Newton's constant and the quantum of action as structural identities of a single elastic medium

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Abstract

Why does gravity have the strength it does, and why is the quantum of action the size it is? In the standard framework these are not questions but inputs: Newton's constant G and the reduced Planck constant $\hbar$ are measured and inserted by hand, and the Planck relation that binds them to the speed of light c is left unexplained. We show that if the vacuum is modelled as a single relativistic elastic medium—a complex scalar field $\Psi = S\,e^{i\theta}$ governed by one Lagrangian, characterised by a stiffness density ρ_P, a coherence length ℓ_P, and the causal structure c—then G and $\hbar$ are not independent inputs but *structural identities* of the medium. Linear response of the field fixes $G = c^2/(\rho_P \ell_P^2)$, a value confirmed independently by the one-loop Sakharov induced action, and canonical quantisation of the phase mode gives $\hbar = \rho_P c\,\ell_P^4$. Eliminating ρ_P between the two returns the Planck relation $\ell_P = \sqrt{\hbar G/c^3}$, so a single pair (ρ_P, ℓ_P) reproduces both constants at once, to better than 0.1%, with no parameter fitted to either. Each identity is moreover the *unique* dimensionally admissible combination of the medium's scales, so the agreement is a forced selection rather than a coincidence. As a physical corollary, the same two identities reproduce the Chandrasekhar mass, unifying quantum degeneracy support and gravitational collapse within one medium. We state plainly the one degree of freedom that remains calibrated rather than derived, and give a falsifiable prediction: a Planck-suppressed correction to the dispersion of gravitational waves.

Keywords: emergent gravity; fundamental constants; Sakharov induced gravity; elastic substrate; Bayesian model selection; Planck scale

1 Introduction

The Standard Model of particle physics and the ΛCDM model of cosmology are, between them, extraordinarily accurate and strikingly incomplete: they contain of order twenty numerical constants that are not predicted but measured, inserted by hand and used as inputs [1]. Two of these constants are more than bookkeeping. Newton's gravitational constant G and the reduced Planck constant $\hbar$ set the strengths of the two interactions—gravity and quantum coherence—whose reconciliation is the central unsolved problem of fundamental physics. That they are treated as brute facts is not a technical inconvenience but a foundational one: a theory that takes G and $\hbar$ as independent primitives cannot, even in principle, say why they have the values they do, nor why they are tied to the speed of light by the Planck relation. If those two constants could instead be *derived*, the derivation would have to explain that relation as well—and any framework that did so would deserve a close look.

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This paper develops one such derivation as a single, self-contained claim. If the physical vacuum is modelled as one relativistic elastic medium—a complex scalar field $\Psi = S\,e^{i\theta}$ governed by a single Lagrangian, in the tradition of induced-gravity, emergent-spacetime, entropic-gravity and superfluid-vacuum programmes [2–5, 9, 11–13]—then G and $\hbar$ are not independent inputs. The medium is characterised by just two substantive properties, a stiffness density ρ_P (setting the energy scale) and a coherence length ℓ_P (setting the ultraviolet grain), together with one causal structure, Lorentz invariance, whose dimensional name is c. The governing Lagrangian is not postulated but constructed as the most general effective field theory the medium's symmetries allow, truncated to the order that governs gravity and quantum coherence, where it is unique (Section 2). From it, the modulus sector yields $G = c^2/(\rho_P \ell_P^2)$ and the phase sector yields $\hbar = \rho_P c\,\ell_P^4$, each a *structural identity* obtained by solving the dynamics.

A sharp objection arises at once, and we meet it head-on: if ℓ_P is taken to be the Planck length, is G not already hidden inside it, making the exercise circular? The answer, developed in Section 5, is that the two identities are not independent—eliminating ρ_P between them *returns* the Planck relation $\ell_P = \sqrt{\hbar G/c^3}$—so that a single pair (ρ_P, ℓ_P) must satisfy the gravitational and the quantum identity at once, which a generic medium does not grant. Moreover, each identity is the unique dimensionally admissible combination of the medium's scales, so the sub-0.1% agreement with observation is a forced selection, not a tautology—a claim we sharpen into a Bayesian test in Section 6.

The programme within which this claim sits, PIU-Ψ [23], is broader in scope; we extract the constants sector because it stands on its own, needing none of the programme's cosmology, particle content, or interpretation of quantum mechanics. Two of its methodological commitments are kept throughout, because they are what make the result non-circular. First, every derivation is carried out in the medium's own *pure ratios*, and human units—metre, second, kilogram—are injected only at the end, as one explicit calibration step, when a dimensionless structural result is compared to a laboratory number; we mark that step where it occurs. Second, every result carries a visible epistemic label—[DERIVED] for a complete derivation, [CALIBRATED] for a value anchored to a datum, [HYPOTHESIS] for a motivated conjecture—so that what is derived (the *form* of each identity, and the selection of a unique combination) is never confused with what is calibrated (the numerical values of ρ_P and ℓ_P, the single degree of freedom that remains open, Section 8). Where a result follows rigorously but only *given* an explicit physical assumption—as with the coherence-cell normalisation in the derivation of $\hbar$—we say so in place, marking it derived under the stated assumption rather than unconditionally, so that the reader can weigh each step for what it is.

The structure of the paper follows the logic just sketched. Section 2 introduces the medium and its Lagrangian. Section 3 derives $G = c^2/(\rho_P \ell_P^2)$ by two independent routes—direct linear response and the one-loop Sakharov induced action—that converge on the same prefactor, with the key point that the discrete grain enters only through a fourth-order fluctuation operator whose Lee–Wick pole regulates the theory without displacing the coefficient of G. Section 4 derives $\hbar = \rho_P c\,\ell_P^4$ by canonical quantisation of the phase rotor, and shows as a corollary that the same two identities reproduce the Chandrasekhar mass. Section 5 shows that the two identities collapse to the Planck relation, settles the circularity objection, and evaluates both constants numerically against CODATA. Section 6 confronts the numerical-coincidence objection, resting it on the dimensional uniqueness of G and $\hbar$ and citing a Bayesian test as corroboration. Section 7 states a falsifiable forward prediction. Section 8 states the one open calibration honestly, and Section 9 concludes.

2 The medium and its Lagrangian

2.1 Ontology: one substrate, two properties, one structure

We posit a single physical medium filling all of space, whose local state is described by a complex scalar field

$$\Psi(x) = S(x)\, e^{i\theta(x)}, \tag{1}$$

where the modulus S is a dimensionless measure of the local deformation of the medium relative to its ground state, and the phase θ is a compact $U(1)$ angle. The field is not a wavefunction on a pre-existing spacetime: it is the state of the substrate itself, and the geometry, causal structure and constants that we usually take as a backdrop are to emerge from its dynamics. This inverts the usual order of explanation, in which spacetime and its constants are given and fields live on top; here the medium is primary and everything else is a reading of it.

The medium carries two substantive properties and one causal structure:

- a *stiffness density* ρ_P, with dimensions of mass per volume, setting the energy scale of deformations;
- a *coherence length* ℓ_P, the medium's structural grain: the scale below which its flexural rigidity suppresses short-wavelength excitations, so that the effective description changes regime rather than breaking down (the ultraviolet behaviour is regulated by the medium itself, not cut off by hand);
- a *causal structure* c. Unlike ρ_P and ℓ_P, this is not a substantive property the medium could carry with one value rather than another. It is the statement that space and time are directions of a single manifold—the medium's causal cone—and its numerical value is an artefact of measuring length and time with separately calibrated rulers. In this sense c is structure, not a tunable property: it is what makes the two substantive properties a pair rather than a triple.

Throughout, the subscript P stands for *Pleno* (the medium): the quantities ρ_P and ℓ_P are the substantive properties of the substrate itself, logically prior to any derived constant. They turn out to be of the order of the Planck density and the Planck length, and we show in Section 5 that they in fact coincide with them numerically; but that coincidence is a *result* of the derivations below, not a definition. In particular ρ_P and ℓ_P must not be read as the Planck density and Planck length *by fiat*: the Planck length, written $\sqrt{\hbar G/c^3}$, is a combination of the derived constants G, $\hbar$, c, whereas ℓ_P is a primary property of the medium from which G and $\hbar$ are obtained. That the substantive freedom of the medium is the pair (ρ_P, ℓ_P) alone—c being structure rather than a property, as just noted—matters for the argument. We show in Section 5 that this pair is irreducible—no internal relation of the programme collapses it to a single number.

2.2 The Lagrangian, and why it takes this form

The dynamics of the medium is governed by a single Lagrangian density. Rather than postulate it, we construct it as the most general effective field theory compatible with the medium's symmetries. Its complete form carries four terms,

$$\mathcal{L} = \tfrac{1}{2}\rho_P c^2 \ell_P^2\, \partial_\mu \Psi^* \partial^\mu \Psi \;-\; V(S) \;-\; \frac{\rho_P c^2 \ell_P^4}{40}\,(\Box S)^2 \;-\; V_q(S), \tag{2}$$

which are, in order: a standard relativistic kinetic term weighted by the elastic modulus $\rho_P c^2 \ell_P^2$; a potential $V(S)$ depending only on the modulus (hence automatically $U(1)$ invariant), which fixes the ground state and need not be specified in detail here; a flexural term quadratic in the d'Alembertian, the leading curvature penalty of the discrete medium, whose coefficient $\rho_P c^2 \ell_P^4/40$ is fixed by an angular average over the substrate's microscopic cell; and an affine (Klauder) term $V_q(S) = 3\hbar^2/(8\rho_P \ell_P^8 S^2)$, an infrared wall that enforces the strict positivity $S > 0$ of the modulus by diverging as $S \to 0$.

Why only two terms enter this paper. Of the four, only the kinetic term and the phase sector it contains enter the derivations of G and $\hbar$. The reason is that both derivations work in the *relaxed regime*, linearising about the ground state S_0, where S is bounded away from zero. There the potential $V(S)$ contributes only its curvature $V''(S_0)$ (the mass of the radial mode, which drops out in the gravitational limit, Section 3.1), the flexural term is suppressed by $(\ell_P/\lambda)^2$ and re-enters only in the gravitational-wave dispersion of Section 7, and the Klauder term $V_q \propto 1/S^2$ is utterly negligible: it is an infrared wall active only as $S \to 0$, and contributes nothing at $S \simeq S_0$. This last point also removes a possible circularity: V_q is the one term of (2) that contains $\hbar$, and if it entered the derivation of $\hbar$ the result would be circular. It does not—the phase-rotor quantisation of Section 4 takes place at $S \simeq S_0$, where V_q is absent—so $\hbar$ emerges from (ρ_P, ℓ_P, c) alone, with no $\hbar$ smuggled in through the Lagrangian. We therefore carry (2) in full for fidelity to the medium's complete dynamics, but use only its first term below.

Construction and uniqueness at leading order

That (2) is a construction rather than a guess is worth making explicit, since it answers the natural objection that the whole edifice rests on an assumed Lagrangian. The construction is the standard effective-field-theory procedure [15, 16]: fix the field, impose the required symmetries, write every term they allow, and order the terms by their number of derivatives. The field is $\Psi = S\, e^{i\theta}$ (Section 2.1). The symmetries are those of the medium: Lorentz invariance in the relaxed regime, global $U(1)$ phase invariance, and the analyticity that restricts the building blocks to Ψ and its derivatives. Every admissible term is then a Lorentz scalar built from $|\Psi|^2$ and derivatives, organised as a derivative expansion:

- *Zero derivatives*: an arbitrary function of $|\Psi|^2 = S^2$, i.e. the potential $V(S)$.
- *Two derivatives*: the unique $U(1)$-invariant scalar $\partial_\mu \Psi^* \partial^\mu \Psi$—the kinetic term.
- *Four derivatives*: the leading operator that penalises curvature and so supplies the ultraviolet regulator; the minimal such scalar is $(\Box S)^2$.

The one term not generated by this classical derivative expansion is the Klauder term V_q. It is not a higher operator in the same series but a consequence of quantising a field whose modulus is restricted to the half-line $S > 0$: affine (rather than canonical) quantisation on a manifold with boundary produces it uniquely, with fixed coefficient $3/8$. It belongs to the zero-derivative sector as a function of S alone, but its origin is the configuration-space boundary, not the symmetry expansion, which is why we list it separately. No other term appears at these orders. In particular, alternative four-derivative structures such as $(\partial S)^4$ or $S^2(\partial\theta)^4$ are not forbidden by symmetry, but they are redundant: the regulator that the medium requires is already supplied, and minimally, by $(\Box S)^2$—the operator with the fewest derivatives that penalises curvature—so adding higher derivative powers regulates nothing new while introducing coefficients that no symmetry fixes. By the same power counting, every term beyond (2) is suppressed at long wavelengths by positive powers of $(\ell_P/\lambda)^2$ and is therefore negligible in the regime where G and $\hbar$ are read off. In this precise sense (2) is not one choice among many but the unique effective Lagrangian of the medium to the order that governs gravity and quantum coherence—the same sense in which the Gross–Pitaevskii or Ginzburg–Landau functional is the unique leading-order description of a condensate.

This also locates honestly what is *not* claimed. The effective-field-theory construction fixes the *form* of (2) from symmetry and power counting; it does not derive that form from a deeper microscopic theory of the substrate, any more than the Gross–Pitaevskii equation is derived, within hydrodynamics, from the atomic Hamiltonian beneath it. That deeper derivation is a separate question, which we state as open in Section 9. The strength of the effective description is precisely that its leading-order consequences do not depend on it: whatever the microphysics, a relativistic elastic medium with these symmetries has (2) as its leading Lagrangian, and the identities that follow are properties of that level of description.

Two further remarks fix the logic. First, the modulus S is dimensionless and normalised so that the undisturbed ground state (the "floor" of the medium) sits at a definite value; deformations are measured as departures from it. Second, the elastic modulus $\rho_P c^2 \ell_P^2$ that weights the kinetic term is, dimensionally, an energy per length—the same dimension as c^4/G. This coincidence of dimension is the first hint of the identity we now derive; the content of Section 3 is that the dynamics forces the coincidence of *value*, not merely of dimension.

3 Newton's constant as a structural identity

We derive $G = c^2/(\rho_P \ell_P^2)$ twice. The first route (Section 3.1) is a direct linear-response calculation: it shows that two deformations of the medium interact with a $1/r$ potential and reads off the prefactor. The second route (Section 3.2) is the Sakharov induced-gravity mechanism: integrating out the medium's fluctuations up to its natural cutoff generates an Einstein–Hilbert term whose coefficient is the same c^4/G. That two independent routes yield the same prefactor is what fixes the identity; a substrate with different dynamics would produce a different prefactor.

3.1 Direct route: linear response and the Green function

We work throughout in the medium's own variables and defer human units to Section 5.

Step 1 — Linearisation about the ground state. Write $S = S_0 + \delta S$ with $|\delta S| \ll S_0$, where S_0 is the ground-state modulus, and keep terms linear in δS. In the static regime ($\partial_t \to 0$) and at scales large compared to ℓ_P (where the flexural term of (2), suppressed by $(\ell_P/L)^2$ relative to the kinetic term, is negligible), the d'Alembertian reduces to the spatial Laplacian, $\Box \to -\nabla^2$. The phase sector does not contribute at linear order for a pure-modulus deformation on a constant-phase background. Expanding the potential $V(S)$ about its minimum—the Klauder term V_q being negligible here, since S_0 is far from the infrared wall at $S \to 0$—the zeroth-order term vanishes (minimum condition) and the quadratic coefficient is the curvature $V''(S_0) = \xi\, \rho_P c^2$, which sets the mass of the radial (Higgs-like) mode of the medium. The linearised field equation is

$$-\rho_P c^2 \ell_P^2\, \nabla^2(\delta S) \;+\; \xi\, \rho_P c^2\, \delta S \;=\; \rho_S(\mathbf{x}), \tag{3}$$

where the source $\rho_S(\mathbf{x})$ is the localised energy density of a matter configuration, itself a bound deformation of the medium: in this framework there is no matter external to the substrate, so a "mass" is a bound configuration of Ψ whose integrated energy density sources the surrounding modulus field. This is why the gravitational charge is the mass–energy itself, as Step 2 makes precise, rather than an independent attribute. In the gravitational regime—distances much larger than the inverse mass of the radial mode—the mass term is negligible against the Laplacian, and (3) reduces to Poisson form:

$$\rho_P c^2 \ell_P^2\, \nabla^2(\delta S) = -\rho_S(\mathbf{x}). \tag{4}$$

Step 2 — The source is the energy density. Before solving, we must say what the source ρ_S is, because this is where the gravitational charge is fixed. In this framework there is no matter external to the substrate: a "mass" is a localised configuration of Ψ, and what gravitates is its energy. The energy density of a modulus deformation follows from the kinetic term of (2) as the canonical T_{00},

$$T_{00} = \tfrac{1}{2}\rho_P c^2 \ell_P^2 \, (\nabla \delta S)^2, \tag{5}$$

and the total energy of the configuration *is* its mass, $Mc^2 = \int T_{00}\, d^3x$. This is a definition, not a calibration: it carries no adjustable factor. The gravitational charge of a body is thus its mass–energy Mc^2, not an independent attribute.

Step 3 — Green function and the interaction energy. For a localised source we write $\rho_S(\mathbf{x}) = q\,\delta^3(\mathbf{x})$, where q is the source strength to be related to Mc^2 below. The Green function of the Laplacian in three dimensions is $\nabla^2(1/r) = -4\pi\,\delta^3(\mathbf{x})$, so the solution of (4) is

$$\delta S_1(r) = \frac{q_1}{\rho_P c^2 \ell_P^2} \cdot \frac{1}{4\pi r}, \tag{6}$$

a $1/r$ deformation—the Newtonian form, obtained rather than assumed. The interaction energy of a second source is the overlap of its density with the first deformation, which for point sources is

$$U_{\text{int}} = \int \delta S_1\, \rho_{S,2}\, d^3x = q_2\, \delta S_1(r_{12}) = \frac{q_1 q_2}{4\pi\, \rho_P c^2 \ell_P^2} \cdot \frac{1}{r_{12}}. \tag{7}$$

Step 4 — Fixing the charge and reading the identity. It remains to relate the source strength q to the physical mass–energy Mc^2. The physical charge is the energy; what is not yet fixed is the multiplicative convention with which that energy enters the *unrationalised* field equation (4), whose Green function carries the geometric 4π of the unit sphere. Requiring that (7) reduce to the Newtonian $U = -GM_1M_2/r_{12}$ with a coupling built from the medium's parameters fixes this convention uniquely. Writing $q_i = \alpha\, M_i c^2$ and substituting into (7),

$$U_{\text{int}} = \frac{\alpha^2 M_1 M_2 c^4}{4\pi\, \rho_P c^2 \ell_P^2\, r_{12}} = \frac{\alpha^2}{4\pi} \frac{M_1 M_2 c^2}{\rho_P \ell_P^2\, r_{12}}, \tag{8}$$

and this equals $-\,c^2/(\rho_P \ell_P^2) \cdot M_1 M_2/r_{12}$ (the attractive sign coming from the gradient energy) precisely when $\alpha^2/4\pi = 1$, i.e.

$$q_i = \sqrt{4\pi}\, M_i c^2. \tag{9}$$

The factor $\sqrt{4\pi}$ is therefore not adjustable and not chosen for elegance: it is forced, and it is the familiar solid-angle factor that distinguishes unrationalised (Gaussian) from rationalised (Heaviside–Lorentz) conventions—exactly as in electromagnetism, where the same $\sqrt{4\pi}$ relates the Gaussian and SI charge. The physical content is that the charge is the energy Mc^2; the $\sqrt{4\pi}$ is a property of the Green function's normalisation, carried by the field convention, not a free coupling. With it, the coefficients match and

$$\boxed{G = \frac{c^2}{\rho_P\, \ell_P^2}} \,. \tag{10}$$

The dimensions check: $[c^2/(\rho_P \ell_P^2)] = (\mathrm{m^2 s^{-2}})/(\mathrm{kg\, m^{-3}} \cdot \mathrm{m^2}) = \mathrm{m^3 kg^{-1} s^{-2}}$, exactly the units of G.

Proposition 1. *The $1/r$ form of the interaction is a derived consequence of the quadratic kinetic term of (2), and the prefactor is fixed without any free normalisation to $c^2/(\rho_P \ell_P^2)$.* Classification: *[DERIVED].*

3.2 Induced route: Sakharov's mechanism in detail

The direct route fixes the prefactor from the tree-level interaction. We now confirm the *same* prefactor from the one-loop effective action, following Sakharov's induced-gravity mechanism [2, 9], of which several modern derivations and thermodynamic reformulations exist [11, 14] applied to the field Ψ. The two routes are logically independent: the first is classical linear response, the second is the integration of quantum fluctuations. Their agreement is what selects the identity.

Step 1 — The gravitational mode is the medium itself. The effective metric $g^{\text{eff}}_{\mu\nu}$ is not an external primitive: it is the transverse–traceless (TT) mode of the perturbed field,

$$g^{\text{eff}}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(\Psi), \tag{11}$$

where $h_{\mu\nu}$ is built from the transverse tensor perturbations of the medium and $\eta_{\mu\nu}$ is the background metric of the relaxed regime. The action induced on this mode is therefore an action of the medium on itself, not of a foreign geometry.

Step 2 — The induced effective action. Integrating out the fluctuations of Ψ produces an effective action [10] with the Einstein–Hilbert structure plus a derived curvature-squared correction,

$$\Gamma_{\text{ind}}[g] = -\frac{c^4}{16\pi G}\int d^4x\,\sqrt{-g}\,R \;+\; \frac{\ell_P^2}{20}\int d^4x\,\sqrt{-g}\,\mathcal{C}[R^2], \tag{12}$$

where the quadratic-curvature term is the geometric translation of the flexural term $\frac{\rho_P c^2 \ell_P^4}{40}(\Box S)^2$ already present in (2), with derived coefficient $\alpha_{R^2} = \ell_P^2/20$ and sign fixed positive by the medium's stability. The number $20 = \ell(\ell+1)|_{\ell=4}$ is the $SO(3)$ relaxation invariant of the substrate; the same $\sqrt{20}$ recurs, as we note below, in the ultraviolet regulator and in the Heisenberg floor.

Step 3 — The heat-kernel scaling of $1/G$. Two features of (12) are robust, independent of the regularisation scheme. First, the *sign*: integrating bosonic fluctuations generates an Einstein–Hilbert term with positive coefficient, i.e. attractive gravity ($G > 0$). Second, the *scale*: by the heat-kernel (Seeley–DeWitt) expansion, the coefficient of $\sqrt{-g}\,R$ is proportional to the square of the ultraviolet cutoff,

$$\frac{1}{G_{\text{ind}}} \;=\; \frac{b}{16\pi^2}\,\frac{\Lambda^2}{c^2}\cdot(\text{units}), \qquad 1/G_{\text{ind}} \propto \Lambda^2. \tag{13}$$

In the present medium the cutoff is not free: it is the microscopic grain $\Lambda = 1/\ell_P$. Restoring the elastic modulus through the native identity $\hbar = \rho_P c\,\ell_P^4$ (Section 4),

$$\frac{c^4}{G_{\text{ind}}} \;\sim\; \frac{\hbar c}{\ell_P^2}\cdot b \;=\; \rho_P c^2 \ell_P^2 \cdot b, \tag{14}$$

with $b = \mathcal{O}(1)$ collecting the Seeley–DeWitt factor and the mode density of the medium. Substituting $\ell_P^2 = \hbar G/c^3$ into $G_{\text{ind}} \sim c^3\ell_P^2/(\hbar b)$ returns $G_{\text{ind}} \sim G/b$: the induced constant reproduces the direct one up to the $\mathcal{O}(1)$ factor b, because both share the single structural scale $\rho_P c^2 \ell_P^2$.

Step 4 — The fixed point $b = 8\pi^2$. The ratio $\gamma_G \equiv G_{\rm ind}/G$ equals unity if and only if $b = 8\pi^2$, obtained by equating the induced coefficient $K_{\rm ind} = b\,\rho_P c^2 \ell_P^2/(16\pi^2)$ with the kinetic coefficient of (2), $K_{F1} = \rho_P c^2 \ell_P^2/2$:

$$\frac{b}{16\pi^2} = \frac{1}{2} \quad \Longleftrightarrow \quad b = 8\pi^2. \tag{15}$$

Both factors of this condition are known structure, not pending physics. The factor $16\pi^2 = (4\pi)^2$ is the four-dimensional phase-space volume of the heat kernel—universal to the method, not a lattice number. The factor $1/2$ is the canonical normalisation of the kinetic term in (2). Thus (15) is not a numerical coincidence to be tuned: b is the phase-space volume of the method divided by the field normalisation of $F1$, and carries no undetermined lattice number. What has genuine physical content is that the two independent routes to G agree—the induced coefficient, divided by its natural phase-space volume, equals the direct kinetic coefficient—and this is falsifiable: a substrate whose gravitational mode carried a scalar component or additional polarisations would give $b \neq 8\pi^2$.

Step 5 — Why the discrete grain does not contaminate the coefficient. One might worry that the microscopic discreteness of the medium spoils the coefficient that fixes G. It does not, and the reason is structural. The TT fluctuation operator is fourth order, because of the flexural term of (2),

$$\mathcal{O}_{\rm TT} = -\Box - \frac{\ell_P^2}{20}\Box^2 = -\Box\left(1 + \frac{\ell_P^2}{20}\Box\right). \tag{16}$$

In momentum space (Euclidean, $\Box \to -k^2$) the associated propagator decomposes into partial fractions in a *forced*—not chosen—way, an exact algebraic identity:

$$\frac{1}{k^2\left(1 - \dfrac{\ell_P^2 k^2}{20}\right)} = \frac{1}{k^2} - \frac{1}{k^2 - \dfrac{20}{\ell_P^2}}. \tag{17}$$

This exhibits two poles with forced residues: a *physical* pole at $k^2 = 0$ (the massless TT mode, the gravitational wave), with residue

$$\operatorname*{Res}_{k^2=0} \frac{1}{k^2(1 - \ell_P^2 k^2/20)} = \lim_{k^2\to 0} \frac{1}{1 - \ell_P^2 k^2/20} = +1, \tag{18}$$

i.e. the standard coupling of a massless mode to curvature; and a regulator (Lee–Wick) pole [17, 18] at $k^2 = 20/\ell_P^2$, of Planck mass, which decouples at scales $L \gg \ell_P$. The heat-kernel bookkeeping then separates cleanly: the regulator pole contributes to the Λ^4 coefficient (a cosmological-constant term), *not* to the Λ^2 coefficient that fixes G. Because the gravitational mode is spin-2 transverse-traceless, with two physical polarisations and representation dimension $5 = 2\ell+1$ ($\ell = 2$)—confirmed both by the medium's spectral lineage (Casimir $\ell(\ell+1) = 6$) and by the standard $D = 4$ graviton count $(D+1)(D-2)/2 = 5$—no scalar or extra polarisation contaminates the coefficient. The condition with physical content, that the integrated TT mode reproduce the stiffness $1/G = \rho_P \ell_P^2$ with no spurious factor, is therefore derived.

It is worth making explicit why the discreteness of the diamond lattice cannot smuggle an extra $\mathcal{O}(1)$ factor into the coefficient of G, since this is the crux of the induced route. The lattice enters the calculation only through the fluctuation operator (16), and it enters in a controlled way: its sole effect is to promote the operator from second to fourth order, adding the flexural term $-(\ell_P^2/20)\Box^2$. That single modification has two consequences, both forced rather than chosen. It shifts the ultraviolet regulator to the Lee–Wick pole at $k^2 = 20/\ell_P^2$, whose contribution is quartic in the cutoff (Λ^4) and so feeds the cosmological constant, not G; and it leaves the residue of the physical $k^2 = 0$ pole—the residue that fixes G—pinned at exactly $+1$ by the partial-fraction identity (17), which is algebraic and admits no free constant. A lattice with a different microscopic geometry would change the number 20 in the flexural coefficient, and hence the mass of the regulator pole, but it could not change the residue of the massless pole away from unity without also changing the mode content away from a pure spin-2 doublet—which the mode count above excludes. The coefficient of G is thus protected by the pole structure, not by an assumption that the medium "behaves exactly right": any admissible single-substrate dynamics of the form (2) yields the same residue, and a substrate that did not (an extra scalar polarisation, say) would announce itself as $b \neq 8\pi^2$ and a measurable deviation from pure spin-2 gravitational radiation.

That the two routes—classical linear response (10) and one-loop induced action (12)–(15)—yield the same prefactor $c^4/G = \rho_P c^2 \ell_P^2$ is the content of the identity. We are careful about what the induced route does and does not establish. It is *not* a first-principles evaluation of the induced coefficient from the full functional integral over the substrate's microscopic degrees of freedom; that computation would require the microscopic dynamics discussed in Section 9 and is not attempted here. But this does not leave b as a free $\mathcal{O}(1)$ number to be tuned. The value $b = 8\pi^2$ factorises, through (15), into two pieces that are both fixed structure: the phase-space volume $16\pi^2 = (4\pi)^2$ of the four-dimensional heat kernel, which is universal to the Seeley–DeWitt method and carries no lattice information, and the factor $\frac{1}{2}$, which is the normalisation convention of the kinetic term in (2). Trying to "derive" the $\frac{1}{2}$ from the diamond lattice would be a category error: it is a convention, not a lattice number. The one piece of (15) that *does* carry physical, falsifiable content—that the integrated TT mode reproduce the stiffness $1/G = \rho_P \ell_P^2$ with no spurious factor—is derived, by the partial-fraction residue $+1$ of (17) and the mode count $2\ell + 1 = 5$ that fixes the gravitational mode as a pure spin-2 doublet (Step 5). *Classification:* the $1/r$ form and the direct coefficient $c^2/(\rho_P \ell_P^2)$ are [DERIVED]; the equality $\gamma_G = 1$, i.e. $b = 8\pi^2$, is [DERIVED] in the specific and limited sense that its only physical content (TT non-contamination) is derived while its remaining factors are method and convention—not, we emphasise, a functional-integral computation of the coefficient, which remains open. The numerical value of G awaits the calibration of Section 5.

3.3 G as the compliance of the medium

The identity (10) admits a physical reading that clarifies why gravity is weak. Inverting it,

$$\frac{1}{G} = \frac{K}{c^4}, \qquad K = \rho_P c^2 \ell_P^2 = \frac{c^4}{G} \approx 1.21 \times 10^{44}\ \mathrm{N}, \tag{19}$$

where K is the structural tension of the medium (the "Planck force"). Thus G is not a coupling strength but the *compliance* (inverse stiffness) of the substrate. Gravity is weak because the medium is extraordinarily dense and hence extraordinarily rigid; the hierarchy between gravity and the other interactions becomes the macroscopic reading of a nearly incompressible medium, rather than an unexplained small number. This reading also points to the calibration route of Section 8: a measurement of the saturation density of a compact object would fix ρ_P without passing through the Planck scale.

4 The reduced Planck constant as a structural identity

Whereas G emerges from the modulus sector, $\hbar$ emerges from the phase sector. The reduced Planck constant quantifies the minimal unit of action of quantum mechanics; in the present framework it arises from the canonical quantisation of the medium's phase mode θ. We again work in the medium's variables and defer human units to Section 5.

Step 1 — Canonical momentum conjugate to the phase. Writing the kinetic term of (2) in the variables (S, θ), the phase sector is $\mathcal{L}_\theta = \frac{1}{2}\rho_P c^2 \ell_P^2 S^2 (\partial_\mu\theta)(\partial^\mu\theta)$. The canonical momentum conjugate to θ is

$$\pi_\theta = \frac{\partial \mathcal{L}_\theta}{\partial(\partial_t\theta)} = \rho_P \ell_P^2 S^2\, \partial_t\theta. \tag{20}$$

Step 2 — Noether charge of the phase. Invariance under $\theta \to \theta + \alpha$ produces, by Noether's theorem, a conserved current $j^\mu = \rho_P c^2 \ell_P^2 S^2 \partial^\mu\theta$, whose integrated time component is the $U(1)$ charge $Q = \int \pi_\theta\, d^3x$ of the medium.

Step 3 — The fundamental mode as a $U(1)$ rotor. The spatially homogeneous phase mode (the zero mode, $\partial_i\theta = 0$) retains only the temporal part of the kinetic term. With the Minkowski contraction $(\partial_\mu\theta)(\partial^\mu\theta) = (\partial_t\theta)^2/c^2 - (\nabla\theta)^2$, the zero mode gives $\mathcal{L}_\theta = \frac{1}{2}\rho_P c^2 \ell_P^2 S^2 \cdot (\partial_t\theta)^2/c^2 = \frac{1}{2}\rho_P \ell_P^2 S^2 \dot{\theta}^2$; the factor c^2 of the modulus cancels against the $1/c^2$ of the temporal contraction. Integrating this density over the support of the mode gives the Lagrangian of a rotor:

$$L_\theta = \tfrac{1}{2}\, I_\theta\, \dot{\theta}^2, \qquad I_\theta = \rho_P \ell_P^2 \int S^2\, d^3x, \tag{21}$$

where I_θ is the *moment of inertia* of the phase rotor, with dimension $[\rho_P \ell_P^2 \cdot \mathrm{m}^3] = \mathrm{kg} \cdot \mathrm{m}^2$ (angular inertia, not mass). Its conjugate momentum $p_\theta = I_\theta \dot{\theta}$ is the angular momentum of the rotor, identical to the Noether charge of Step 2.

Step 4 — Normalisation of the fundamental mode (derived, not postulated). The integral $\int S^2\, d^3x$ is neither a free parameter nor a convention; it is fixed by two properties of the medium already established. First, the modulus S is dimensionless and, through the constitutive identification $S^2 = n/n_0$ (the local excitation density relative to the ground-state density), is normalised to unity at saturation: the fundamental quantum is the minimal-saturation excitation, $n = n_0$, i.e. $S = 1$. Second—and this is the physical content the normalisation rests on, not a fitted choice—the support of the fundamental phase mode is one coherence cell, of volume ℓ_P^3. The reason is definitional rather than assumed: ℓ_P *is* the coherence length of the medium, the scale over which the phase θ maintains rigidity (Section 2); below ℓ_P there is no phase structure to excite, because the flexural term of (2) penalises curvature of Ψ on shorter scales with an energy cost that diverges as the scale shrinks. A phase excitation therefore cannot be localised more tightly than one coherence cell, and its minimal, lowest action support is exactly that cell. This is the same ℓ_P that sets the ultraviolet regulator of the Lagrangian and the Lee–Wick pole of Section 3.2; the mode normalisation is not an independent postulate but a reading of the medium's grain. Hence

$$\int S^2\, d^3x = \underbrace{S^2}_{=1} \cdot \underbrace{V_{\text{cell}}}_{=\ell_P^3} = \ell_P^3, \qquad \text{so} \qquad I_\theta = \rho_P \ell_P^2 \cdot \ell_P^3 = \rho_P \ell_P^5. \tag{22}$$

Crucially, this normalisation is computed with no reference to $\hbar$ or to the quantisation of charge; it uses only the saturation value $S = 1$ and the coherence volume ℓ_P^3, so the value of $\hbar$ that emerges in Step 5 is not circular.

Step 5 — Quantisation of the rotor and the value of $\hbar$. The phase lives on the circle $\theta \in [0, 2\pi)$, so the zero mode is a planar rotor. The angular-momentum spectrum of a planar rotor is an exact canonical theorem—the eigenvalues of $\hat{p}_\theta = -i\hbar\, \partial_\theta$ under periodicity are

$$p_\theta = n\hbar, \qquad n \in \mathbb{Z}. \tag{23}$$

The fundamental quantum ($n = 1$) carries $p_\theta = \hbar$. That same fundamental mode rotates at the coherence frequency of the medium, $\omega_P = c/\ell_P$. This is not an extra assumption or a zone-boundary estimate: it is the *definition* of ℓ_P as the coherence length—the scale over which the phase maintains rigidity while propagating at speed c, so that $\omega_P \ell_P = c$ identically. The physical angular momentum of the rotor, in the medium's parameters, is $p_\theta = I_\theta\, \omega_P$. This is the one physical input of the derivation: the canonical spectrum gives $p_\theta = \hbar$ for the fundamental mode, and the structural dynamics gives that same mode a physical angular momentum $I_\theta\, \omega_P$; identifying the two—the statement that the fundamental $U(1)$ excitation of the medium *is* the quantum of action—and using (22),

$$\hbar = I_\theta\, \omega_P = (\rho_P \ell_P^5) \cdot \frac{c}{\ell_P} = \rho_P c\, \ell_P^4. \tag{24}$$

This yields the identity

$$\boxed{\hbar = \rho_P\, c\, \ell_P^4\ .} \tag{25}$$

The dimensions check: $[\rho_P c\, \ell_P^4] = \mathrm{kg\, m^{-3} \cdot m\, s^{-1} \cdot m^4} = \mathrm{kg\, m^2 s^{-1}} = \mathrm{J\, s}$, exactly the units of $\hbar$.

Proposition 2. *The reduced Planck constant is a structural identity of the medium, $\hbar = \rho_P c\, \ell_P^4$, derived by canonical quantisation of the homogeneous phase rotor.* Classification: *the functional form $\hbar \propto \rho_P c\, \ell_P^4$ is [DERIVED] from the rotor spectrum and the medium's parameters; the unit prefactor rests on identifying the fundamental mode's support with one coherence cell of volume ℓ_P^3, which we have argued follows from the definition of ℓ_P as the coherence length (below which the flexural term forbids phase structure) rather than being an independent postulate. A reader who regards that identification as an assumption should read the unit prefactor as* derived under that stated assumption*; the functional form is unconditional.*

The same value of $\hbar$ is reached by routes internal to the framework that are independent of the rotor quantisation—a Madelung–Bohm reading of the phase field, and a stochastic derivation via the Einstein relation $D = k_B T/\gamma$ using the medium's structural temperature and viscosity, in the spirit of Nelson's stochastic mechanics [20]. These agree with (25) because they are all readings of the one medium, so the value is not an artefact of a single quantisation scheme. Crucially, the stochastic route obtains $D = \hbar/(2m)$ from ingredients that do not themselves contain $\hbar$ (the structural temperature $k_B T_P = \rho_P c^2 \ell_P^3$ and a Stokes drag at scale ℓ_P), which fixes the causal direction as $\rho_P, \ell_P, c \to \hbar$ rather than the reverse.

4.1 A physical corollary: the Chandrasekhar mass

Because G and $\hbar$ are now identities of one medium, quantities that in standard physics require both gravity and quantum mechanics—and hence the marriage of two independent theories—become internal balances of the substrate. The clearest example is the Chandrasekhar mass, the upper limit to the mass a degenerate star can support. In the standard treatment it combines three ingredients drawn from three theories: Pauli exclusion (quantum statistics), relativistic degeneracy pressure (quantum mechanics), and hydrostatic equilibrium (gravity). In the present framework all three are behaviours of the same medium: the fermionic statistics follow from the topological spin–statistics closure of the medium's solitons, the degeneracy pressure is the flexural rigidity that also sets the Heisenberg floor, and gravity is the macroscopic response of Section 3. Balancing degeneracy pressure against self-gravity for a relativistic degenerate gas gives the $n = 3$ polytrope, whose Lane–Emden structure fixes the numerical coefficient, and yields a radius-independent limiting mass [19]

$$M_{\mathrm{Ch}} = \frac{\sqrt{3\pi}}{2} \left(-\xi^2 \theta'\right)_{n=3} \frac{(\hbar c/G)^{3/2}}{(\mu_e m_u)^2}, \tag{26}$$

with $(-\xi^2\theta')_{n=3} = 2.0182$ the surface constant of the $n = 3$ Lane–Emden polytrope, μ_e the number of nucleons per electron ($\mu_e = 2$ for carbon–oxygen matter) and m_u the atomic mass unit. The purely geometric coefficient $\frac{\sqrt{3\pi}}{2} \cdot 2.0182 \approx 3.098$ is fixed by the polytrope, with nothing adjusted. Writing $m_P = \sqrt{\hbar c/G}$, this is $M_{\text{Ch}} = 3.098\, m_P (m_P/\mu_e m_u)^2$, which evaluates to

$$M_{\text{Ch}} \approx 1.45\, M_\odot, \tag{27}$$

the observed white-dwarf limit. The dimensions check: $[(\hbar c/G)^{3/2}/m_u^2] = [\text{kg}^3]/[\text{kg}^2] = \text{kg}$. The significance for this paper is not a new stellar formula but a unification: the same two identities $G = c^2/(\rho_P \ell_P^2)$ and $\hbar = \rho_P c\, \ell_P^4$, read together with the medium's spin–statistics and rigidity, reproduce a quantity that otherwise demands two separate theories. *Classification:* the functional form and coefficient are [DERIVED] from the polytrope and the medium's established identities; the identification of the degeneracy pressure with the medium's flexural rigidity is [DERIVED] in the wider corpus [23].

5 The link between the two identities and numerical evaluation

5.1 The two identities are not independent

The identities (10) and (25) are not logically independent. Eliminating ρ_P between them,

$$G = \frac{c^2}{\rho_P \ell_P^2} \quad \text{and} \quad \hbar = \rho_P c\, \ell_P^4 \implies \rho_P = \frac{c^2}{G\ell_P^2} \implies \hbar = \frac{c^2}{G\ell_P^2}\, c\, \ell_P^4 = \frac{c^3 \ell_P^2}{G}, \tag{28}$$

so that

$$\boxed{\ell_P = \sqrt{\frac{\hbar G}{c^3}}\,.} \tag{29}$$

The right-hand side is exactly the Planck length; the coherence length ℓ_P of the medium therefore *coincides numerically* with $\sqrt{\hbar G/c^3}$ as an algebraic consequence of the two identities, rather than being defined as it. This is the crux of the non-circularity argument. One might object that if ℓ_P is simply set equal to the Planck length, then G is already "inside" ℓ_P and recovering it is trivial. The reply is that the two identities are not independent: fixing the two substantive properties (ρ_P, ℓ_P) satisfies *both* the gravitational and the quantum identity simultaneously. The non-trivial content is not "recovering G" but that a *single* length ℓ_P and a *single* density ρ_P reproduce G and $\hbar$ at once—something a generic medium does not guarantee. The triplet (ρ_P, ℓ_P, c) is dimensionally independent (three distinct physical dimensions that no identity of the framework collapses), and among all combinations of them with the correct units the dynamics of (2) selects a definite one for each constant—G via the Sakharov/linear-response route, $\hbar$ via phase quantisation. In fact, as Section 6 shows, restricting to integer exponents leaves *exactly one* admissible combination per constant, so the selection is not among many but among essentially one; either way, it is the dynamics—not unit-matching—that fixes the value, and this is what makes the sub-0.1% agreement falsifiable rather than tautological.

Claim 1. No relation internal to the framework reduces the two substantive properties (ρ_P, ℓ_P) to one. The natural candidate—identifying ρ_P with a "Planck density" m_P/ℓ_P^3—is circular: using (10) and (25), the Planck mass is $m_P = \sqrt{\hbar c/G} = \rho_P \ell_P^3$, so $m_P/\ell_P^3 = \rho_P$ *identically.* The identity carries no new information; the medium retains exactly one uncalibrated structural degree of freedom (see Section 8).

5.2 Injecting the human yardstick: numerical evaluation

Up to this point every result has been a structural identity in the medium's own ratios. We now perform the single calibration step: we introduce the human units—metre, second, kilogram—and the calibrated values of the medium's properties, in order to compare with laboratory

Table 1: Structural identities evaluated at the calibrated values (30), compared with CODATA. No parameter is fitted to G or $\hbar$; the agreement is a consequence of the identities.

Constant	Structural identity	Value (this work)	Rel. error
Gravitational constant G	$c^2/(\rho_P \ell_P^2)$	6.6765×10^{-11}	3.3×10^{-4}
Reduced Planck constant $\hbar$	$\rho_P c\, \ell_P^4$	1.0540×10^{-34}	6.0×10^{-4}
Planck relation	$\ell_P = \sqrt{\hbar G/c^3}$	(derived link)	—

constants. This is the only place where human units enter, and it enters as an explicit act of measurement, not as an assumption buried in the derivation.

The calibrated values of the medium's properties are

$$\rho_P = 5.155 \times 10^{96}\ \mathrm{kg\,m^{-3}}, \quad \ell_P = 1.616 \times 10^{-35}\ \mathrm{m}, \quad c = 2.998 \times 10^{8}\ \mathrm{m\,s^{-1}}. \tag{30}$$

Evaluating (10):

$$\begin{aligned} c^2 &= 8.9880 \times 10^{16}\ \mathrm{m^2 s^{-2}}, \\ \ell_P^2 &= 2.6115 \times 10^{-70}\ \mathrm{m^2}, \\ \rho_P \ell_P^2 &= 1.3462 \times 10^{27}\ \mathrm{kg\,m^{-1}}, \\ G = \frac{c^2}{\rho_P \ell_P^2} &= 6.6765 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}. \end{aligned} \tag{31}$$

Compared with $G_{\mathrm{CODATA}} = 6.6743 \times 10^{-11}$ [21] $\mathrm{m^3 kg^{-1} s^{-2}}$, the relative discrepancy is 3.3×10^{-4}. Evaluating (25):

$$\begin{aligned} \ell_P^4 &= 6.8197 \times 10^{-140}\ \mathrm{m^4}, \\ \rho_P c &= 1.5455 \times 10^{105}\ \mathrm{kg\,m^{-2} s^{-1}}, \\ \hbar = \rho_P c\, \ell_P^4 &= 1.0540 \times 10^{-34}\ \mathrm{J\,s}. \end{aligned} \tag{32}$$

Compared with $\hbar_{\mathrm{CODATA}} = 1.0546 \times 10^{-34}$ J s [21], the relative discrepancy is 6.0×10^{-4}. Both identities reproduce the measured constants to better than 0.1% with no adjustable parameter in the identity itself.

6 Coincidence, dimensional uniqueness, and a statistical test

The identities of Table 1 invite an obvious objection: that the agreement of G and $\hbar$ with observation could be a fortuitous algebraic coincidence—that among the many ways of combining the medium's scales, some combination was bound to land near the measured value [7]. The strongest reply is not statistical but structural, and it is already implicit in Section 5: *each constant is the unique dimensionally admissible combination of the medium's scales.*

Enumerating all combinations $c^a \rho_P^b \ell_P^d$ with integer exponents (we scan $|a|, |b|, |d| \leq 6$; the result is stable), the number carrying the units of G ($\mathrm{m^3 kg^{-1} s^{-2}}$) is exactly one, $G \propto c^2 \rho_P^{-1} \ell_P^{-2}$, and the number carrying the units of $\hbar$ ($\mathrm{kg\,m^2 s^{-1}}$) is exactly one, $\hbar \propto \rho_P c\, \ell_P^4$. There is therefore no combinatorial freedom to exploit: dimensional analysis alone fixes the functional form of each identity and excludes every alternative. What the dynamics of Sections 3 and 4 supplies is that this single admissible combination, with no adjustable prefactor, reproduces the measured value to better than 0.1%. A coincidence can, at most, place one isolated constant; it does not explain why the two core constants are *each* the unique combination of the same two primaries, why both hit their measured values at once, and why the single pair (ρ_P, ℓ_P) that does so also returns the Planck relation. That is structural coherence, not tuning.

This qualitative argument can be made quantitative with Bayesian model selection, comparing the framework against the null hypothesis that the agreements are numerical coincidences. Carried out for the dimensionless quantities derived in the programme—the Chandrasekhar mass and, in the wider corpus, the cosmological ratios $\sqrt{3}\,\pi$ and f_c (matched against Planck 2018 data [22])—the analysis returns a Bayes factor of order 10^{11} against the coincidence hypothesis, "decisive" on the Jeffreys scale [6, 8]. We do not reproduce that computation in the body of the paper, for two reasons. First, its null model rests on a choice of algebraic alphabet and operator set, which is a construct of the description language rather than a feature of nature; the resulting number, though reproducible, is not an objective probability and should not be read as one. Second, for the two core constants the decisive statement is the dimensional uniqueness above, which needs no alphabet at all. We therefore state the Bayesian result only as corroboration, and refer the reader to the self-contained verification script provided as Supplementary Information and to the full treatment in the deposited corpus [23] (Zenodo, DOI 10.5281/zenodo.21205881), where the enumeration, the likelihoods, and the robustness analysis are given in full. The weight of the present paper rests on the derivations of Sections 3 and 4, not on this factor.

7 A falsifiable forward prediction

The framework is not merely retrodictive. The flexural term of (2) modifies the propagation of the transverse–traceless (TT) modes of the medium—the gravitational waves. Their derived dispersion relation is

$$\omega^2 = c^2 k^2 \left[1 + \frac{(k\ell_P)^2}{20}\right], \tag{33}$$

where the coefficient 1/20 is the flexural phase coefficient 1/40 of (2) multiplied by the factor 2 from varying $(\Box h_+)^2$. The TT mode is massless at zeroth order (the potential does not couple to a modulus-preserving configuration), so its propagation speed is exactly c; the sole departure from luminal propagation is the Planck-scale correction in (33). For frequencies accessible to current detectors ($k\ell_P \sim 3 \times 10^{-41}$ at $\sim$ 100 Hz), the correction $(k\ell_P)^2/20 \sim 6 \times 10^{-83}$ lies some sixty-seven orders of magnitude below the present observational bound $\sim 10^{-15}$ on anomalous gravitational-wave dispersion, and is consistent with the multimessenger event GW170817. We state plainly that this places the prediction far beyond the reach of any foreseeable experiment: it is not a near-term observational test, and we do not present it as one. Its value is of a different kind. First, it is a genuine forward prediction in the Popperian sense—the framework fixes the *sign* and *magnitude* of the correction with no freedom, so the identification of gravitational waves as flexural phase modes of the medium is falsifiable in principle: a measured gravitational-wave dispersion *larger* than (33) at accessible frequencies, or a TT mode with a rest mass, would refute it. Second, the smallness is itself a consistency check: a substrate model that predicted a correction anywhere near current bounds would already be excluded, and the framework instead reproduces exact luminal propagation to the precision of every existing measurement. *Classification:* [DERIVED] for the dispersion relation; [HYPOTHESIS] for the primordial-spectrum suppression it implies at $k \sim \ell_P^{-1}$.

8 What remains open: the calibration degree of freedom

Honesty requires stating precisely what is *not* derived. The functional forms of both identities are derived, and the selection of a unique dimensional combination for each is derived. What is *calibrated*, not derived, is the numerical value of the pair (ρ_P, ℓ_P)—the substantive properties of the medium. As shown in Section 5, these two properties are irreducible within the framework, so at least one of them must be fixed by anchoring to a datum, the single unavoidable act of introducing a human unit into a medium that works in pure ratios.

A clarification is essential here, because it is where the appearance of circularity is sharpest. When we assign ℓ_P and ρ_P their numerical values (30), we are anchoring the *primary properties of the Pleno*, and they happen to take the same numbers as the Planck length and Planck density. This is not the same as *defining* ℓ_P to be $\sqrt{\hbar G/c^3}$ and thereby smuggling G back in. The non-circular content, established in Section 5, is that a *single* coherence length and a *single* stiffness density reproduce G *and* $\hbar$ simultaneously; the numerical coincidence of those anchored values with the Planck scales is a consequence of that structure, not an input to it. What we have not done, and what would remove the last trace of the objection, is calibrate (ρ_P, ℓ_P) from a source independent of G and $\hbar$ themselves.

That independent calibration is the genuinely open problem, and we state it as the central limitation of this work: a measurement or derivation of the medium's stiffness density or coherence length from physics that is neither gravitational nor quantum—for instance from its non-linear elastic response, or from the saturation density of a compact object, routes indicated in Section 3.3—which would then *predict* G and $\hbar$ rather than being anchored alongside them. Until that calibration exists, the status of the identities is: the *form* is derived and falsifiable; the *value* rests on a one-property calibration. We regard this as the honest boundary of the present claim, and as the sharpest target for future work.

9 Conclusion

We opened with two questions the standard framework cannot ask: why gravity has the strength it does, and why the quantum of action has the size it does. Within a single-substrate model of the vacuum governed by one Lagrangian, both questions have an answer. Newton's constant and the reduced Planck constant are not independent inputs but structural identities of the medium: $G = c^2/(\rho_P \ell_P^2)$ from the modulus sector, obtained by two convergent routes—classical linear response and the one-loop Sakharov induced action, which agree on the prefactor at the derived fixed point $b = 8\pi^2$—and $\hbar = \rho_P c\, \ell_P^4$ from canonical quantisation of the phase rotor, with a mode normalisation that is itself derived rather than assumed.

The two identities are bound together. Eliminating ρ_P between them returns the Planck relation $\ell_P = \sqrt{\hbar G/c^3}$, so a single pair (ρ_P, ℓ_P) reproduces both constants at once, to better than 0.1%, with no parameter fitted to either. This is the crux of the answer to the circularity objection raised in the introduction: the recovery of G is not smuggled in through ℓ_P, because the same two properties must *simultaneously* satisfy the quantum identity, and a generic medium does not arrange that. Each identity is, moreover, the unique dimensionally admissible combination of the medium's scales, so the match to observation is a forced selection rather than a coincidence; a Bayesian test (Section 6) corroborates this, though the decisive point is the dimensional uniqueness itself. As a physical corollary, the two identities together reproduce the Chandrasekhar mass, unifying within one medium the quantum degeneracy support and the gravitational collapse that standard physics must treat with two separate theories.

What has been established, then, is the *form* of both identities and the dynamical selection that fixes their combination; both are derived and both are falsifiable.

A word on what the medium *is*, since the derivations treat it as a relativistic elastic substrate without specifying its microscopic constitution. In this framework the medium is not an analogy but the single substrate of which matter and energy are two configurations, so that $E = mc^2$ is the statement that both are states of one thing. Its closest realised counterparts are the macroscopic quantum media of condensed-matter physics—a Bose–Einstein condensate or a superconductor, each described by a complex order parameter $\Psi = S\, e^{i\theta}$ formally identical to the field used here, with a measurable stiffness, a coherence length, and propagating modes.

The present work uses only this effective, order-parameter level of description, at which the Lagrangian (2) is fixed by symmetry and power counting (Section 2) and the two identities follow. What it does *not* supply is the deeper microscopic dynamics from which (2) would itself be derived—the analogue of the atomic theory underlying a condensate's Gross–Pitaevskii description. We regard this not as a gap to be papered over but as the defining feature of an effective-field-theory result: the leading-order Lagrangian, and hence the identities for G and $\hbar$, are *universal*—they follow from the symmetries alone and are the same whatever the microphysics turns out to be. Identifying that microphysics is a genuine open question, and a worthwhile one, but the value of the present result does not wait on it, just as the Gross–Pitaevskii description of a condensate was correct and useful before, and independently of, its derivation from the many-body Hamiltonian. The claim of this paper is accordingly the sharp one that *given* the effective elastic description—itself not assumed but constructed from the medium's symmetries—G and $\hbar$ are structural identities rather than inputs.

What remains open beyond that is stated plainly and is the single quantitative boundary of the claim: the numerical value of the pair (ρ_P, ℓ_P) is, at present, calibrated rather than derived, anchored to the Planck scale for want of an independent handle. The decisive step for future work is therefore sharply defined—an independent calibration of the medium's stiffness density or grain length from non-gravitational, non-quantum physics, for instance from the saturation density of a compact object. Supply that one number, and the identities of this paper stop being retrodictions of G and $\hbar$ and become predictions of them from the properties of the medium alone. That is the sense in which the constants of nature may not be constants at all, but readings of a single elastic thing.

Declarations

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Competing interests. The author declares no competing interests.

Data availability. No datasets were generated. A self-contained Python script (standard library only) is provided as Supplementary Information; it reproduces the identities of Section 5, the dimensional-uniqueness counts for G and $\hbar$, and the full enumerative Bayesian test summarised in Section 6. This preprint, together with the verification script, is deposited at Zenodo, 10.5281/zenodo.21286066. The complete programme corpus from which this result is extracted is archived separately at Zenodo, 10.5281/zenodo.21205881.

Use of AI tools. A large language model was used for AI-assisted copy-editing (language, style, and consistency) and to assist in organising and cross-checking the LaTeX presentation of derivations that are the author's own. All physical claims, derivations, and numerical evaluations are the author's, who takes full responsibility for the content.

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