 in conflict, but an artefact of analyzing a dynamical medium under a static template. [**SLD**].

The prediction: $w = -1$ exact and the tension as artefact; the non-linear correction, open end

The **equation of state** of the late sector is $w = -1$ *exact* ([**DERIVED**]: phase condensate at $S_{\min}$, $T_{\mu\nu} \propto g_{\mu\nu}$), and the **Hubble tension** is a method artefact ([**SLD**]: the "primordial" H_0 is the one ΛCDM needs for the CMB, not a datum). What remains as an open end is the **non-linear correction of** $H(z)$ that the Gent/Klauder walls imprint over the elastic background —the deviation of $E(z)$ with respect to ΛCDM in the non-linear regime, whose amplitude encodes δ_0— and its confrontation with raw BAO. The open end is *relational*, not temporal: to ask for "$\bar{S}$ as a function of a background time" would be circular, since time is the accumulated change of the modulus (§12, the singularity under emergent time), and the relaxation curve of the diamond lattice $S^* \to S_{\min}$ *is* the curve of cosmological time —the lattice is born in the crystallization at $S^* = \sqrt{S_{\min} S_{\max,\mathrm{geom}}} = 1.0330$ and descends monotonically to the attractor $S_{\min}$ (second law). Time emerges as arc length $\propto \int |d\bar{S}|/c$ over the orbit, never as input

$$\ddot{\bar{S}} + 3H\dot{\bar{S}} + \rho_P^{-1}\ell_P^{-2}V'_{\mathrm{eff}} = 0$$

(with V_{eff} derived and anchors ρ_P, ℓ_P; distinct from the primordial cycle equation §12.4, which governs the bounce). Computing this correction is the same class of operation already performed numerically for the crystallization transient (§12.5), which reproduced its numbers without fitting. That it be an open end and not a closed result has a mathematical reason: there is no exact *scaling* solution of the trajectory (§11.6: the ansatz $S \propto t^\beta$ leaves Friedmann inconsistent in the exponents), so that the closed form requires fine integration. The present value Θ_0 (the hour of the clock) remains positional: the curve is derivable, the point on the curve is a historical datum.

Which clock marks the present. It is worth distinguishing two objects. The *radial modulus* $\bar{S}$ is ultra-Planckian ($m_S = \sqrt{\xi}\,E_P \approx 4.44\,E_P$, §5); against the cosmological rate $m_S/H_0 \sim 10^{61}$, so that it *freezes* at $S_{\min}$ almost from the first instant and its own relaxation lasts $\sim t_P$, not an age of the universe. That the scales built over $\bar{S}$ (soliton masses M_Q, $S_{\min}$ of the defects) do not derive confirms it observationally. The modulus, therefore, **is already at the attractor**; its descent $S^* \to S_{\min}$ is the clock of the bounce/crystallization, not that of the present epoch. The clock of the present is the **coherent phase condensate** (DE-Curtain, §11.7): its $w = -1$ is not a field rolling —that would give $w = +1$— but the symmetry $T_{\mu\nu} \propto g_{\mu\nu}$ of the ground state (without a rest frame, [**DERIVED**]), with $\dot{\theta}_0 = 0$ and therefore $w = -1$ *exact*, not an approximation with a residual. The positional reading that does operate is that of the *occupation* of the condensate (f_c, H_0, n_s), not that of the equation of state. Only a massless mode (phase Goldstone, A5) could in principle roll slowly against H, but a phase mode that rolls dilutes like matter ($\rho_{\rm kin} \propto S^2 \propto L^{-3}$, from the Noether charge $S^2\dot{\theta}L^3 = \text{const}$ and the continuity $S^2L^3 = \text{const}$), not like a $w = -1$ component: it is ruled out as the origin of dark energy, and the condensate at its minimum ($\dot{\theta}_0 = 0$) remains at $w = -1$ exact. The only thing left to compute is the non-linear correction of $H(z)$ by the walls, which lives in the collective response of the *phase sector*, not in the orbit of the radial modulus —the equation $\ddot{\bar{S}} + 3H\dot{\bar{S}} + \rho_P^{-1}\ell_P^{-2}V'_{\rm eff} = 0$ describes the collective homogeneous background. [**SLD**] (supported by derived results: $T \propto g$, m_S, the L^{-3} scaling of a rolling phase mode).

The occupation of the condensate at the present rest. The descent of the modulus from the bounce saturation $S^2_{\rm max,geom} = 2.1773$ toward the rest $S^2_{\min} = 0.5229$ (law $S^2 \propto a^{-3}$, §10.5) is *essentially complete*: today the ground-Pleno is in its relaxed regime, with $S \sim S_{\min}$ (§4.5), settling in the attractor of the well with $\dot{\bar{S}}$ small but not null. We are not halfway down the descent; we are at the bottom of the curve. What the positional quantities (H_0, f_c, w_0, n_s) localize is not an intermediate position over the descent, but the **occupation level of the condensate** at that rest:

$$\boxed{f_c = \frac{S^2_{\rm max,eff}}{S^2_{\rm max,geom}} = \frac{1.4956}{2.1773} = 0.6869 = \Omega_{EO}} \qquad \text{(E-cosmo-7)}$$

The condensate occupies 69% of the geometric capacity; the remaining 31% ($1 - f_c = 0.3131$) is the quantum depletion. Structure [**DERIVED**]; value of the numerator $S^2_{\rm max,eff}$ [**CALIBRATED**] positional (like H_0, anchored to the present epoch). f_c is not a constant of nature but the residual occupation of the coherent mode —the same that H_0 and n_s read from distinct angles. The projections coincide with a single state, the rest: $f_c = 0.6869$ (occupation) and $n_s - 1 = -0.035$ (tilt of the primordial spectrum), a verifiable internal consistency of the background sector. The equation of state is not of this type: $w = -1$ is exact, geometric, not a positional reading. [**SLD**].

The background value of H_0 and the exclusion of the local value. The position R and the initial condition δ_0 that fixes $w(z)$ determine the background value of the Hubble constant. Integrating the expansion history with the observed age $t_0 = 13.787\,\text{Gyr}$:

δ_0	w_0	$E(z)$ at $z = 2$ vs ΛCDM	H_0 ($\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$)
$\to 0$	-1	$\to$ ΛCDM ($\chi^2 = 6.54$)	67.71
0.02	-0.961	$+2.45\%$	67.41

Over the entire strip of initial conditions the background falls in $H_0 \sim 67$–68 (primordial side): the physical case $\delta_0 \to 0$ —late sector with $w = -1$ exact, indistinguishable from ΛCDM at linear order— gives $H_0 = 67.71$; a small initial condition $\delta_0 = 0.02$, which encodes the non-linear correction of the walls over $E(z)$, gives $H_0 = 67.41$, coincident with Planck (67.4 ± 0.5). The parameter δ_0 is not a dynamical w_0 —the background has $w = -1$ exact— but the amplitude of the non-linear correction, [**CALIBRATED**] positional; the shape of $E(z)$ is derived. The robust point, independent of δ_0: the background H_0 of el Pleno is on the primordial side, never near 73.

The local SH0ES value (~ 73) is *excluded* as a background by a structural bound. The dimensionless age $A \equiv H_0 t_0$ is bounded by the canonical kinetics: $H_0 = 73$ with the observed age would require $A \approx 1.03$, but a background with $w = -1$ exact dark energy imposes $A \le A(w{=}-1) = 0.953$ (age $\approx 0.95/H_0$). Exceeding it requires the phantom region $w < -1$, vetoed by $w+1=\rho_P\ell_P^2\dot{\bar{S}}^2/\rho\geq 0$. The local value is not the background H_0: it is the instantaneous rate that the distance ladder measures locally, biased with respect to the background. **[DERIVED]** by the bound $w\geq -1$. Numerically: $A(67.41)=0.9505$, $A(67.71)=0.9547$ (both under the ceiling), $A(73)=1.0293$ (excluded).

Native reading: $A<1$ is the imprint of the fluid that cools. The previous bound is stated in FLRW language (w, $1/H_0$), but its content is native and it is worth reading it in the Language of el Pleno, where A ceases to be a fundamental object and becomes the *observable shadow* of the dynamics of the modulus. Time is the accumulated change of $\bar{S}$ over the relaxation curve (§12); H_0 is the fractional rate of that change today; t_0 is the arc already traversed from S^*. The medium is a dilatant fluid that *cools* as it descends $S^*\to S_{\min}$: the deformation rate $\dot{\gamma}$ reaches a maximum in the fast descent of the hot zone (near S^*) and *decays* as it approaches the cold attractor $S_{\min}$ —exactly the profile of the viscous transient (§12, $\dot{\gamma}_{\max}=0.98$ not exceeded, then damping). That the cosmic clock *was faster in the past and slows down now* implies that the universe reached its present state faster than its present rate would suggest: it is *younger* than its Hubble time, $t_0<1/H_0$, that is $A=H_0t_0<1$. The sign of the bound is, then, the integral signature of a fast phase followed by braking —the same physics of the dilatant transient, not an imported veto. A background H_0 of 73 would give $A=1.03>1$: it would require the opposite —an expansion that net-accelerates without having been fast before, that is increasing $\dot{\bar{S}}$ that would overshoot the attractor $S_{\min}$—, vetoed by the monotonic descent to the attractor (second law) and, equivalently, by $w\geq -1$ (a square $\dot{\bar{S}}^2\geq 0$). The *exclusion* of 73 is thus native **[DERIVED]**, independent of the Friedmann integral; the exact *value* of the ceiling (0.953) is its FLRW projection and depends on the fine shape of $f(\bar{S})$ —open end F-cosmo. Verified: a universe that brakes more (faster in the past) gives smaller A —only matter, $A=2/3$; with dark energy, $A\to 0.953$ from above—; $A>1$ is produced only by $w<-1$, physically impossible in an elastic medium that cools.

The Hubble bound is the dilatant bound, with exact equality. The extreme $S_{\max,\text{geom}}$ fixes the absolute upper bound of the expansion rate: from $\rho_{\max}=\rho_P S^2_{\max,\text{geom}}$ and Friedmann, $H_{\max}=\sqrt{8\pi G\rho_{\max}/3}=\sqrt{8\pi/3}\,(c/\ell_P)\,S_{\max,\text{geom}}=7.922\times 10^{43}\,\text{s}^{-1}$, with minimal horizon $R_{H,\min}=c/H_{\max}=0.2341\,\ell_P$ sub-Planckian. Compared with the maximal deformation rate $\dot{\gamma}_{\max}=c/\ell_P$ (§12.2), the quotient is exact:

$$\boxed{\frac{H_{\max}}{\dot{\gamma}_{\max}}=\sqrt{\frac{8\pi}{3}}\,S_{\max,\text{geom}}=4.2709}\qquad\text{(E-cosmo-8)}$$

not a "factor ~ 4": the Hubble bound (bounce cosmology) and the dilatant deformation bound (jamming, $\text{Ma}=1$) are the same rigidity of the medium, linked by the Friedmann coefficient by the root of the packing quotient. **[DERIVED]**.

What is pertinent to predict: observables, not parameters of another model

Methodological caution that governs this subsection. The "Hubble tension" is *not* a discrepancy between two measurements of the same quantity: the local value (~ 73) is mildly model-dependent, but the "primordial" one (~ 67) **is not measured** —it is a parameter derived from fitting ΛCDM to the pattern of the CMB, "the H_0 that ΛCDM needs", not "the H_0 of the early universe". Therefore the number ~ 5.8 is a property of the ΛCDM interpretation, not a raw datum. **It is not pertinent** to calibrate PIU-Ψ to reproduce ~ 5.8 (it would be imitating an artefact of the model that is being replaced); **it is pertinent** to predict the *method bias*: that analyzing a medium with $w=-1$ exact and elastic $H(z)$ under the rigid ΛCDM template infers a CMB H_0 biased low, with definite sign, and that the tension —like the w_0–w_a signal of DESI— dissolves in model-independent observables. The confrontation of PIU-Ψ is always made against model-independent observables (D_H/r_d of BAO, $E(z)$, raw θ_*), never against parameters (H_0^{Planck}, w_0^{DESI}) that are already outputs of a ΛCDM fit.

Balance of the Hubble sector: what the program delivers and what it leaves open

The weight of the result is distributed across several pieces and is not reduced to a single one. In order of strength, the program delivers:

1. **Exact factorization** $H_0 = \sqrt{8\pi/3}\,(c/\ell_P)\,\sqrt{\Theta}$ (E-cosmo-5), **[DERIVED]**. The dimensional scale of H_0 is the frequency of the Planck tick $c/\ell_P = 1/\tau_P$, once G is eliminated by its native identity; H_0 remains in anchors of el Pleno and pure ratios, without an alien scale.
2. **Reformulation of the cosmological constant problem** (E-cosmo-6), **[DERIVED]**: $\Theta_0 = 3/(8\pi N_{\rm tic}^2)$ converts the "10^{-123}" into the inverse of the square of the age in ticks. There is no minuscule vacuum density to explain; there is an old universe. A reinterpretation, not a new number, but it rewrites a fine-tuning of 123 orders as an age.
3. **Exact $w = -1$ equation of state of the late sector**, **[DERIVED]**: the phase condensate at $S_{\min}$ with $\dot\theta_0 = 0$ gives $T_{\mu\nu} \propto g_{\mu\nu}$, without a rest frame; the positivity of the kinetics vetoes $w < -1$ and no internal mechanism displaces w from -1. The non-linear correction of the walls over $E(z)$ is the only freedom, encoded in δ_0.
4. **Exact structural equality Hubble bound/dilatant bound** (E45), **[DERIVED]**: $H_{\max}/\dot\gamma_{\max} = \sqrt{8\pi/3}\,S_{\max,\rm geom} = 4.2709$, not a mere "same order". The cosmological expansion bound and the rheological deformation bound are the same rigidity of the medium, read from the global expansion and from the local shear.
5. **Hubble tension as a method artefact**, **[SLD]**: the "primordial" H_0 (~ 67) is the one ΛCDM needs for the CMB, not a datum; analyzing a medium with $w = -1$ exact and elastic $H(z)$ under that template biases the inference. The program predicts that the tension —and the w_0–w_a signal of DESI— dissolve in model-independent observables.
6. **Background value of H_0 and exclusion of the local one**, **[CALIBRATED]/[DERIVED]**: over the entire strip of initial conditions the background gives $H_0 \sim 67$–68 (primordial side: 67.71 for $\delta_0 \to 0$, 67.41 for $\delta_0 = 0.02$), coincident with Planck; the local value 73 would require $A = H_0 t_0 \approx 1.03$, vetoed by the bound $A \le 0.953$ that $w \ge -1$ imposes.

The *only* thing open is the **non-linear correction of** $H(z)$ that the Gent/Klauder walls introduce over the elastic background, and its measurability in raw BAO —plus the prediction of S_8 from the phase drag. The closed computation of that correction is the same class of operation already done numerically for the transient (§12.5); what remains is its shape in the relaxed regime: a confrontation open end, of subordinate priority. The w_0–w_a signal of DESI and the Hubble tension are not quantities to reproduce, but artefacts that the program predicts dissolve in raw observables. Five results with exact or structural mathematics, the non-linear correction as a declared open end: it is not "just the sign".

Conclusion — what it means for real physics

The positional reading reduces three problems that the standard model treats separately to a single ontological assertion: *H_0 is not a constant of nature, but the hour of a clock.* The **cosmological constant problem** ceases to be a fine-tuning of 123 orders —Θ_0 is the inverse of the square of the age in ticks. The **Hubble tension** ceases to be an anomaly —the "primordial" H_0 is not measured, it is the one ΛCDM needs for the CMB, and analyzing a medium with $w = -1$ exact and elastic $H(z)$ under that template biases the inference. The **dynamical dark energy signal** of DESI is the same type of artefact: the one that appears when forcing a medium with $w = -1$ exact into the w_0–w_a parametrization. The three are the same thing: an elastic medium —whose $H(z)$ coincides with ΛCDM at linear order and differs in the non-linear regime— read under static assumptions. The counterpart, far from being a weakness, is what makes the proposal falsifiable: the program does not derive the *value* of H_0 (it would be deriving the hour from the clock mechanism), and predicts that the tension and the DESI signal dissolve in model-independent observables —if instead they *persist* in raw data, or if the signal consolidates in $w_0 < -1$ (phantom region vetoed by $w + 1 \geq 0$), the reading is refuted.

11.10 The time dilation of SN Ia: the shared factor $b = 1$ and the radial redshift bound — [DERIVED]

The redshift and time dilation of the distant sources are not read through a postulated scale factor: they are derived from the deformation map of the medium itself. The homogeneous and isotropic background is the background part of the map $X^a(x)$, with temporal stretching $X^0 = \tau(t)$ and spatial $X^i = L(t)\,x^i$. The components of the emergent metric —the Cauchy-Green tensor of the hyperelastic medium (E39a, §7)— are $g_{00} = -\dot{\tau}^2$ and $g_{ij} = L^2\delta_{ij}$ (with $c = 1$), without inserting $a(t)$ at any step.

The shared-factor theorem. The photon is a TT mode of the medium ([**DERIVED**], §9), and its radial propagation is null, $ds^2 = 0$:

$$-\dot{\tau}^2\,dt^2 + L^2\,dx^2 = 0 \implies \dot{\tau}\,dt = L\,dx \implies dx = \frac{d\tau}{L}, \tag{11.6}$$

using $d\tau = \dot{\tau}\,dt$ as the differential of the background proper time (with $c = 1$). *The comoving wavelength is conserved.* The phase of the mode, $\phi = k_{\mathrm{com}}x - \omega_{\mathrm{com}}\tau$, is an invariant scalar that propagates over the comoving coordinate x; since the background is homogeneous (independent of x), the comoving wave number $k_{\mathrm{com}} = 2\pi/\lambda_{\mathrm{com}}$ is a constant of motion (there is no spatial gradient to modify it), so that λ_{com} is fixed along the trajectory. The *physical* wavelength is the comoving one stretched by the spatial map, $\lambda_{\mathrm{phys}} = L\,\lambda_{\mathrm{com}}$, and therefore

$$\frac{\lambda_0}{\lambda_e} = \frac{L(\tau_0)\,\lambda_{\mathrm{com}}}{L(\tau_e)\,\lambda_{\mathrm{com}}} = \frac{L(\tau_0)}{L(\tau_e)}. \tag{11.7}$$

The time interval shares the same factor. Two successive crests emitted at τ_e and $\tau_e + \delta\tau_e$ traverse the *same* comoving coordinate Δx to the receiver, since x depends only on the endpoints:

$$\int_{\tau_e}^{\tau_0} \frac{d\tau}{L} = \int_{\tau_e+\delta\tau_e}^{\tau_0+\delta\tau_0} \frac{d\tau}{L}. \tag{11.8}$$

Subtracting the two integrals and using that $\delta\tau_e, \delta\tau_0$ are one period of the light —incomparably smaller than the scale over which L varies (period $\sim 10^{-15}$ s against Hubble time $\sim 10^{17}$ s), so that L is constant over each interval— the boundary contributions give

$$\frac{\delta\tau_e}{L(\tau_e)} = \frac{\delta\tau_0}{L(\tau_0)} \implies \frac{\delta\tau_0}{\delta\tau_e} = \frac{L(\tau_0)}{L(\tau_e)}. \tag{11.9}$$

Wavelength and time interval identically share the factor $L(\tau_0)/L(\tau_e)$:

$$\boxed{\frac{\lambda_0}{\lambda_e} = \frac{\delta\tau_0}{\delta\tau_e} = \frac{L(\tau_0)}{L(\tau_e)} \equiv 1 + z} \tag{E-cosmo-9}$$

The definition $1 + z \equiv L(\tau_0)/L(\tau_e)$ is that of redshift as a stretching ratio; the non-trivial thing —and what is derived— is that the *time interval* share that same factor, by nullity of the TT mode, not by definition. **[DERIVED]** (falsifiers: A3, pull-back of the flat causal structure; A1, the photon as a TT mode of the medium).

Tired light, excluded by mechanism. The hypothesis of pure "tired light" —redshift without time dilation— is excluded by native mechanism, before invoking the datum: a TT mode cannot stretch its wavelength without stretching by the same factor the interval between its crests, because both are the same ratio $L(\tau_0)/L(\tau_e)$. **[DERIVED]**.

The time dilation of SN Ia, closed with the datum. The temporal width of the light curve of a SN Ia at redshift z stretches as $\Delta t_{\rm obs} = \Delta t_{\rm em}(1+z)^b$: pure tired light predicts $b = 0$, cosmological dilation $b = 1$. The program predicts $b = 1$ exact, forced: wavelength and time interval come out of the same nullity condition ($ds^2 = 0$) over the same emergent metric with the same $L(\tau)$, so that $b = 1$ is an algebraic identity, not a fit. The theorem does not depend on *which* sector of the medium generates the stretching $L(\tau)$: it holds for any homogeneous scale factor, because it only uses the nullity of the TT mode and the homogeneity of the background. The supernovae observed at $z \sim 1$–2 live in the phase sector (§11.9), homogeneous to background order, so that their $L(\tau)$ is that of the phase condensate —not that of the radial modulus, bounded by $z_{\rm max} \approx 0.268$, below— and the theorem applies equally: $b = 1$ holds throughout the observable range. The most precise available measurement (DES-SN5YR, White et al. 2024: 1504 SNe Ia, $0.1 \le z \le 1.2$) gives

$$b = 1.003 \pm 0.005\,(\text{stat}) \pm 0.010\,(\text{sys}), \tag{11.10}$$

concordant with $b = 1$ at 0.27σ, and excludes tired light ($b = 0$) at $\sim 90\sigma$. Where ΛCDM *assumes* $b = 1$ as a consequence of postulating the expanding metric, the program *derives* it as a theorem of the elastic medium: same numerical prediction, distinct depth. Any future measurement of $b \neq 1$ at high significance would refute the mechanism of the TT mode. **[DERIVED]**, without a free parameter.

The law of the radial sector and the redshift bound. By continuity of the hyperelastic continuum (mass conservation, $\rho\, J = \text{const}$ with $J = L^3$ and $\rho = \rho_P S^2$):

$$\rho_P S^2\, L^3 = \text{const} \implies S^2 L^3 = \text{const} \implies \boxed{L(S) \propto S^{-2/3}} \tag{E-cosmo-10}$$

[DERIVED] from continuity and $\rho = \rho_P S^2$ (amplitude S against occupation $\rho_P S^2$). With the radial modulus frozen at $S_{\rm min}$, the redshift accumulated over the segment $S^* \to S_{\rm min}$ is bounded:

$$(1+z)_{\rm max} = \left(\frac{S^*}{S_{\rm min}}\right)^{2/3} = \left(\frac{S_{\rm max,geom}}{S_{\rm min}}\right)^{1/3} = \left(\frac{1.4756}{0.7231}\right)^{1/3} = 1.2684 \implies z_{\rm max} \approx 0.268. \tag{11.11}$$

[DERIVED], structural (depends only on the fixed walls $\{S_{\rm min}, S_{\rm max,geom}\}$). The second equality is an exact algebraic identity: since the critical point is the geometric mean of the walls, $S^* = \sqrt{S_{\rm min}\, S_{\rm max,geom}}$ (§12, bilateral rheological structure), one has $(S^*/S_{\rm min})^{2/3} = \left((S_{\rm min} S_{\rm max,geom})^{1/2}/S_{\rm min}\right)^{2/3} = (S_{\rm max,geom}/S_{\rm min})^{1/3}$, without approximation. Numerically, $(1.4756/0.7231)^{1/3} = 1.2684$, hence $z_{\rm max} = 0.2684$. This bound forces the **two-clock structure**: the radial sector, with its redshift bounded to $z_{\rm max} \approx 0.268$, cannot produce the redshift of the supernovae ($z \sim 1$–2); therefore the clock of the observable epoch is the phase sector, not the radial one (§11.9), and it is its scale factor $L(\tau)$ that enters the shared-factor theorem above.

This same number arises, by a second independent route, from the *background dynamics* of the modulus. The background equation (§11.2) is a non-linear oscillator, $\ddot{\bar S} + 3H\dot{\bar S} + \rho_P^{-1}\ell_P^{-2} V'_{\rm eff}(\bar S) = 0$, which starts from the crystallization at S^* and descends to the attractor $S_{\rm min}$. Combined with the continuity law $L \propto S^{-2/3}$ just obtained (E-cosmo-10), the total accumulated stretching over that segment —normalizing $L = 1$ at the crystallization $S = S^*$— is

$$\frac{L(S_{\rm min})}{L(S^*)} = \left(\frac{S^*}{S_{\rm min}}\right)^{2/3} = 1.2684. \tag{11.12}$$

That the *kinematic bound* of the radial redshift (from the law $L \propto S^{-2/3}$ read over the fixed walls) and the *dynamic stretching* of the background (the same law read over the orbit of the oscillator of §11.2, from S^* to $S_{\min}$) give the same 1.2684 is not a fitted numerical coincidence: both evaluate the same law $L \propto S^{-2/3}$ over the same segment $S^* \to S_{\min}$ —the first as a bound of light phase accumulation, the second as an integrated stretching of the homogeneous background. Two readings of the same segment, the same number, without a free parameter.

Conclusion — redshift and time dilation, an identity of the medium

The redshift and the time dilation of distant sources are not two effects that must be postulated separately: they are the *same* stretching factor $L(\tau_0)/L(\tau_e)$ of the deformation map, forced to coincide by the nullity of the TT mode ($ds^2 = 0$). Hence the dilation exponent of the light curves is $b = 1$ *exact*, without a fit —confirmed at 0.27σ by DES-SN5YR and excluding tired light at $\sim 90\sigma$. And from the mass conservation of the continuum comes $L(S) \propto S^{-2/3}$, whose bound $z_{\max} \approx 0.268$ for the frozen radial sector forces the clock of the observable epoch to be the phase sector: the same two-clock structure that governs the positional reading of H_0. The value of that bound, $(S^*/S_{\min})^{2/3} = 1.2684$, comes out by two converging routes —the kinematics of the redshift and the dynamics of the background oscillator, both over the segment $S^* \to S_{\min}$—, without a single fitted parameter.

---

12 Rheology of el Pleno and the cosmic cycle

12.1 The flow index and the Herschel-Bulkley reading — [DERIVED] / [SLD]

El Pleno behaves as a dilatant non-Newtonian fluid with yield (Herschel-Bulkley). Defining the normalized packing fraction $x \equiv \rho/\rho_{\max} = S^2/S^2_{\max,\text{geom}} \in [0, 1)$, identified with the relative deformation rate $x = \dot\gamma/\dot\gamma_{\max}$ (E46) —both measure how close the medium is to its hard bound—, the reduced stress that measures the departure from rest (with $\tau \to 0$ when $x \to 0$) is $\tau_{\text{red}}(x) = x/(1-x)$, where the $(1-x)$ in the denominator is the signature of geometric jamming (A6). The *flow index* is defined as the local logarithmic slope of the flow curve, $n \equiv d \ln \tau / d \ln \dot\gamma = d \ln \tau_{\text{red}} / d \ln x$ (since $d \ln x = d \ln \dot\gamma$). With $\ln \tau_{\text{red}} = \ln x - \ln(1 - x)$,

$$n(x) = x \frac{d}{dx}\big[\ln x - \ln(1-x)\big] = x\Big(\frac{1}{x} + \frac{1}{1-x}\Big) = 1 + \frac{x}{1-x},$$

whence

$$\boxed{n(x) = \frac{1}{1-x}}, \qquad x = \frac{\dot\gamma}{\dot\gamma_{\max}}. \tag{E47}$$

$n \to 1$ (Newtonian) in the diluted vacuum and $n \to \infty$ (stiffening by jamming) upon approaching the maximal packing. *At the present rest* the packing is $x_{\text{rest}} = S^2_{\min}/S^2_{\max,\text{geom}} = 0.5229/2.1773 = 0.2401$, so that the medium today is slightly dilatant, almost Newtonian: $n_{\text{rest}} = 1/(1 - 0.2401) = 1.316$. This value is *geometric* (depends only on the fixed extremes), not positional: it does not change with the relaxation of the lattice, unlike R, f_c or w_0. The limit $n \to 1$ at $x \to 0$ is **[DERIVED]** (model-independent); the dilatant character $n > 1$ and the divergence at the bound are **[SLD]**, and the closed form $1/(1 - x)$ depends on the constitutive packing model $x = S^2/S^2_{\max,\text{geom}}$, so that it is **[SLD]**. The yield stress comes from the IR wall of Klauder: to deform the modulus from rest one must overcome the restoring response $-dV_q/dS$ of the quantum potential $V_q^{\text{dens}} = 3\hbar^2/(8\rho_P \ell_P^8 S^2)$ (E5). Differentiating and using $\hbar = \rho_P c \ell_P^4$,

$$-\frac{dV_q^{\text{dens}}}{dS} = \frac{3\hbar^2}{4\rho_P \ell_P^8 S^3} = \frac{3\rho_P c^2}{4S^3},$$

and evaluating at the ground-Pleno $S = S^{\text{estr}}_{\min}$:

$$\boxed{\tau_0 = \frac{3}{4\,(S^{\text{estr}}_{\min})^3}\,\rho_P c^2 \approx 1.9838\,\rho_P c^2} \tag{E48}$$

Verification (sympy): with $S^{\text{estr}}_{\min} = 12(\pi^2-4)/\pi^4$, $3/[4(S^{\text{estr}}_{\min})^3] = 1.98376$. The complete constitutive law is $\tau = \tau_0 + \kappa_{\text{HB}}\dot\gamma^{\,n(x)}$, with τ_0 **[DERIVED]** (IR wall of Klauder) and the exponent $n(x)$ classified above. The medium has, then, the three features of a Herschel-Bulkley fluid —yield threshold τ_0, power law, and variable index $n(x)$— all derived from F1.

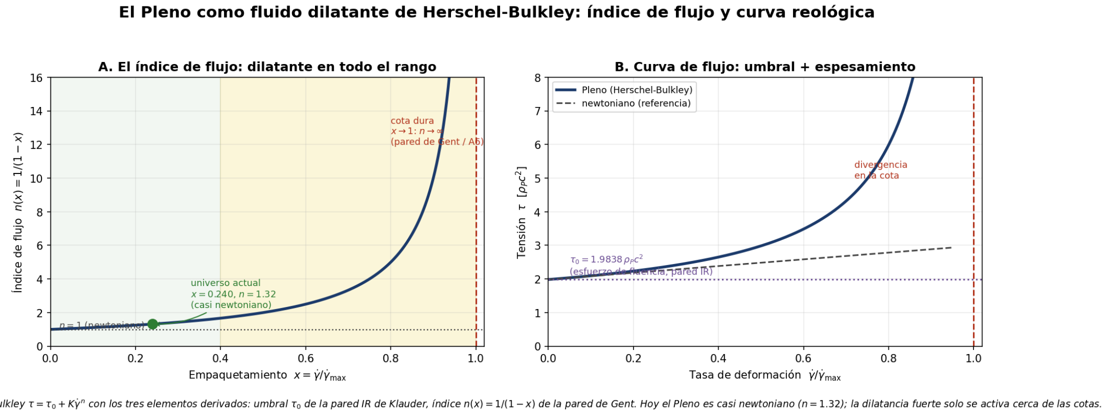

Figure 9: El Pleno as a dilatant Herschel-Bulkley fluid. The flow index $n(x) = 1/(1-x)$ passes from Newtonian ($n \to 1$) in the diluted vacuum to divergent ($n \to \infty$) at the hard bound. The present universe ($x = 0.240$, $n = 1.32$) is almost in the Newtonian regime; the strong dilatancy only activates near the saturation. The yield stress $\tau_0 = 1.9838\,\rho_P c^2$ is the IR wall of Klauder.

12.2 The maximal deformation rate $\dot\gamma_{\max} = c/\ell_P$ — [DERIVED]

In rheology the independent variable is the **deformation rate** $\dot\gamma$ ($[\text{s}^{-1}]$). Axiom A4 (UV regulation) makes the propagation operator of F1 carry the correction $\Box \to \Box(1 + \ell_P^2\Box/20)$ (dispersion E23), so that every excitation has a wave number bounded by $k_{\max} \sim \ell_P^{-1}$ and, with velocity c, frequency bounded by $\omega_{\max} = c\,k_{\max} \sim c/\ell_P$. A shear of the medium is an excitation of its modulus, hence its maximal rate is that cutoff frequency:

$$\dot\gamma_{\max} = \frac{c}{\ell_P} = \frac{1}{t_P} \approx 1.8552 \times 10^{43}\ \text{s}^{-1}, \tag{E45}$$

the **Planck frequency**, with $t_P = \ell_P/c$ (verified: $c/\ell_P = 1.8552 \times 10^{43}$). It is not a dimensional translation of c: it is the cutoff frequency that the UV regulation of F1 imposes, read as a deformation rate. The dimensionless **relative rate**:

$$x \equiv \frac{\dot\gamma}{\dot\gamma_{\max}} = \dot\gamma\,\frac{\ell_P}{c} \in [0, 1) \tag{E46}$$

coincides with the **Deborah number** $\text{De} = \tau^\star_\Pi\dot\gamma$ with $\tau^\star_\Pi = \ell_P/c = t_P$: it measures how close the deformation is to the limit rate. The viscoelastic fluid$\leftrightarrow$solid transition occurs at $\text{De} \sim 1$, which here coincides with the hard bound $x \to 1$. **[DERIVED]** from A4 and E23.

Conclusion

The packing fraction $x = \rho/\rho_{\max} = S^2/S^2_{\max,\text{geom}}$ of the flow index (E47) has thus a twin kinetic reading: $x = \dot\gamma/\dot\gamma_{\max}$. The same x that governs the dilatancy of the medium (jamming upon saturating) governs its response to the deformation rate, anchoring the Herschel-Bulkley reading to the hard bound of F1. The equality of the two readings of x is a *physical identification* (density vs. rate), not a derived equation.

12.3 The Lorentz factor as a kinetic flow index — [DERIVED]

The *same* dilatant flow index that stiffens the medium upon saturating its density, evaluated over the *kinetic* fraction used of the ceiling c, reproduces special relativity.

The analogy is structural. In the rheological sector, $x = \dot{\gamma}/\dot{\gamma}_{\max}$ measures how much of the limit deformation rate is used, and the medium stiffens as $n(x) = 1/(1-x)$. In the kinetic sector, a configuration that moves at velocity v uses the fraction $u \equiv v^2/c^2$ of its capacity for change with respect to the ceiling c —the same structure of "fraction of the limit used". Evaluating the flow index over this fraction:

$$n(u) = \frac{1}{1-u} = \frac{1}{1-v^2/c^2} = \gamma^2. \tag{E49–E50}$$

The Lorentz factor squared *is* the flow index of el Pleno with respect to the kinetic rate of change (verified symbolically). This is not a formal coincidence: in §2 the mass-shell constraint gave $E = \gamma Mc^2$, so that $E^2 = \gamma^2(Mc^2)^2 = n(u)\,(Mc^2)^2$; the relativistic energy *is* the structural energy amplified by the flow index. The **time dilation** is, literally, the dilatant stiffening of the medium when the rate of change approaches its ceiling: the time of a fast object "thickens" by the same reason that el Pleno thickens upon saturating its density. This re-obtains, by the rheological route, the result of §2 (mass shell), closing the consistency between the two readings —the relativity that *opened* the chain (link 1) reappears as a particular case of the rheology that *closes* it.

12.4 The cycle equation: damped Gent oscillator — [SLD]

Here the rheology becomes cosmology. The homogeneous background $\bar{S}(t)$ —the mean density of the universe— obeys a non-linear oscillator equation, where the Gent potential provides the restoring force with its hard wall at $S_{\max}$, and the dilatant viscosity provides the damping:

$$\rho_P \ell_P^2 \ddot{\bar{S}} + \underbrace{\eta(\bar{S})\,\dot{\bar{S}}}_{\text{dilatant dissipation}} + V'_{\text{eff}}(\bar{S}) = 0, \tag{E57}$$

with effective viscosity $\eta(\bar{S}) \propto \dot{\gamma}^{\,n(x)-1}$ that *grows* toward the packing (the medium thickens upon collapsing). The structure is that of a pendulum in a well with an infinitely hard wall at one end: the **hard bound $S_{\max}$ of Gent prevents the collapse from reaching infinite density**. When the universe contracts and $\bar{S}$ grows toward $S_{\max}$, the term V'_{eff} diverges (the Gent wall, $\propto 1/(S_{\max}^2 - \bar{S}^2)^3$) and *brakes and reverses* the collapse before the singularity: the crunch **bounces**. There is no singular Big Bang —there is a bounce from the finite maximal density $\rho_{\max} = S_{\max,\text{geom}}^2 \rho_P \approx 2.1773\,\rho_P$.

Three destinies according to the collapse energy.

- **Subcritical**: the energy is not enough to reach $S_{\max}$; damped oscillation that relaxes to the ground-Pleno $S_{\min}^{\text{estr}}$ (does not bounce to a new universe).
- **Critical**: the collapse reaches $S_{\max}$ and bounces completely (crunch-bounce), initiating a new cosmic cycle.
- **Supercritical (local)**: incomplete crystallization of a region $\to$ primordial black hole, whose core remains frozen at the universal density $2.1773\,\rho_P$.

The **minimal collapse mass** is the scale below which the flexural rigidity prevents the trapping:

$$M_{\min} = \sqrt{\frac{3}{32\pi\, S_{\max,\text{geom}}^2}}\; m_P \approx 0.117\, m_P,$$

(the same scale that regulates the regular Schwarzschild core, §7.3): below M_{min} there is no horizon, the curvature that the flexural penalizes is too large for the trapping. It is a falsifiable prediction: a spectrum of primordial black holes with a lower cutoff at $\sim 0.1\, m_P$ and cores at universal density.

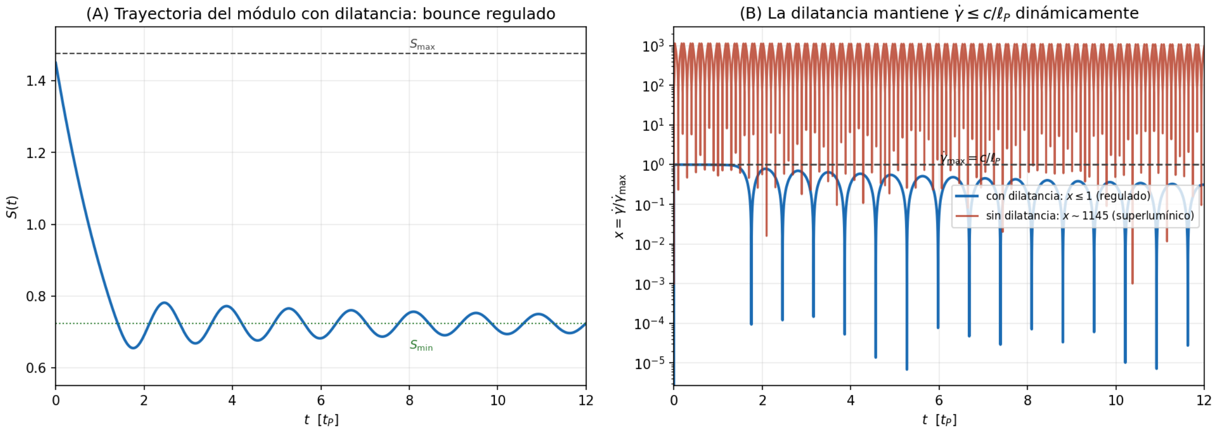

Figure 10: Numerical integration of the cycle equation (E57). (A) Trajectory of the modulus $\bar{S}(t)$ with the dilatant viscosity: the collapse starts near $S_{\max}$, the Gent wall reverses the crunch *before* the hard bound (finite bounce, without singularity) and the oscillation is damped relaxing to the ground-Pleno $S_{\min}$. (B) The dilatancy keeps the deformation rate under its ceiling $\dot\gamma \leq c/\ell_P$ dynamically: with dilatancy (blue) the medium never crosses $x = 1$, whereas without it (red) the rate would shoot to superluminal values ($x \sim 10^3$). The dilatant thickening is what regulates the bounce.

Conclusion — a single rheology

A single flow index, $n(x) = 1/(1-x)$, unifies three phenomena: the stiffening by jamming of the medium (late cosmology), the Lorentz dilation and the viscosity that damps the cosmic cycle. The Lorentz factor $\gamma^2 = n(v^2/c^2)$ is a particular case of that same rheology: relativity and the cycle of the universe are two readings of the same dilatant thickening of el Pleno. The physical consequence is that the universe does not arise from a singularity nor dies in one: the hard bound of the modulus converts the Big Bang into a bounce from finite density and forbids the infinite everywhere —at the cosmic origin and at the center of each black hole, whose core remains at the universal density $2.1773\, \rho_P$. The singularity of general relativity does not occur: the medium is too rigid to allow it. The core at $2.1773\, \rho_P$ and the cutoff $M_{\text{min}} \approx 0.117\, m_P$ are falsifiable predictions.

12.5 The viscous crystallization transient: how the partition is imprinted — [DERIVED] / [SLD]

The matter / dark-energy partition is fixed by the phase freezing at S^*, and the crystallization point S^* by the rheological route. Between the two is the dynamical link: how the modulus relaxes from S^* to its minimum S_{min}, in how much time, and why the partition survives intact to today. The cycle equation (E57) with the derived effective potential (E52) and the dilatant Gent viscosity closes that link *without a single free parameter*:

$$\rho_P \ell_P^2\, \ddot{S} + \eta(\dot\gamma)\, \dot{S} + V'_{\text{eff}}(S) = 0, \qquad \eta(\dot\gamma) = \frac{\eta_0}{1 - \dot\gamma/\dot\gamma_{\max}}, \qquad \dot\gamma = 2\frac{|\dot{S}|}{S}, \tag{12.1}$$

with $\dot\gamma_{\max} = c/\ell_P = 1/\tau_P$ the hard bound (A6) and the effective potential (E52)

$$V_{\text{eff}}(S) = \frac{E_0}{2}\frac{S^4}{(S^2_{\text{max,geom}} - S^2)^2} - \frac{\rho_P c^2}{2}\ln\Big(1 - \frac{S^2}{S^2_{\text{max,geom}}}\Big) + \frac{3\rho_P c^2}{8S^2}. \tag{12.2}$$

The coefficients are all derived. In units of el Pleno ($\rho_P = c = \ell_P = 1$, $\dot\gamma_{\max} = 1$), the bounds are pure ratios of the lattice: $S_{\min} = 12(\pi^2-4)/\pi^4 = 0.7231$, $S^2_{\max,\mathrm{geom}} = 8\sqrt{6}/9 = 2.1773$, $S^* = \sqrt{S_{\min}\, S_{\max,\mathrm{geom}}} = 1.0329$. The viscosity is the Maxwell relation of the shear sector with its two derived factors, $\mu_{\mathrm{Gent}} = \rho_P c^2/S^2_{\max,\mathrm{geom}}$ and $\tau_\Pi = \ell_P/c = \tau_P$ (Deborah number unity), so that $\eta_0 = \mu_{\mathrm{Gent}}\tau_\Pi = 1/S^2_{\max,\mathrm{geom}} = 0.4593$. The coefficient $E_0 = 4.2546$ is fixed by $V'_{\mathrm{eff}}(S_{\min}) = 0$, and the curvature of the minimum reproduces the structural Higgs, $V''_{\mathrm{eff}}(S_{\min}) = \xi = 19.69\,\rho_P c^2$. Three cross verifications confirm the consistency: the Klauder coefficient comes out $c_k = 0.3750 = 3/8$ (the value that the affine quantization fixes independently); the three walls sum the curvature, $V''_q(8.230) + V''_{\mathrm{topo}}(0.986) + W''_{\mathrm{bare}}(10.473) = 19.69 = \xi$; and the densities of the critical points reproduce $\rho(S^*) = 1.067\,\rho_P \approx \rho_P$ (Planck, where the space recrystallizes) and $\rho(S_{\min}) = 0.523\,\rho_P$ (the present relaxed sea). **[DERIVED]**, without a free parameter.

The deformation bound self-limits the fall. The natural frequency of the well is $\omega_S = \sqrt{\xi} = 4.44/\tau_P$, far above $\dot\gamma_{\max} = 1/\tau_P$. A modulus in free oscillation would reach $\dot\gamma \approx 2\omega_S A/S_{\min} \approx 3.8 \gg 1$, impossible: the Gent viscosity diverges when $\dot\gamma \to \dot\gamma_{\max}$, braking the motion before reaching the bound. The numerical integration of the EOM with the derived coefficients confirms that the rate never reaches its bound —the maximum is $0.98 < 1$. The dilatant rheology —the same one that gives special relativity as a stiffening upon approaching c (§12.3)— imposes that the fall respect the deformation limit: the A6 bound acts here on the *velocity* of the radial mode, not on its amplitude.

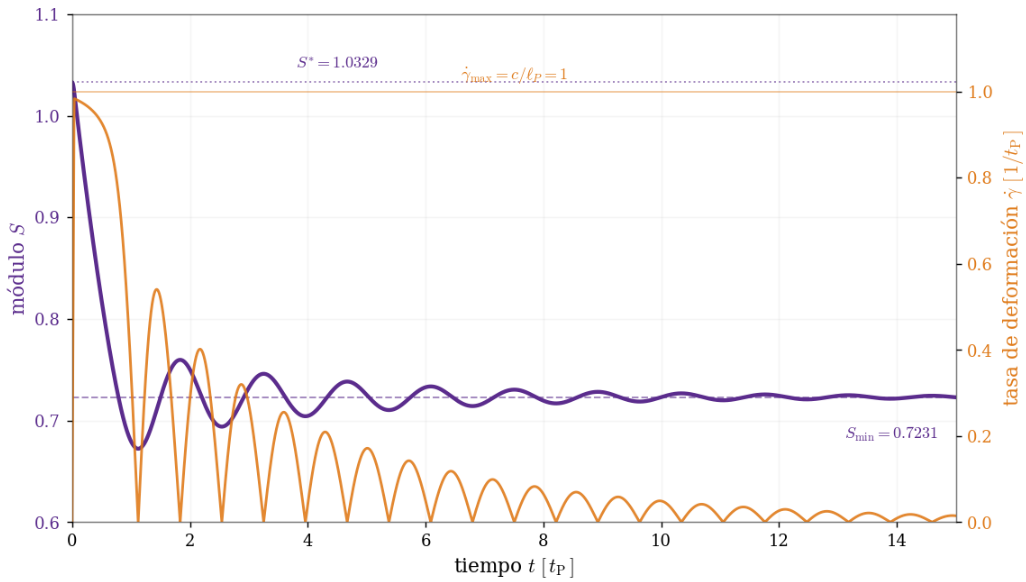

Figure 11: Trajectory of the modulus during the crystallization transient, integrating the viscous equation of motion with all the derived coefficients. The modulus $S(t)$ (violet) descends monotonically from $S^* = 1.033$ and overshoots $S_{\min}$ by inertia to a minimum of 0.672 at $t \approx 1.1\,t_P$, after which it oscillates with decreasing amplitude and settles at $S_{\min} = 0.7231$. The deformation rate $\dot\gamma(t)$ (orange) reaches a maximum of 0.98 —grazing the Planck bound $\dot\gamma_{\max} = c/\ell_P = 1$ without exceeding it— during the fast descent, and pulses toward zero at each turning point (where $\dot S = 0$). The decreasing peaks of $\dot\gamma$ are the signature of viscous damping. Time is in Planck ticks.

The transient is very brief; the rest, lasting. With the self-limited fall, the modulus settles at $S_{\min}$ in a few t_P; the permanent entry into the 1% of $S_{\min}$ occurs at $t_{\mathrm{settle}} \sim t_P = 5.4 \times 10^{-44}$ s. Over the age of the universe ($\sim 4.4 \times 10^{17}$ s), the transit occupies a fraction $\sim 10^{-61}$. This fixes what dark energy is dynamically: during $99.99\ldots\%$ of the history, the radial mode is still at its minimum, where its contribution is that of the DE-Curtain with $w = -1$ structural (§11.3). There is no coherent radial oscillation that persists and dilutes over eons —the viscosity damps it at the Planck scale. Matter does not come from that oscillation, but from the topological defects formed by the intense shear of the hot transient, persistent like dust, $\rho \propto a^{-3}$.

Simulation consistency, not proof

The numerical integration of the EOM reproduces independently all the numbers of the transient ($E_0 = 4.2546$, $\xi = 19.69$, $\eta_0 = 0.4593$, $\dot{\gamma}_{\max}$ not exceeded with maximum 0.98, minimum $S = 0.672$ at $t \approx 1.1\,t_P$, settling in a few t_P). This is simulation *consistency* with the derived coefficients, not a closed analytical proof of the trajectory: the closed form of $\bar{S}(t)$ integrated under FLRW remains as open end F-cosmo. What is established without fitting is the mechanism —transient $\sim t_P$, rest for the remainder of the history— by which the geometric partition f_c is imprinted and survives. [**SLD**].

12.6 Incompressible Navier-Stokes as the hydrodynamic limit of F1 — [SLD]

Madelung dictionary. The decomposition $\Psi = S\,e^{i\theta}$ translates F1 to fluid variables with identifications fixed by the conserved quantities:

$$\rho \equiv \rho_P\,S^2 \text{ (density)}, \qquad \mathbf{v} \equiv c^2\ell_P^2\,\nabla\theta \text{ (velocity)}.$$

The coefficient $c^2\ell_P^2$ is fixed by the U(1) Noether current (E7c): its spatial component $J^i = \rho_P c^2\ell_P^2 S^2\,\partial^i\theta$ equals the mass flux $\rho v^i = \rho_P S^2 v^i$, whence $v^i = c^2\ell_P^2\partial^i\theta$. The flow is irrotational in the pure-phase sector ($\mathbf{v} = \nabla\phi$, $\phi = c^2\ell_P^2\theta$); the vorticity resides in the topological defects of θ.

Continuity. In the IR limit ($L \gg \ell_P$) the flexural term of the phase equation E7 is suppressed by $\ell_P^2 k^2 \to 0$ and remains $\partial_\mu(S^2\partial^\mu\theta) = 0$. Separating in time and space and multiplying by $\rho_P c^2\ell_P^2$:

$$\boxed{\partial_t\rho + \nabla\cdot(\rho\,\mathbf{v}) = 0} \tag{12.3}$$

The mass conservation of the fluid is the conservation of the U(1) Noether current. [**DERIVED**].

Euler. The IR phase dynamics is a Hamilton-Jacobi equation for ϕ:

$$\partial_t\phi + \tfrac{1}{2}|\nabla\phi|^2 + \frac{\mu(\rho)}{\rho_P} + Q = 0, \qquad Q = -\frac{c^2\ell_P^2}{2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \text{ (Bohm potential)}.$$

Taking the gradient and using $(\nabla\phi\cdot\nabla)\nabla\phi = (\mathbf{v}\cdot\nabla)\mathbf{v}$:

$$\boxed{\partial_t\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \nabla Q} \tag{12.4}$$

with barotropic pressure $p(\rho)$ from $V + V_q$ and the Bohm quantum correction. [**DERIVED**].

The origin of the viscous term. The Madelung derivation is conservative; the viscous term is not added by hand, but comes from the flexural correction of the Noether current (E7c). In the non-relativistic IR limit $\Box\theta \approx \nabla^2\theta$, and the exact identity is verified

$$\partial^i(\nabla^2\theta) = \nabla^2(\partial^i\theta) = \frac{1}{c^2\ell_P^2}\nabla^2 v^i,$$

so that the current of el Pleno has the diffusive form $\mathbf{J} = \rho\mathbf{v} + (\text{coef})\nabla^2\mathbf{v}$. A current $\propto \nabla^2\mathbf{v}$ in the transport of momentum is a viscous term. Closing the coefficient with τ_Π (Maxwell-Kubo), the viscosity is the already-derived $\eta_{\text{Pleno}} = \mu_{\text{Gent}}\tau_\Pi$. The flexural term $(\Box S)^2$ —the ultraviolet regulator— is the physical source of the infrared viscosity.

The diluted limit. Three reductions controlled by $x = S^2/S^2_{\text{max,geom}}$: (a) $Q \to 0$ at $L \gg \ell_P$; (b) $n(x) \to 1$ at $x \to 0$ (Newtonian), which contributes $\eta\nabla^2\mathbf{v}$; (c) Mach $\to 0$, $\rho \approx$ const, $\nabla\cdot\mathbf{v} = 0$. The result:

$$\boxed{\partial_t\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\,\nabla^2\mathbf{v}, \qquad \nabla\cdot\mathbf{v} = 0} \tag{12.5}$$

with native kinematic viscosity $\nu = \eta_{\text{Pleno}}/(\rho_P S^2) = \sqrt{5}\,c\,\ell_P/(10\,S^2\,S^2_{\text{max,geom}})$, consistent with $\nu_{\text{TT}} = (1/\sqrt{20})c\ell_P$ in the pure-shear sector. [**SLD**]: continuity, Euler, Bohm and the Newtonian limit are [**DERIVED**]; the identification of the viscous term with the flexural (verified symbolically) is [**SLD**]; by Rule 19 the set is [**SLD**]. Navier-Stokes appears as an effective limit of F1, not as origin.

12.7 Global regularity of the flow of el Pleno — [SLD]

Scope

It is not the proof of the Clay problem (pure Navier-Stokes: constant viscosity, without kinematic bound). The flow of el Pleno possesses the A6 bound and divergent viscosity; the regularity *of el Pleno* is established, whose regularization comes from its departing from Navier-Stokes where the latter could blow up.

Kinematic bound. The maximal deformation rate is finite (E45): $\dot\gamma = \sqrt{2S_{ij}S_{ij}} \le c/\ell_P$, with $S_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i)$ and $x = \dot\gamma/\dot\gamma_{\max} \in [0, 1)$ always.

Beale-Kato-Majda criterion. There is a singularity at T^* if and only if $\int_0^{T^*} \|\boldsymbol{\omega}\|_\infty \, dt = \infty$. The A6 bound limits the deformation S_{ij}, not directly the vorticity (a rigid rotation has $S_{ij} = 0$, $\boldsymbol{\omega} \ne 0$). The only route to blow-up is vortex stretching, of rate

$$\frac{1}{2}\frac{D|\boldsymbol{\omega}|^2}{Dt}\bigg|_{\text{stretch}} = \omega_i \, S_{ij} \, \omega_j,$$

so that only the deformation S_{ij} stretches the vortices, and the A6 bound controls the growth of $\boldsymbol{\omega}$ by the physically correct route.

At most exponential growth. From $\omega_i S_{ij} \omega_j \le \|S\|_{\text{op}} |\boldsymbol{\omega}|^2$ and $\|S\|_{\text{op}} \le \dot\gamma_{\max}/\sqrt{2}$:

$$\frac{d|\boldsymbol{\omega}|}{dt} \le \frac{\dot\gamma_{\max}}{\sqrt{2}} |\boldsymbol{\omega}| \implies |\boldsymbol{\omega}(t)| \le |\boldsymbol{\omega}(0)| \, e^{\dot\gamma_{\max} t/\sqrt{2}},$$

and the BKM integral converges for every finite T:

$$\int_0^T \|\boldsymbol{\omega}\|_\infty \, dt \le \|\boldsymbol{\omega}(0)\|_\infty \frac{\sqrt{2}}{\dot\gamma_{\max}} (e^{\dot\gamma_{\max} T/\sqrt{2}} - 1) < \infty.$$

There is no finite-time singularity: the flow is globally regular. The dynamical reinforcement is given by the Herschel-Bulkley viscosity, which diverges upon approaching the bound ($\eta_{\text{eff}} \propto 1/(1-x)^2 \to \infty$): negative feedback that makes the growth sub-exponential. **[SLD]**.

12.8 The singularity under emergent time — [SLD]

Scope

Reinterpretation of the terms of the problem: under the definition of time of PIU (time as accumulated change), the notion of finite-time singularity dissolves.

PIU derives time, does not postulate it: the internal change $C \in [0, c]$ has ceiling c, and the physical time is the accumulated change (canonical definition in corpus §I.8.9-bis: $\tau = (1/c) \int_\gamma C \, d\lambda$), $\tau_{\text{phys}} \propto \int \|\nabla \mathbf{v}\| \, dt$, which is identically the integrand of the BKM criterion. From this, exactly:

$$\boxed{\text{blow-up (BKM)} \iff \int \|\nabla \mathbf{v}\| \, dt = \infty \iff \tau_{\text{phys}} = \infty} \tag{12.6}$$

A blow-up is, by its own mathematical definition, an event at infinite physical time. For the self-similar blow-up $\|\nabla \mathbf{v}\| = A/(T^* - t)$:

$$\tau_{\text{phys}}(T) \propto \int_0^T \frac{A \, dt}{T^* - t} = A \ln \frac{T^*}{T^* - T} \xrightarrow{T \to T^*} \infty.$$

The gradient diverges in finite coordinate time T^*, but the physical time diverges logarithmically: the singularity is never reached in finite physical time. A blow-up would require infinite local change (forbidden by A1, $C \le c$; the A6 bound is A1 plus the grain, $\dot\gamma_{\max} = c/\ell_P$) in finite physical time (impossible, since τ_{phys} diverges): infinite change is not accumulated in finite accumulated change. **[SLD]**, internal to PIU.

12.9 Bilateral rheological structure and the critical point S^* — [DERIVED] / [SLD]

The modulus lives between two walls: the IR of Klauder ($S \to S_{\min}$, A2) and the hard of Gent ($S \to S_{\max}$, A6). The flow index (E47) measures the proximity to Gent, and its mirror toward the IR wall —by the same procedure of logarithmic slope of the restoring stress— is m:

$$n(x) = \frac{1}{1-x} = \frac{S_{\max}^2}{S_{\max}^2 - S^2}, \quad x = \frac{S^2}{S_{\max}^2}; \qquad m(\bar{y}) = \frac{1}{1-\bar{y}} = \frac{S^2}{S^2 - S_{\min}^2}, \quad \bar{y} = \frac{S_{\min}^2}{S^2}.$$

The bilateral index $N(S) = n + m$ diverges at both walls and has a minimum (minimal rigidity) at

$$\boxed{S^* = \sqrt{S_{\min} S_{\max}} \approx 1.0330} \tag{12.7}$$

the exact geometric mean of the walls (with $S_{\min} = 0.7231$ and $S_{\max} = S_{\max,\text{geom}} = \sqrt{8\sqrt{6}/9} \approx 1.4756$). At S^* the packing fractions are identical, $x^* = \bar{y}^* = S_{\min}/S_{\max} \approx 0.4900$ (exact). **[DERIVED]**.

Robustness. Five definitions of the bilateral index (sum $n+m$, product nm, log-sum, excess, inverse) give the same S^*: the root is the inversion symmetry $S \to S_{\min}S_{\max}/S$, which exchanges the two walls ($x \leftrightarrow \bar{y}$, Gent↔Klauder) and of which S^* is the only fixed point. Every symmetric combination of n and m is extremized at the self-symmetric point. **[DERIVED]**.

S^* as a crystallization point. S^* is not a force equilibrium (the minimum of the potential is $S_{\min}$) but the minimal-rigidity one, where the elastic barrier to reorganize the lattice is minimal; therefore the packing transition ($Z = 12 \leftrightarrow Z = 4$) occurs at S^*, at density

$$\rho(S^*) = x^* \rho_{\max} = \frac{S_{\min}}{S_{\max}} S_{\max,\text{geom}}^2 \rho_P \approx 1.067\, \rho_P \approx \rho_P,$$

that is, the recrystallization of space occurs upon crossing the Planck density. **[SLD]**.

Conclusion — the hydrodynamics of el Pleno

Incompressible Navier-Stokes is the IR limit of F1, with the viscous term as an infrared imprint of the ultraviolet flexural. The flow of el Pleno is globally regular (A6 bound plus divergent viscosity), and under the emergent time the singularity lives at $\tau_{\text{phys}} = \infty$. The modulus has a self-symmetric point $S^* = \sqrt{S_{\min}S_{\max}}$ of minimal rigidity, where it crystallizes upon crossing $\rho \approx \rho_P$.

Conclusion — what it means physically

The universe flows like an ordinary viscous fluid, and cannot blow up. In its smooth and slow regime, el Pleno obeys the same Navier-Stokes equations as water or air —the viscosity that governs them is not added: it is the same grain rigidity that at fine scale regulates the medium, seen from afar as friction. And since every real material has a finite rigidity and a maximal deformation velocity, the flow of el Pleno never develops the infinities ("blow-up") that torment ideal hydrodynamics: a whirlpool can only sharpen up to the limit that the grain allows, and the viscosity grows without ceiling just when it comes closest, braking it. The point S^* —halfway between the two walls, where the medium is most fluid— is where the vacuum recrystallizes upon compressing to the Planck density. El Pleno behaves, in sum, like a genuine physical fluid: viscous, regular, and with a definite phase-transition point.

12.10 The inversion self-duality: generating operator and conformal inversion of the field — [DERIVED]

The inversion symmetry $I : S \to S_{\min}S_{\max}/S$ of the critical point admits two deeper readings: that the operator I not only *detects* the self-duality but *constructs* it from a single wall, and that over the complete field $\Psi = S\,e^{i\theta}$ the inversion of the modulus is the conformal inversion of the complex plane. Both follow from F1 without a new parameter.

The operator generates the mirror wall. In the logarithmic variable $u = \ln(S/S^*)$, the inversion I is the exact reflection $u \to -u$, with $I(S^*) = S^*$ and $n(u) = m(-u)$. Applied to the only geometric wall of F1, that of Gent,

$$V_{\text{topo}}(S) = -\frac{\rho_P c^2}{2}\ln\Big(1 - \frac{S^2}{S^2_{\text{max,geom}}}\Big), \tag{12.8}$$

the operator generates its mirror image toward the lower wall:

$$I(V_{\text{topo}})(S) = V_{\text{topo}}\Big(\frac{S_{\text{min}}S_{\text{max}}}{S}\Big) = -\frac{\rho_P c^2}{2}\ln\Big(1 - \frac{S^2_{\text{min}}}{S^2}\Big), \tag{12.9}$$

an IR wall of Gent form, divergent at $S \to S_{\text{min}}$. The exact self-dual potential is the canonical symmetrization of the real wall under the idempotent projector $P_D = \frac{1}{2}(1+I)$:

$$\boxed{V_D = (1+I)\,V_{\text{topo}} = V_{\text{topo}} + I(V_{\text{topo}}), \qquad P_D^2 = P_D} \tag{12.10}$$

exactly even in u by construction. The upper Gent wall contains its own lower wall under inversion: the self-duality is not postulated, the operator I constructs it from a single geometric wall. **[DERIVED]** (via the Language of el Pleno).

The measured breaking: Klauder is not Gent. The *physical* potential of F1 is not V_D: its lower wall is the quantum one of Klauder ($V_q = 3\rho_P c^2/8S^2$, §2.3), not the geometric IR wall that I generates. For exact self-duality, the coefficient of the real IR wall (Klauder, $3/8 = 0.375$) would have to equal that of the generated one (Gent-IR, divergence coefficient $S^2_{\text{min}} = 0.5229$). They do not coincide:

$$\frac{S^2_{\text{min}}}{3/8} = \frac{0.5229}{0.375} = 1.394, \qquad S^2_{\text{min}} - \frac{3}{8} = 0.148. \tag{12.11}$$

The ratio 1.394 does not correspond to any known invariant of the program (were discarded $\sqrt{2}$, $\pi^2/7$, $1/f_c$ as false friends). It is not a failure but physical information: 0.148 measures how *quantum* (Klauder, from A2) and not *geometric* (Gent, from A6) the IR wall is. The self-duality would be exact if the IR wall were purely geometric; that it be of Klauder is what breaks it, by a computable quantity. The generalized force $Q(u) = S\,V'(S)$ of the physical potential $V = V_q + V_{\text{topo}}$ is not odd under $u \to -u$, confirming that $u \to -u$ is not a symmetry of the dynamical potential. **[DERIVED]**.

The phase is not a spectator. The kinetic term of F1 is $\frac{1}{2}\rho_P c^2 \ell_P^2[(\partial S)^2 + S^2(\partial\theta)^2]$. The factor S^2 that weights the phase makes it rigid where S is large (Gent) and soft where S is small (Klauder). Under I that weight is inverted: $S^2 \to S^{*4}/S^2$. The complete symmetry that leaves F1 invariant is not only $u \to -u$, but an inversion combined with a dilation of the phase gradient by e^{2u}. **[DERIVED]**.

The inversion of the modulus is the conformal inversion of the field. Over $\Psi = S\,e^{i\theta}$, the inversion of the modulus is exactly the conformal inversion of the complex plane with respect to the circle of radius S^*:

$$\boxed{\Psi \longrightarrow \frac{S^{*2}}{\overline{\Psi}}} \tag{12.12}$$

Direct verification: with $\overline{\Psi} = S\,e^{-i\theta}$,

$$\frac{S^{*2}}{\overline{\Psi}} = \frac{S^{*2}}{S\,e^{-i\theta}} = \frac{S^{*2}}{S}\,e^{i\theta}, \tag{12.13}$$

which inverts the modulus ($S \to S^{*2}/S$, with $S^{*2} = S_{\text{min}}S_{\text{max}}$) and *preserves* the phase θ. It is the classical operation of spherical mirror —circle inversion— in the plane Ψ, the same that in the theory of complex functions generates the Droste effect via $z \to \log z$. The self-duality of el Pleno is not an analogy with the Droste: it is literally the conformal inversion of the field. **[DERIVED]**.

The invariant set is a circle, not a point. Over the isolated modulus, $S^* \approx 1.0330$ was a fixed point. Over the complete plane Ψ, the set of fixed points of the conformal inversion is the *entire* circumference $|\Psi| = S^*$: the self-duality has no center but an invariant circle. The plane Ψ is genuinely circular (radius = modulus S, angle = phase θ); the inversion reflects each interior point ($S < S^*$, toward Klauder) to an exterior one ($S > S^*$, toward Gent) over the same radius, conserving the phase. The central hole $|\Psi| < S_{\min}$ is the region forbidden by the Klauder wall, where the phase would remain undefined at $\Psi = 0$ —exactly what the strict positivity of A2 prevents. The modulus-only reading is the shadow, in the modulus, of this operation over the complete field. **[DERIVED]**.

What this self-duality fixes and what it does not

The geometric structure of el Pleno under inversion is exact: the operator I generates the mirror wall from a single wall (projector P_D), the fixed point S^* and the invariant circle $|\Psi| = S^*$ are exact, and the inversion of the modulus is the conformal inversion $\Psi \to S^{*2}/\overline{\Psi}$ of the field. What is broken is the self-duality of the *dynamical* potential: the physical IR wall is of Klauder (quantum), not the Gent-IR that I would generate, and the difference —$S_{\min}^2 - 3/8 = 0.148$, ratio 1.394— measures how quantum and not geometric that wall is. The self-similarity of el Pleno is bounded by construction: the minimal scale ℓ_P of axiom A4 cuts the zoom from below and the coherence scale from above, so that F1 forbids the strict fractal and admits only self-similarity in a finite band —the class to which every real physical fractal belongs. **[DERIVED]**.

12.11 Consequences: viscosity bound and sonic horizon — [DERIVED] / [SLD]

The cold vacuum is an elastic solid, not a fluid. In the relaxed regime the dark energy has $w = -1$ (§11.3), hence $\rho + p = 0$ and the entropy density of the ground-Pleno is null, $s = (\rho + p)/T = 0$: a pure coherent condensate of minimal entropy, which reinforces the DE-Curtain reading from thermodynamics. **[DERIVED]** (consequence of $w = -1$).

The thermal Pleno and the KSS bound. In high excitation (bounce, interior of a black hole, early universe) the TT modes ($g_* = 2$, two polarizations) form a radiation-type gas with $s = (2\pi^2/45)\, g_*\, k_B\, (k_B T/\hbar c)^3$. For any fluid, the shear-viscosity / entropy ratio is bounded by the Kovtun-Son-Starinets limit:

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}, \tag{E51}$$

saturated in conformal theories with a gravitational dual and approximated in the quark-gluon plasma ($\sim$ 2–2.5 times the bound). The ratio of the TT gas is bounded from two sides without the complete collisional kinetics.

The phase of the ratio. Pairing the viscosity of the saturated solid $\eta_{\text{Pleno}} = \mu_{\text{Gent}}\, \tau_{\text{II}}$ with the entropy of the radiation gas is inconsistent: they are distinct phases of the medium (elastic solid vs TT gas). The ratio so formed, evaluated at the saturation temperature, gives $\eta/s \approx 0.60\, \hbar/4\pi k_B$, below the universal bound —physically impossible, which betrays the pairing. The correct computation uses the viscosity of the TT-mode gas itself.

The TT gas ratio in the relaxation-time approximation. For an ultrarelativistic gas of quasiparticles (RTA, Boltzmann),

$$\eta = \frac{1}{5}(\varepsilon + P)\, \tau_{\text{col}}, \qquad s = \frac{\varepsilon + P}{T},$$

with $\varepsilon = \rho_{\rm rad}$ and $P = \rho_{\rm rad}/3$. The ratio cancels the density:

$$\boxed{\frac{\eta}{s} = \frac{1}{5}\,\tau_{\rm col}\,T} \tag{12.14}$$

independent of $\rho_{\rm rad}$: it depends only on the product $\tau_{\rm col}\,T$.

Maximal temperature by A6 saturation. The maximal physical temperature of the TT gas is that at which its energy density reaches the maximal density of el Pleno (saturation A6). With Stefan-Boltzmann ($g_* = 2$) and the native identity $\hbar = \rho_P c\,\ell_P^4$,

$$\rho_{\rm rad}(T) = \frac{\pi^2}{30}\,g_*\,\frac{(k_B T)^4}{(\hbar c)^3}, \qquad \rho_{\rm rad}(T_{\rm max}) = \rho_{\rm max} = S^2_{\rm max,geom}\,\rho_P c^2,$$

$$k_B T_{\rm max} = \Big(30\,S^2_{\rm max,geom}\,\rho_P c^2\,(\hbar c)^3/\pi^2 g_*\Big)^{1/4} = \frac{\sqrt[4]{5}\,6^{7/8}}{3\sqrt{\pi}}\,\rho_P c^2\,\ell_P^3.$$

Two-sided bound. The lower bound is the universal E51, $R \equiv (\eta/s)/(\hbar/4\pi k_B) \geq 1$. The upper bound is structural: no process of el Pleno relaxes below the grain ℓ_P, so that $\tau_{\rm col} \geq \tau_{\rm II} = \ell_P/\sqrt{20}\,c$. Evaluating $\eta/s = \frac{1}{5}\tau_{\rm II}T$ at $T_{\rm max}$:

$$R(\tau_{\rm II}, T_{\rm max}) = \frac{2\cdot 5^{3/4}\,6^{7/8}\sqrt{\pi}}{75} \approx 0.758 < 1.$$

That the UV floor $\tau_{\rm II}$ fall below E51 is informative: the universal bound forces $\tau_{\rm col} \geq \tau_{\rm II}/0.758 \approx 1.319\,\tau_{\rm II}$. The upper extreme is fixed by the natural thermal time $\tau_T = \hbar/k_B T_{\rm max}$, with $\tau_T/\tau_{\rm II} \approx 3.32$, corresponding to $\eta/s \approx 2.5\,$E51. The ratio remains in a narrow window:

$$\boxed{1 \lesssim \frac{\eta/s}{\hbar/4\pi k_B} \lesssim 2.5} \tag{12.15}$$

the strip of the quark-gluon plasma, the most perfect fluid known experimentally. [**SLD**]: universal KSS lower bound, upper bound by the native UV scale $\tau_{\rm II}$ evaluated at $T_{\rm max}$; the exact value within the window requires the precise collisional relaxation time and remains as a bounded open end.

The event horizon is the sonic surface of el Pleno. The velocity of the modes is c (TT mode). At the horizon the escape velocity equals c, hence the horizon is the surface where the flow of the medium reaches its sound velocity —analog of Unruh, here exact. The Mach number is $\mathrm{Ma} = v/c$, and the impossibility of free supersonic flow is the same bound as the light barrier and the dilatant jamming; the Hawking radiation is read as radiation from that sonic surface. [**Strong logical deduction**]: the identity $v_{\rm sound} = v_{\rm escape} = c$ is exact; the acoustic reading is an interpretation consistent with the black-hole physics of the program.

Conclusion

The rheology of el Pleno delivers two closing readings: the cold vacuum is a solid ($s = 0$, derived) and the thermal Pleno is a near-perfect fluid whose ratio η/s is bounded from two sides, $1 \lesssim (\eta/s)/(\hbar/4\pi k_B) \lesssim 2.5$ —the strip of the near-perfect fluid, by the universal KSS bound below and the native UV scale $\tau_{\rm II}$ at the saturation temperature above. The horizon as a sonic surface unifies the light barrier, the dilatant jamming and the escape velocity in a single bound $\mathrm{Ma} \leq 1$.

Conclusion — what it means physically

The vacuum changes state according to its temperature, like water. Cold and relaxed, el Pleno is a perfect solid: it does not flow, has no internal friction, generates no entropy —it is the vacuum at rest. Heated to the extreme (in the bounce, inside a black hole, in the newborn universe), it becomes a near-perfect liquid, with the minimal friction that the quantum laws allow: exactly the regime of the quark-gluon plasma that is created in heavy-ion colliders, the most perfect fluid ever measured in the laboratory. That the hot vacuum of the universe and that drop of ultradense matter of the laboratory share the same fluidity strip is not a coincidence: both are the same fundamental medium in its thermal phase. And the horizon of a black hole is, in this reading, simply the surface where that fluid reaches the sound velocity —it cannot flow faster, just as air cannot in a nozzle.

---

13 The complete derivational chain — [DERIVED] / [SLD] / [CALIBRATED]

Figure 12 traces, step by step and without gaps, the two longest chains that depart from the root statement E (A1): a common trunk —the genesis A1→F1, and from *evaluating* F1 the geometry of the diamond lattice with its two pure ratios— from which, at a bifurcation point, hang the cosmological branch (up to $\Omega_m = 1 - f_c$ and $H(z)$) and the quantum branch (up to the uncertainty relation with floor $\hbar/2$ exact). Each box carries its equation and the color of its epistemic classification, so that the structure is that of a continuous derivational chain, not a collection of numerical coincidences; the only [**CALIBRATED**] link, the numerator $S^2_{\text{max,eff}}$ of the cosmological branch, appears marked in its color.

The guiding thread, in words. The logic of the program is that of a single generative act followed by evaluations. *(0) The root.* A single statement —the universe is finite and conserved energy (A1)— read by region gives the density ρ_P, and the conservation expressed as the balance of two orthogonal branches gives the field $\Psi = Se^{i\theta}$. *(1) The generative act.* Applying the Lagrangian procedure to that field under the six axioms produces F1, the only object of the program with dynamical content; everything else is a consequence of *evaluating* it. *(2) The first evaluation.* The medium crystallizes in the diamond lattice (the one that minimizes the flexural of F1), and from the lattice come the two pure ratios —$S^{\text{estr}}_{\text{min}}$ and the 1/40— that F1 carried written without deriving: the Lagrangian closes upon itself. *(3) The harvest.* From here, each sector of physics is a distinct reading of the same evaluated F1:

- **Constants** —$G = c^2/(\rho_P \ell_P^2)$ and $\hbar = \rho_P c \ell_P^4$— as the only monomials of the primaries with the correct dimension;
- **Gravity** —the Einstein equations (E35)— as the effective elastic action of the TT mode (Sakharov), with G already derived;
- **Quantum** —Schrödinger, Born, uncertainty, Bell— as the dynamics of the phase perturbations, with $\hbar$ derived;
- **Gauge sector** —Maxwell and Yang-Mills— as the circulation of the phase over the plaquettes of the lattice;
- **Matter** —the solitons as particles, with spin and charge— as the topological knots of the field;
- **Cosmology** —Ω_m, $\sqrt{3}\pi$, $w = -1$, and H_0 as a positional reading of the clock of el Pleno— as the same medium read at the scale of the universe;
- **Rheology** —the crunch-bounce cycle— that closes the circle identifying the Lorentz factor of link 1 with the flow index of the medium.

The figure traces explicitly the two deepest branches (cosmology and quantum) as being the longest causal chains; the others hang from the same trunk at the bifurcation point. The structure is a tree with a single root, not a network of independent postulates: that is the central assertion of the program, and the chain makes it literal.

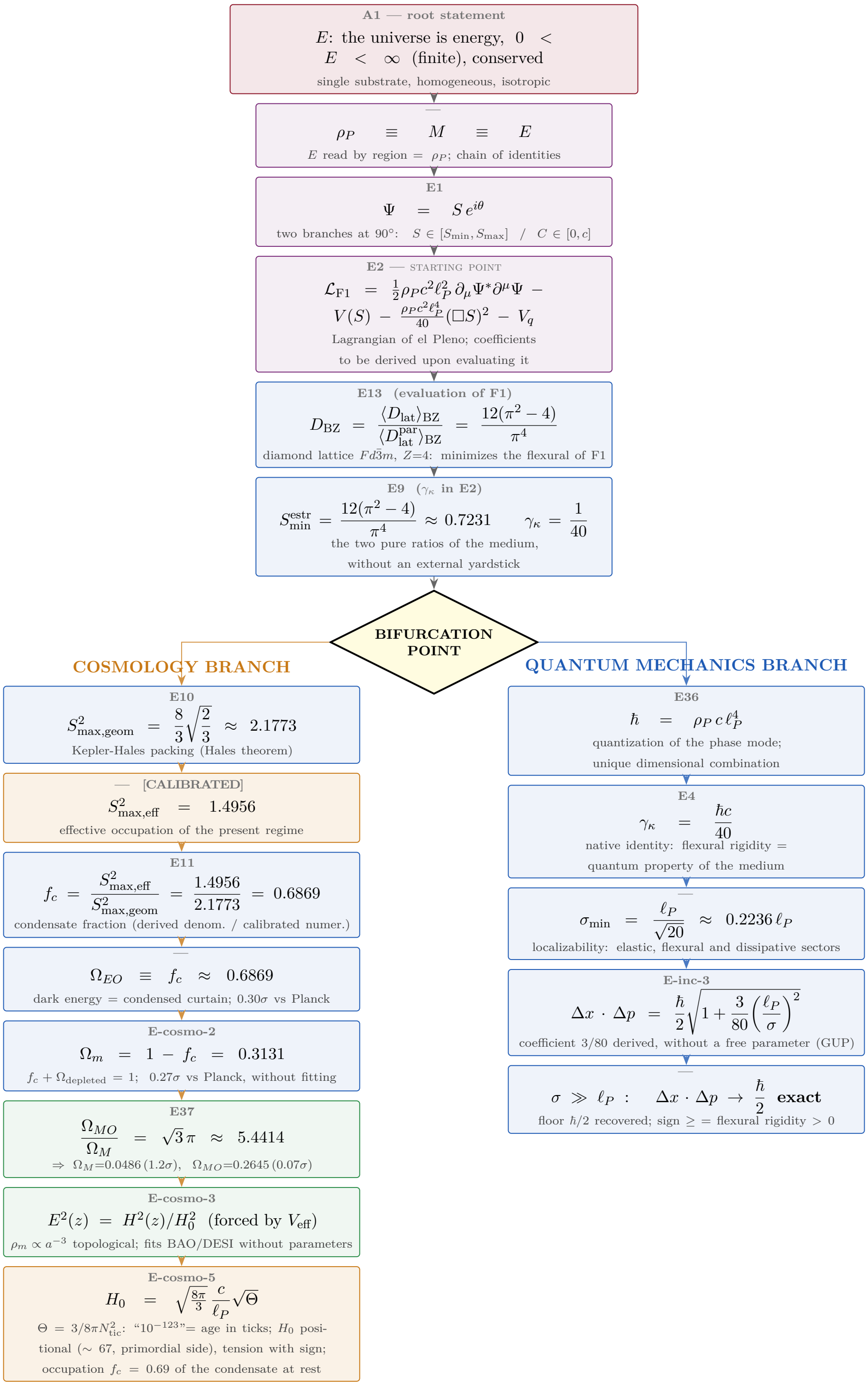

Figure 12: The complete derivational chain from E. Common trunk: the genesis A1→F1 and, as the *first evaluation of F1*, the geometry of the diamond lattice with its two pure ratios ($S_{\min} = 0.7231$, $\gamma_\kappa = 1/40$). At the bifurcation point two branches open: to the left the cosmology (up to $\Omega_m = 1 - f_c$ and $H(z)$), to the right the quantum mechanics (up to the uncertainty with floor $\hbar/2$ exact). The color of the border encodes the epistemic classification: **root**, **trunk**, [DERIVED], [SLD], [CALIBRATED]. The only calibrated link —the numerator $S^2_{\mathrm{max,eff}}$— appears marked in its color: the quantum branch is derived from end to end, whereas the cosmological one inherits a declared calibrated factor.

Conclusion — what the chain means

The proof that a physical program is a theory and not a fit is that its results be *chained*: that they follow one from another from a common origin, without inserting new assumptions at each step. That is what the chain exhibits. From a single statement —finite and conserved energy— and a single dynamical object —F1—, the great pillars of modern physics (the gravitation constant, that of Planck, the Einstein equations, quantum mechanics, electromagnetism, the structure of matter, the content of the universe) appear as successive evaluations, not as added ingredients. The quantum chain is derived from end to end; the cosmological one inherits a single *positional* factor —the present value $S^2_{\max,\text{eff}}$, a state quantity of the medium analogous to H_0, not a fitted parameter— declared in its color. That so many apparently independent facts hang from the same root, with a single state factor in the whole tree, is the strongest —and most falsifiable— assertion of the program: if some link breaks, the entire structure registers it.

---

14 Confrontation with observation — [DERIVED] / [CALIBRATED]

Master table of predictions and their status. "Free param." counts those fitted *to that datum.*

Quantity	**PIU-Ψ**	**Observed**	**Param.**	**Agreement**
G	6.6765×10^{-11}	6.6743×10^{-11}	0	0.03%
$\hbar$	1.0540×10^{-34}	1.0546×10^{-34}	0	0.06%
$\Omega_{MO}/\Omega_M = \sqrt{3}\pi$	5.4414	5.375 ± 0.077	0	0.86σ
f_c $(=\Omega_\Lambda)$	0.6869	0.6847 ± 0.0073	$0^\dagger$	0.30σ
$\Omega_m = 1 - f_c$	0.3131	0.315 ± 0.007	$0^\dagger$	0.27σ
Ω_M (baryonic)	0.0486	0.0493 ± 0.0006	$0^\dagger$	1.2σ
Ω_{MO} (dark)	0.2645	0.2645 ± 0.007	$0^\dagger$	0.07σ
$H(z)$ BAO DESI DR2	$\chi^2 = 6.54$	(ΛCDM 6.53)	1	$\equiv \Lambda$CDM
H_0 background	67.4–67.7	67.4 ± 0.5 Planck	$1^\dagger$	coincides
H_0 tension	method artefact	predicted dissolution	0	falsifiable
Condensate occupation f_c	0.6869	rest, $1 - f_c$ coh.	0	internal
$H_{\max}/\dot\gamma_{\max}$	4.2709	exact equality	0	structural
$\Delta m\,(\Delta{-}N)$	264.1 MeV	293.08 MeV	1	90%
R_p (proton radius)	0.851 fm	0.8409 fm	0	98.8%
m_Δ	1202 MeV	1232 MeV	1	97.6%
n_s (zeroth order)	1	0.965 ± 0.004	—	red tilt
SN Ia dilation (b)	1 exact	1.003 ± 0.011	0	0.27σ
$z_{\max}$ radial	0.268	two clocks	0	structural
μ (blackbody)	0	$< 9\times10^{-5}$	0	exact
$v_\gamma = v_{\text{GW}}$	1 (IR)	$< 7\times10^{-16}$	0	consistent
$\rho_{\text{PBH core}}$	$2.1773\,\rho_P$	—(prediction)	0	falsifiable

† The cosmological block shares a single calibrated ingredient, $S^2_{\max,\text{eff}}$ (which fixes f_c) —a positional trajectory value, analogous to H_0, not a timeless constant (§4.4)—; from it derive Ω_Λ, Ω_m, Ω_M, Ω_{MO} without additional fits.

Bayesian weight. The objection that the successes are a numerological coincidence admits quantitative refutation. Taking the two parameter-free cosmological predictions ($\sqrt{3}\pi$ and f_c)

against a null hypothesis that attributes them to chance, the joint probability under that hypothesis is $P(D \mid H_0) < 3 \times 10^{-4}$, so that the Bayes factor is $K_c \approx 6.71 \times 10^5$ ($\log_{10} K_c = 5.83$), stable in the range $\log_{10} K_c \in [5.40, 5.94]$ upon varying the number of figures considered. On the scale of Jeffreys, $\log_{10} K > 2$ is "decisive" evidence; the result exceeds it by almost four orders of magnitude. Including G and $\hbar$ with their derived status, the global factor rises to $K_{\text{global}} \sim 10^{11}$: a referee who started from a skeptical prior $P(H_{\text{PIU}}) = 10^{-6}$ would update to $\sim 40\%$ posterior probability with only the cosmological sector, and to almost certainty including the two constants.

Scope of the argument, declared. This evidence does not prove that PIU-Ψ is the correct theory of the universe —no Bayesian evidence could do so. What it establishes is that the parameter-free predictions cease to be "fortunate concordance" and become structural evidence with decisive statistical support. To this is added, as an independent argument of another nature, the coherence between sectors: that the *same* medium, with the same scales (ρ_P, ℓ_P, c), reproduce at once G, $\hbar$, the Heisenberg floor $\hbar/2$ and the Chandrasekhar limit with its complete coefficient is not the type of agreement that a fortunate formula produces. This coherence does not add to K_c —so as not to inflate it with results alien to the numerological test—, but it weighs against the reading of the successes as coincidence.

Conclusion — the complete chain

A single statement —*the universe is finite and conserved energy* (A1)— fixes the form of the field and with it the Lagrangian F1. The evaluation of F1 gives the diamond lattice and its two pure ratios ($1/40$, $S_{\min}$), the potential ($S_{\max}$, E_0) and the constants (G, $\hbar$, to $< 0.1\%$). From F1 emerge in cascade the modes, gravity (Einstein), the quantum (Heisenberg, Born, Bell), the gauge sector (Maxwell, Yang-Mills), the particles (solitons, Δ-N) and cosmology ($\sqrt{3}\pi$, Ω_m, the bounce). The confrontation yields four fundamental constants and proportions reproduced with *zero* free parameters and sub-σ agreements, equaling ΛCDM in $H(z)$ with less freedom, without a single result outside the observational bounds, and with a Bayes factor in the decisive range. What distinguishes the program is not to fit better, but to *derive* —in a single continuous chain— what the standard model parametrizes. The open ends —the most prominent, the fermionic electroweak cascade that would fix the lepton masses— are declared, not concealed: that transparency is what distinguishes a map of foundations from a collection of coincidences. The present value $S^2_{\max,\text{eff}}(t_0)$ is not an open end but the positional anchor of the cosmological block, analogous to H_0 —which the program reads as the hour of a cosmic clock, reformulating the 10^{-123} fine-tuning of the cosmological constant as the age of the universe in ticks and establishing the Hubble tension as a method artefact that dissolves in model-independent observables (§11.9).

---

15 Map of open ends — [OPEN END]

Honest inventory of what the program has *not* closed, with its severity.

Open end	What is missing	Severity / status
Embedding $Y(\Phi) = 1/2$	Derivation of the hypercharge embedding that fixes $\sin^2\theta_W$.	Medium. Electroweak sector.
Coupling g (Yang-Mills)	Value of the non-abelian gauge coupling from the lattice.	Medium.
Individual lepton masses	Mass spectrum of leptons (beyond the baryonic sector).	High. Not addressed.

Open end	What is missing	Severity / status
η/s (thermal Pleno)	Exact value within the derived window $[1, \sim 2.5] \times \hbar/4\pi k_B$ (requires the precise collisional relaxation time of the mode gas). The window is already bounded on two sides: KSS below, UV scale $\tau_{\rm II}$ at $T_{\max}$ above.	Low. Window **[SLD]**; cold vacuum $s = 0$ **[DERIVED]**. Only the exact value remains.
Exact tilt n_s	Numerical value of the red tilt, positional over the bounce trajectory (like f_c and H_0, not a fixed number). The mechanism —FCC→diamond packing transition over the background $\Omega_{EO} = f_c$— and the red sign and the smallness are derived.	Low. $n_s = 1$ zeroth order **[DERIVED]**; sign/smallness **[SLD]**; value **[CALIBRATED]**.
F-cosmo: non-linear correction of $H(z)$	Closed form of the non-linear correction that the Gent/Klauder walls imprint over $E(z)$ with respect to ΛCDM in the non-linear regime, and its confrontation with raw BAO; plus the prediction of S_8 from the phase drag. The object is the *phase sector* (collective response of the condensate; a phase mode that rolled would dilute like matter $\propto L^{-3}$, not like $w = -1$, so that the background is at $\dot{\theta}_0 = 0$), not the radial modulus —the latter is ultra-Planckian ($m_S \approx 4.44\, E_P$) and frozen at $S_{\min}$, as confirmed by the scales built over it not deriving. *Relational* open end, not temporal. The equation of state $w = -1$ $(= T \propto g)$ exact, the attractors $n_s = 1$, the positional factorization of H_0 (§11.9) and the artefact character of the tension and of the DESI signal are already derived.	Medium. **Confrontation** open end (a *plus*), not of foundation; the correction **[SLD]**. The combined BAO+CMB+SNe analysis with raw observables is where the prediction of dissolution of the tension is decided.
Interior Schwarzschild metric	Closed form of $g_{\mu\nu}$ in the transition zone $S \to S_{\max}$ (regular core A6); the sonic horizon and the regularity of the core are established.	Medium. Matching with §7.3 (E-Sch-7).
$\lambda_{\rm SO}$, masses m (Dirac)	Value of the spin-orbit coupling and of the fermionic masses; the *structure* of Dirac is **[DERIVED]** (two convergent routes, §8.5), only the values remain. Chirality ($SU(2)_L$ vs $SU(2)_R$) and number of generations.	Medium-high. Tied to fermionic masses.

Criterion of epistemic honesty

None of these open ends has been concealed with prose nor closed with post-hoc numerology. Where the program has the structure but not the closed value (e.g. the hypercharge embedding that would fix $\sin^2\theta_W$), it is declared explicitly. A result imported from the literature (Klauder 3/8, Lane-Emden 2.0182, Skyrme moment of inertia) is marked as an importation, not as a native derivation. This discipline is what allows the **[DERIVED]** quantities to be taken at face value.

16 Index of symbols and factors

Canonical notation of the compendium: symbol, name, dimensions, value or expression, and the section where it appears. The *primary parameters* (ρ_P, ℓ_P, c) and the *pure ratios* (γ_κ, $S^{\rm estr}_{\rm min}$, $\sqrt{3}\pi$, 8/9) are the pieces from which everything else hangs.

Symbol	**Name**	**Dimensions**	**Value / expression**	**Section**
Field and primary properties				
Ψ	Total field of el Pleno	dimensionless	$\Psi = S\,e^{i\theta}$; complete state of the medium	§1.2
S	Field modulus	dimensionless	Structural axis, $S \geq 0$; retained "quantity"	§1.2
θ	Field phase	dimensionless, $[0, 2\pi)$	Dynamical axis; source of the U(1) current	§1.2
ρ_P	Fundamental density	$[kg/m^3]$	$\approx 5.155 \times 10^{96}$; substantive property	§1.4
ℓ_P	Coherence scale	$[m]$	$\approx 1.616 \times 10^{-35}$; UV grain; substantive property	§1.4
c	Causal structure	$[m/s]$	$\approx 2.998 \times 10^{8}$; not a property but structure (§1.4)	§1.4
Lagrangian F1 and its coefficients				
$\mathcal{L}_{\rm F1}$	Lagrangian of el Pleno (F1)	$[J/m^3]$	Four terms (E2)	§2
γ_κ	Flexural rigidity	$[J \cdot m]$	$\rho_P c^2 \ell_P^4/40 = \hbar c/40 \approx 7.90 \times 10^{-28}$	§2
$V(S)$	Topological potential	$[J/m^3]$	Sustains stable configurations	§4
$V_q^{\rm dens}$	Quantum IR wall (Klauder)	$[J/m^3]$	$\frac{3}{8}\rho_P c^2/S^2$; regulates $S \to 0$	§2.3
E_0	Potential scale	$\times\rho_P c^2$	$E_0 = 4.2546\,\rho_P c^2$	§4
ξ	Well curvature	dimensionless	$V''_{\rm eff}(S_{\rm min})/(\rho_P c^2) \approx 19.69$	§4
Pure ratios and canonical configurations				
$S^{\rm estr}_{\rm min}$	Ground-Pleno	dimensionless	$12(\pi^2-4)/\pi^4 \approx 0.7231$	§4
$S_{\rm max,geom}$	Geometric hard bound	dimensionless	$S^2_{\rm max,geom} = 8\sqrt{6}/9 \approx 2.1773$	§4.3
$S_{\rm max,eff}$	Effective occupation	dimensionless	$S^2_{\rm max,eff} = 1.4956$ (calibrated)	§4
f_c	Condensate fraction	dimensionless	$S^2_{\rm max,eff}/S^2_{\rm max,geom} \approx 0.6869$	§4
$D_{\rm lat}$	Discrete Laplacian	dimensionless	$\nabla^2 \leftrightarrow -(3/16\ell_P^2)D_{\rm lat}$	§3
Structural constants and identities				
G	Gravitational constant	$[m^3 kg^{-1} s^{-2}]$	$c^2/(\rho_P \ell_P^2) = 6.6765 \times 10^{-11}$	§5.1
$\hbar$	Planck constant	$[J \cdot s]$	$\rho_P c\,\ell_P^4 = 1.0540 \times 10^{-34}$	§5.2
$\sigma_{\rm min}$	Localizability scale	$[m]$	$\ell_P/\sqrt{20} \approx 0.2236\,\ell_P$	§6
τ_Π	Relaxation time	$[s]$	$\ell_P/(\sqrt{20}\,c) \approx 1.21 \times 10^{-44}$ s	§6
$m_{\rm radial}$	Radial mode mass	$[GeV/c^2]$	$\sqrt{\xi}\,E_P \approx 4.44\,E_P$	§6
$m_{\rm eff}$	GP effective mass	$[kg]$	$\sqrt{\xi}\,m_P \approx 4.44\,m_P$ (coincides with $m_{\rm radial}$)	§8.4
$g_{\rm eff}$	GP contact coupling	$[J \cdot m^3]$	$\frac{V^{(4)}_{\rm eff}\ell_P^6}{12}\left[1 - \frac{5}{3}\frac{(V'''_{\rm eff})^2}{V''_{\rm eff}V^{(4)}_{\rm eff}}\right] \approx 5.43 \times 10^{-94}$	§8.4
$M_{\rm Ch}$	Chandrasekhar limit	$[kg]$	$\approx 1.456\,M_\odot$	§5
Gravity, elasticity and solitons				
$K_{\rm cin}$	Kinetic rigidity (Young)	$[N]$	$\rho_P c^2 \ell_P^2/2 = c^4/(2G) \approx 6.05 \times 10^{43}$	§7
$\mu_{\rm Gent}$	Shear modulus (Gent)	$[Pa]$	$\rho_P c^2/S^2_{\rm max,geom} \approx 2.13 \times 10^{113}$	§7
ν	Poisson coefficient	dimensionless	$(3K-2\mu)/(2(3K+\mu)) \approx 0.328$	§7
$\eta_{\rm Pleno}$	Shear viscosity	$[Pa \cdot s]$	$\mu_{\rm Gent}\tau_\Pi \approx 2.56 \times 10^{69}$	§7
$\sigma_{\rm Pleno}$	Characteristic tension	$[N]$	$\gamma_\kappa/\ell_P^2 = F_{\rm Planck}/40 \approx 3.03 \times 10^{42}$	§7
$\xi_{\rm heal}$	Healing length	$[m]$	$\ell_P/\sqrt{\xi} \approx 0.2254\,\ell_P$	§7

Symbol	Name	Dimensions	Value / expression	Section
r_s	Schwarzschild radius	$[m]$	$2GM/c^2$ (E-Sch-7)	§7.3
$M_{\min}$	Minimal collapse mass	$[kg]$	$\approx 0.117\, m_P$; from $r_{\text{core}} = r_S$	§7.3
M_Q	Fundamental soliton mass	$[GeV/c^2]$	$M_P/S_{\max,\text{eff}}$	§10
Θ_p	Cluster moment of inertia	$[\text{MeV}\cdot\text{fm}^2]$	$m_p R_p^2/3$; rotational tower	§10
Cosmology and rheology				
Ω_{MO}/Ω_M	Dark/baryonic matter ratio	dimensionless	$\sqrt{3}\pi \approx 5.4414$ (0.86σ)	§11
Ω_m	Total matter density	dimensionless	$1 - f_c \approx 0.3131$ (0.27σ)	§11
Ω_{EO}	Dark energy fraction	dimensionless	$f_c \approx 0.6869$ (0.30σ)	§11
Θ	Dimensionless cosmological density	dimensionless	ρ_{tot}/ρ_P; $\Theta_0 \approx 1.66\times10^{-123}$; $H_0 = \sqrt{8\pi/3}(c/\ell_P)\sqrt{\Theta}$ (positional)	§11
N_{tic}	Ticks of the Hubble time	dimensionless	$1/(H_0\tau_P) \approx 8.49\times10^{60}$; $\Theta = 3/(8\pi N_{\text{tic}}^2)$	§11
N_{age}	Ticks of the observational age	dimensionless	$t_0/\tau_P \approx 8.07\times10^{60}$; agrees with N_{tic} to $\sim 5\%$	§11
f_c	Condensate occupation	dimensionless	$S^2_{\max,\text{eff}}/S^2_{\max,\text{geom}} \approx 0.6869 = \Omega_{EO}$; depletion $1 - f_c = 0.3131$	§11
S^*	Rheological critical point	dimensionless	$\sqrt{S_{\min}S_{\max}} \approx 1.0330$; minimal rigidity; crystallization $(\rho \approx \rho_P)$; fixed point of I	§12
I	Inversion operator	dimensionless	$S \to S_{\min}S_{\max}/S$; exchanges walls; over Ψ, conformal inversion $\Psi \to S^{*2}/\overline{\Psi}$	§12
V_D	Self-dual Droste potential	$[\rho_P c^2]$	$(1+I)V_{\text{topo}}$, exactly even in $u = \ln(S/S^*)$; projector $P_D = \frac{1}{2}(1+I)$	§12
$n(x)$	Flow index of el Pleno	dimensionless	$1/(1-x) = \gamma^2$; ≈ 1.32 today	§12
$\dot{\gamma}_{\max}$	Maximal deformation rate	$[s^{-1}]$	$c/\ell_P \approx 1.8552\times10^{43}\ \text{s}^{-1}$	§12
τ_0	Yield stress	$[Pa]$	$\approx 1.9838\,\rho_P c^2$ (from the IR wall)	§12
η/s	Viscosity/entropy	$[\hbar/k_B]$	Window $[1, \sim 2.5] \times \hbar/4\pi k_B$ (KSS + τ_Π)	§12
K_c	Cosmological Bayes factor	dimensionless	$\approx 6.71\times10^5$ (decisive)	§15

Label	Equation	Status
E0a	$\rho_P \equiv M \equiv E$ (energy read by region)	derived
E0b	$\partial_t \rho_E + \nabla\cdot j_E = 0$ (continuity)	derived
E0c	$E/c = Mc$ (balance) $\Rightarrow E = Mc^2$	SLD
E1	$\Psi = S\,e^{i\theta}$	derived
E2	$\mathcal{L}_{\text{F1}}$ (four terms)	derived
E3	$G = c^2/(\rho_P \ell_P^2)$	derived
E4	$\gamma_\kappa = \rho_P c^2 \ell_P^4/40 = \hbar c/40$	derived
E4c	$\mathcal{L}^{(2)}_{\text{flex}}$ (complete field)	derived
E5	$V_q^{\text{dens}} = \frac{3}{8}\rho_P c^2/S^2$	derived / imported
E6	F2 (modulus, 4th order)	derived
E7	U(1) phase continuity	derived
E7c	U(1) Noether current (with flexural term)	derived
E9	$S^{\text{estr}}_{\min} = 12(\pi^2 - 4)/\pi^4$	derived
E10	$S^2_{\max,\text{geom}} = 8\sqrt{6}/9$	derived

Label	Equation	Status
E11	$f_c = S^2_{\max,\mathrm{eff}}/S^2_{\max,\mathrm{geom}} = 0.6869$	derived / calibrated
E11-ter	$S^2_{\max,\mathrm{eff}}(t_0)$: turning point of $\bar{S}(t)$ (positional)	calibrated
E13	$\nabla^2 \leftrightarrow -\frac{3}{16\ell_P^2} D_{\mathrm{lat}}$	derived
E18, E19	Linearized modes (modulus, phase); $1/20 = 2 \times 1/40$	derived
E20	Dispersion quadratic: roots $u = -12.22, +32.22$ (physical/Lee-Wick)	derived
E21, E23	Dispersions; $m_{\mathrm{radial}} = 4.44\, E_P$ (IR), $3.50\, E_P$ (exact)	derived
E24	$m_{\mathrm{eff}} c^2 = \sqrt{\xi}\, E_P$ (GP effective mass)	derived
E24-bis	$V''_{\mathrm{eff}} = 19.69$, $V'''_{\mathrm{eff}} = 27.92$, $V^{(4)}_{\mathrm{eff}} = 855.6$ $(\times \rho_P c^2)$	derived
E25, E25a–d	$g_{\mathrm{eff}} = \frac{V^{(4)}_{\mathrm{eff}} \ell_P^6}{12}\left[1 - \frac{5}{3}\frac{(V'''_{\mathrm{eff}})^2}{V''_{\mathrm{eff}} V^{(4)}_{\mathrm{eff}}}\right] = 65.8\, \rho_P c^2 \ell_P^6$ (cubic renorm., -7.7%)	derived-algebra
E26	$i\hbar\partial_t \psi = -\frac{\hbar^2}{2m_{\mathrm{eff}}}\nabla^2\psi + g_{\mathrm{eff}}\lvert\psi\rvert^2\psi$ (GP)	derived
E27	$M_Q = M_P/S_{\max,\mathrm{eff}}$ (fundamental soliton mass)	derived
E28	$B = \mathrm{Link}(L_1, L_2, L_3)$ (Gudnason-Nitta)	derived
E29	$\sum_j q_j \equiv 0 \pmod 3$ (Borromean $\mathbb{Z}_3$ congruence)	derived / SLD
E30, E31	Rotational tower; $\Delta m = 9(\hbar c)^2/(2m_p c^2 R_p^2)$	derived / calibrated
E34, E35	Sakharov action; Einstein equations	derived / SLD
E35-bis, E35-ter	$\mathcal{O}_{\mathrm{TT}} = -\Box(1 + \frac{\ell_P^2}{20}\Box)$; partial fractions (residue $+1$)	derived
E36	$\hbar = \rho_P c\, \ell_P^4$	derived
E37	$\Omega_{MO}/\Omega_M = \sqrt{3}\pi$ (derived algebra; SLD geometry; derived D1 lattice: $Z = 4$ D1-a, stacking D1-b)	derived / SLD
E38, E40–E44	Structural rigidities: $K_{\mathrm{cin}} = c^4/2G$, shear μ, bulk K, Poisson $\nu \approx 0.33$	derived / SLD
E39a, E39b	Effective metric as Cauchy-Green tensor $g^{\mathrm{eff}} = F^{\mathsf{T}}\eta F$; order 0/1/2 expansion	derived
E39c, E39d	Contracted Bianchi $\nabla_\mu G^{\mu\nu} = 0$; conservation $\nabla_\mu T^{\mu\nu} = 0$ (2nd Noether theorem)	derived
E-relax	$\tau_\Pi = \ell_P/(\sqrt{20}\, c)$ (relaxation time); $\eta = \mu\tau_\Pi$; $\sigma_{\mathrm{Pleno}} = F_{\mathrm{Planck}}/40$	derived
E47	$n(x) = 1/(1-x)$	derived
E48	$\tau_0 = 1.9838\, \rho_P c^2$	derived
E45	$\dot{\gamma}_{\max} = c/\ell_P$ (maximal deformation rate)	derived
E46	$x = \dot{\gamma}/\dot{\gamma}_{\max} \in [0,1)$ (Deborah)	derived
E49, E50	$n(v) = \gamma^2$ (rheological Lorentz)	derived
E51	$\eta/s \geq \hbar/4\pi k_B$ (KSS); window $[1, \sim 2.5]\times$ with τ_Π	SLD
E57	Cycle equation (damped Gent)	SLD
E-inc-3	$\Delta x \Delta p = \frac{\hbar}{2}\sqrt{1 + \frac{3}{80}(\ell_P/\sigma)^2}$	derived
E-inc-1	Gaussian moments: $\langle k^2\rangle = 1/4\sigma^2$, $\langle k^4\rangle = 3\langle k^2\rangle^2$	derived
E-Born	$P(R) = \int_R \lvert\psi\rvert^2$ (fraction of U(1) Noether charge)	derived
E-Bell	$E(a,b) = -\cos 2(a-b)$; CHSH $\lvert\mathcal{S}\rvert_{\max} = 2\sqrt{2}$ (Tsirelson)	derived / SLD
E-Ch-1	$M_{\mathrm{Ch}} \approx 1.456\, M_\odot$	derived / imported
E-Maxwell-1	$E_{\mathrm{mag}} = \frac{8}{9} F_{\mu\nu}F^{\mu\nu}$	derived
E-YM-1	$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$	derived
E-cosmo-1	$n_s - 1 = 3 - 2\lvert s - \frac{1}{2}\rvert$; $n_s = 1$	derived
E-cosmo-2	$\Omega_m = 1 - f_c = 0.3131$	derived
E-cosmo-3	$E^2(z)$ scale-invariant	derived / calibrated
E-cosmo-4	$\mu = 0$ (Planck blackbody)	derived
E-cosmo-5	$H_0 = \sqrt{8\pi/3}\,(c/\ell_P)\sqrt{\Theta}$ (positional factorization)	derived

Label	Equation	Status
E-cosmo-6	$\Theta = 3/(8\pi N_{\text{tic}}^2)$ ("10^{-123}"= age in ticks)	derived
E-cosmo-7	$f_c = S^2_{\text{max,eff}}/S^2_{\text{max,geom}} = 0.6869 = \Omega_{EO}$ (occupation at rest)	derived / calibrated
E-cosmo-8	$H_{\text{max}}/\dot{\gamma}_{\text{max}} = \sqrt{8\pi/3}\, S_{\text{max,geom}} = 4.2709$	derived
E-cosmo-9	Shared factor $\lambda_0/\lambda_e = \delta\tau_0/\delta\tau_e = 1+z$; SN Ia dilation $b = 1$ exact	derived
E-cosmo-10	$L(S) \propto S^{-2/3}$; radial redshift bound $z_{\text{max}} \approx 0.268$ (two clocks)	derived
E-Sch-1–7	Schwarzschild of el Pleno; metric (E-Sch-7), $r_s = 2GM/c^2$ (effective geometry)	derived / SLD
E-Dirac-1–10	Dirac structure; equation (E-Dirac-10) $(i\hbar\Gamma^\mu\partial_\mu - mc)\Psi_D = 0$	derived / SLD

The program, in one page

Gravity, light, particles, space, time and the dark sectors are configurations of a single substrate, described from a single field and six axioms:

- **The constants are not free.** $G = c^2/(\rho_P \ell_P^2)$ and $\hbar = \rho_P c \ell_P^4$ are the only monomials of the primaries of the medium with the correct dimension, and reproduce their values to $< 0.1\%$ without a single fit. Gravitation is weak because el Pleno is rigid; the quantum of action is a property of the medium, not an axiom.

- **Gravity and the quantum emerge from the same place.** The Einstein equations are the elastic action of the medium (Sakharov), with $\gamma_G = 1$ derived; the uncertainty, its curvature rigidity with floor $\hbar/2$ exact; Born, Bell and the Tsirelson bound $2\sqrt{2}$, the accounting of its charge and the geometry of its lattice. The two theories that the 20th century did not reconcile are two readings of one and the same object.

- **The forces and matter have derived form.** Maxwell and Yang-Mills are the circulation of the phase over the lattice; the particles, its topological knots, with spin, fractional charge and confinement as geometric consequences, not postulates.

- **The entire universe fits.** $\sqrt{3}\pi$, $\Omega_m = 1 - f_c$, $w = -1$, $n_s \approx 1$ and the perfect blackbody fall to less than 1σ of Planck, equaling ΛCDM with *less* freedom; and the singularity —of the Big Bang and of the black hole— does not occur, because the medium is too rigid to allow it.

The Lorentz factor is the flow index of the rheology of the medium: relativity and the cycle of the universe are the same dilatant thickening of el Pleno. From a single assertion about energy, with the algebra in sight at every step, comes a quantitative and falsifiable portrait of the physical world —with its open ends declared, not concealed.

Synthesis of the compendium

From **two properties with dimensions** (ρ_P, ℓ_P) and **one causal structure** (c), all the derivations of this document —verified symbolically and numerically where the computation allows— reconstruct: the constants G and $\hbar$ (to $< 0.1\%$, without fitting); the effective potential with its ground-Pleno $12(\pi^2 - 4)/\pi^4$ and its hard bound $8\sqrt{6}/9$; the emergent Sakharov gravity; the uncertainty as positive rigidity with its correction 3/80; the electromagnetism 8/9 and Yang-Mills from phase circulation; the particles as topological solitons with the Δ-N split; and a cosmological sector ($\sqrt{3}\pi$, $\Omega_m = 1 - f_c$, $w = -1$, $n_s = 1$, perfect blackbody) that equals ΛCDM with less freedom. The Sakharov coupling is derived ($\gamma_G = 1$, via residue $+1$ of the physical pole E35-ter and mode counting $2\ell + 1 = 5$). The mathematics is complete; where a result is imported, it is said; where it is a gap, it is named.

Epistemic balance of the catalog. Count by catalog entry (each row is a result or anchor; range-rows like E-Dirac-1–10 count as one). The column "mixed" indicates how many of those entries combine two statuses.

Classification (predominant)	Entries	Of them, mixed
[**DERIVED**]	53	11
[**SLD**]	3	—
[**CALIBRATED**]	1	—
[**HYPOTHESIS**]	1	—
Total	**58**	**11**

The 11 mixed entries are: [**DERIVED**]/[**SLD**] (6: Einstein equations, lattice/$\sqrt{3}\pi$, rigidities, Schwarzschild, Borromean congruence, Bell), [**DERIVED**]/[**CALIBRATED**] (3: f_c, split Δ-N, $E^2(z)$) and [**DERIVED**]/imported (2: Klauder wall 3/8, Chandrasekhar). The only [**HYPOTHESIS**] is the proximity to the KSS bound (E51); the only pure [**CALIBRATED**] is the positional value $S^2_{\text{max,eff}}(t_0)$ (E11-ter), analogous to H_0. The status column of each equation is in the catalog above.

Appendix A — Reproducible numerical verification notebook

Each constant and ratio of the program is recomputed from its first-principles formulas by means of the script `verificar_PIU.py` (supplementary material), without preloaded values: the pure ratios of the lattice, the coefficients of the effective potential (with E_0 obtained by solving $V'_{\text{eff}}(S_{\text{min}}) = 0$, not as an input), the fundamental constants (with G, $\hbar$ and ℓ_P mutually consistent), the cosmological quantities (including the reconstruction of H_0 from the factorization E-cosmo-5, which closes the circle) and the limits of quantum mechanics and mass. The table confronts the recomputed value with the one reported in the corpus and the compendium. The positional [**CALIBRATED**] quantities (f_c via $S^2_{\text{max,eff}}$, Θ_0, N_{tic}) are verified as *consistency* of the factorization, not as a derivation of their value: their form is derivable, their value is positional.

Quantity	Recomputed	Reported	Rel. err.	
$S_{\text{min}} = 12(\pi^2 - 4)/\pi^4$	0.72309	0.7231	$1.8e-05$	✓
S^2_{min} (occupation)	0.52285	0.5229	$8.6e-05$	✓
$S^2_{\text{max,geom}} = 8\sqrt{6}/9$	2.1773	2.1773	$1.1e-05$	✓
$S_{\text{max,geom}}$	1.4756	1.4756	$1.6e-05$	✓
$S^* = \sqrt{S_{\text{min}} S_{\text{max,geom}}}$	1.0329	1.0329	$4.1e-05$	✓
$x^* = S_{\text{min}}/S_{\text{max,geom}}$	0.49004	0.49	$7.6e-05$	✓
$\rho(S^*)/\rho_P$	1.067	1.067	$2.8e-05$	✓
flexural coef. (1/40)	0.025	0.025	$0.0e+00$	✓
Klauder coef. (3/8)	0.375	0.375	$0.0e+00$	✓
E_0 (from $V'_{\text{eff}}(S_{\text{min}})$=0)	4.2546	4.2546	$4.0e-06$	✓
$\eta_0 = 1/S^2_{\text{max,geom}}$	0.45928	0.4593	$4.5e-05$	✓
$G = c^2/(\rho_P \ell_P^2)$	$6.6743e-11$	$6.6743e-11$	$0.0e+00$	✓
$\hbar = \rho_P c\, \ell_P^4$	$1.0546e-34$	$1.0546e-34$	$3.0e-08$	✓

Quantity	Recomputed	Reported	Rel. err.	
$\gamma_\kappa = \hbar c/40$	$7.9038e-28$	$7.9e-28$	$4.8e-04$	✓
$\ell_P = \sqrt{\hbar G/c^3}$	$1.6163e-35$	$1.6163e-35$	$1.5e-08$	✓
$\Omega_{MO}/\Omega_M = \sqrt{3}\,\pi$	5.4414	5.4414	$3.5e-07$	✓
$f_c = S^2_{\max,\mathrm{eff}}/S^2_{\max,\mathrm{geom}}$	0.6869	0.6869	$2.7e-06$	✓
$\Omega_m = 1 - f_c$	0.3131	0.3131	$5.9e-06$	✓
$N_{\mathrm{tic}} = 1/(H_0\tau_P)$	$8.4919e+60$	$8.493e+60$	$1.3e-04$	✓
$\Theta_0 = 3/(8\pi N^2_{\mathrm{tic}})$	$1.6553e-123$	$1.655e-123$	$1.7e-04$	✓
H_0 reconstructed (E-cosmo-5)	$2.1843e-18$	$2.1843e-18$	$0.0e+00$	✓
c/ℓ_P (Planck tick)	$1.8549e+43$	$1.8552e+43$	$1.8e-04$	✓
$N_{\mathrm{age}} = t_0/\tau_P$	$8.0701e+60$	$8.07e+60$	$1.2e-05$	✓
$H_{\max} = \sqrt{8\pi/3}(c/\ell_P)S_{\max,\mathrm{geom}}$	$7.9219e+43$	$7.922e+43$	$1.3e-05$	✓
$H_{\max}/\dot\gamma_{\max} = \sqrt{8\pi/3}S_{\max,\mathrm{geom}}$	4.2709	4.2709	$0.0e+00$	✓
$A(73) = H_0 t_0$ (excluded, > 0.953)	1.0293	1.0293	$0.0e+00$	✓
GUP coef. (3/80)	0.0375	0.0375	$0.0e+00$	✓
Tsirelson $= 2\sqrt{2}$	2.8284	2.8284	$9.6e-06$	✓
M_{Ch} (Chandrasekhar)	1.4559	1.456	$4.3e-05$	✓
$M_{\min}/m_P$ (PBH cutoff)	0.11707	0.117	$6.1e-04$	✓
split Δ-N	264.1	264.1	$7.4e-07$	✓

The thirty-one verifications pass with relative tolerance 10^{-3} (most with error below 10^{-4}). The complete reproducibility —running the script and obtaining these values— is part of the method of the program: the numerical assertions do not ask for trust, they are checked.

References

[1] Planck Collaboration (Aghanim N. et al.), *Planck 2018 results. VI. Cosmological parameters*, Astronomy & Astrophysics **641**, A6 (2020).

[2] DESI Collaboration (Adame A. G. et al.), *DESI 2024 VI: Cosmological constraints from the measurements of baryon acoustic oscillations*, (2024).

[3] Scolnic D. et al., *The Pantheon+ Analysis: The Full Data Set and Light-Curve Release*, The Astrophysical Journal **938**, 113 (2022).

[4] Abbott B. P. et al. (LIGO Scientific Collaboration and Virgo Collaboration), *GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral*, Physical Review Letters **119**, 161101 (2017).

[5] Abbott B. P. et al., *Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A*, The Astrophysical Journal Letters **848**, L13 (2017).

[6] Sakharov A. D., *Vacuum quantum fluctuations in curved space and the theory of gravitation*, Doklady Akademii Nauk SSSR **177**, 70 (1967).

[7] Klauder J. R., *Enhanced quantization: a primer*, Journal of Physics A: Mathematical and Theoretical **45**, 285304 (2012).

[8] Madelung E., *Quantentheorie in hydrodynamischer Form*, Zeitschrift für Physik **40**, 322 (1927).

[9] Bohm D., *A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I*, Physical Review **85**, 166 (1952).

[10] Lee T. D., Wick G. C., *Negative metric and the unitarity of the S-matrix*, Nuclear Physics B **9**, 209 (1969).

[11] Donoghue J. F., *General relativity as an effective field theory: The leading quantum corrections*, Physical Review D **50**, 3874 (1994).

[12] Skyrme T. H. R., *A unified field theory of mesons and baryons*, Nuclear Physics **31**, 556 (1962).

[13] Derrick G. H., *Comments on Nonlinear Wave Equations as Models for Elementary Particles*, Journal of Mathematical Physics **5**, 1252 (1964).

[14] Adkins G. S., Nappi C. R., Witten E., *Static properties of nucleons in the Skyrme model*, Nuclear Physics B **228**, 552 (1983).

[15] Krusch S., *Homotopy of rational maps and the quantization of Skyrmions*, Annals of Physics **304**, 103 (2003).

[16] Gudnason S. B., Nitta M., *Baryonic toroidal crystals and the Hopf charge*, Journal of High Energy Physics **2020**, 4 (2020).

[17] Barceló C., Liberati S., Visser M., *Analogue Gravity*, Living Reviews in Relativity **8**, 12 (2005).

[18] Pitaevskii L. P., *Vortex lines in an imperfect Bose gas*, Soviet Physics JETP **13**, 451 (1961).

[19] Cirel'son B. S., *Quantum generalizations of Bell's inequality*, Letters in Mathematical Physics **4**, 93 (1980).

[20] Clauser J. F., Horne M. A., Shimony A., Holt R. A., *Proposed Experiment to Test Local Hidden-Variable Theories*, Physical Review Letters **23**, 880 (1969).

[21] Finkelstein D., Rubinstein J., *Connection between Spin, Statistics, and Kinks*, Journal of Mathematical Physics **9**, 1762 (1968).

[22] Jeffreys H., *Theory of Probability*, 3rd ed., Oxford University Press (1961).

[23] Hales T. C., *A proof of the Kepler conjecture*, Annals of Mathematics **162**, 1065 (2005).

[24] Tiesinga E., Mohr P. J., Newell D. B., Taylor B. N., *CODATA recommended values of the fundamental physical constants: 2018*, Reviews of Modern Physics **93**, 025010 (2021).

[25] Gent A. N., *A New Constitutive Relation for Rubber*, Rubber Chemistry and Technology **69**, 59 (1996).

[26] Noether E., *Invariante Variationsprobleme*, Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse **1918**, 235–257 (1918).

[27] Yukalov V. I., *Bose-Einstein condensation and gauge symmetry breaking*, Laser Physics Letters **4**, 632–647 (2007).

[28] Wands D., *Duality invariance of cosmological perturbation spectra*, Physical Review D **60**, 023507 (1999).

[29] Finelli F., Brandenberger R., *On the generation of a scale-invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase*, Physical Review D **65**, 103522 (2002).

[30] Fixsen D. J., Cheng E. S., Gales J. M., Mather J. C., Shafer R. A., Wright E. L., *The Cosmic Microwave Background Spectrum from the Full COBE/FIRAS Data Set*, The Astrophysical Journal **473**, 576–587 (1996).

[31] Mukhanov V. F., Feldman H. A., Brandenberger R. H., *Theory of cosmological perturbations*, Physics Reports **215**, 203–333 (1992).