PIU: A Universal Physical Substrate Proposal

Axiomatic Foundations and Derivation of G and $\hbar$

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Abstract. We present the axiomatic framework of the Principle of Universal Integrity (PIU): a fundamental postulate and six axioms describing the Pleno as a continuous physical substrate. The canonical Lagrangian density $\mathcal{L}_{F1}$, composed of three structurally fixed terms (kinetic, potential, and bending), determines the fundamental constants G and $\hbar$ as native structural identities of the substrate, with errors below 0.1% relative to CODATA 2018 and zero adjustable parameters. The gravitational constant emerges from the kinetic–coefficient identity $\frac{1}{2}\rho_P c^2 \ell_P^2 = c^4/(2G)$ via Sakharov's induced–gravity mechanism, while the reduced Planck constant arises from canonical quantization of the Madelung–Bohm flow associated with $\mathcal{L}_{F1}$. The simultaneous derivation of both constants from a single Lagrangian reduces the foundational dimensional input of physics to three Planckian scales (ρ_P, ℓ_P, c), replacing the $\sim$ 19 free parameters of the Standard Model and the $\sim$ 6 of ΛCDM with a structurally closed set.

1. Introduction

Contemporary fundamental physics faces a structural crisis, characterized by the persistent incompatibility between General Relativity and Quantum Field Theory, the unexplained ontological status of dark matter and dark energy, and the proliferation of $\sim$ 25 adjustable free parameters across the Standard Model and ΛCDM cosmology. The Principle of Universal Integrity (PIU) is introduced as a structural response to this crisis. Rather than treating spacetime as a passive geometric background or postulating discrete localized particles as fundamental entities, PIU proposes a single, continuous, hyperelastic physical substrate called the *Pleno*.

This paper establishes the formal axiomatic foundations of the PIU framework (Version 31.10) and details the derivation of two fundamental constants—the universal gravitational constant G and the reduced Planck constant $\hbar$—directly from the Pleno's structural scales, with no adjustable parameters. By grounding the physical universe in a strictly defined continuous medium, we recover both the spacetime metric (as an emergent effective field) and quantum mechanics (via exact Madelung–Bohm equivalence applied to the Pleno field) as macroscopic phenomena of a single underlying substrate.

2. Axiomatic Framework

The PIU framework is built upon a reduced logical structure, minimizing the required assumptions to a single core postulate, six axioms, and three dimensional calibrations (ρ_P, ℓ_P, c).

2.1 The Fundamental Postulate (P-FUND)

Postulate of Universal Integrity. *The universe is not an empty void populated by isolated objects, but a continuous, indivisible, and dynamically active physical substrate (the Pleno). All physical entities, interactions, and observable phenomena—including particles, gauge fields, and spacetime curvature—are strictly emergent manifestations of the local topological and dynamic states of this single underlying medium.*

2.2 The Six Axioms of the Pleno

To formalize the Fundamental Postulate into a mathematically tractable field theory, we define the Pleno as a complex scalar field $\Psi = S\,e^{i\theta}$ governed by the following six axioms:

1. **Cosmological Substrate (A1).** The Pleno acts as the continuous cosmological background of the universe, existing everywhere with a nonzero baseline density. All other physical objects are local modes of this single field.
2. **Ground State and Planckian Scales (A2).** The Pleno possesses inherent, invariant fundamental scales characterized by three Planckian dimensional constants: density ρ_P, length ℓ_P, and the speed of wave propagation c. The vacuum corresponds to a nonzero minimal structural density $S_{\min}$.
3. **Locality and Lorentz Invariance (A3).** The dynamics of the Pleno strictly obey local interactions and Lorentz invariance at macroscopic scales, precluding any nonlocal algebraic operators in its fundamental Lagrangian.
4. **Maximum Curvature (A4).** The substrate possesses a fundamental structural rigidity that imposes a maximum allowable curvature $\kappa_{\max} = 1/\ell_P$, intrinsically regularizing ultraviolet divergences without external prescription.
5. **Topological Conservation (A5).** Quantum numbers and discrete particle charges are topological invariants of the Pleno's phase field θ, enforcing absolute conservation laws without ad-hoc internal symmetries.

6. **Maximum Density Bound (A6).** The Pleno exhibits a saturation limit—a hard upper bound on the structural density $S^2_{\max}$—which structurally blocks singularities inside black holes and at the Big Bang.

These axioms restrict the allowed mathematical forms for the Pleno's Lagrangian, enforcing a unique, structurally rigid equation of motion (cf. Section 3 below).

3. The Fundamental Lagrangian $\mathcal{L}_{F1}$

Under the axiomatic constraints A1–A6, the most general stable Lorentz-invariant Lagrangian density for the scalar field $\Psi(x^\mu)$ admits a strict tripartite form. Excluding non-local operators (forbidden by A3), explicit mass terms (excluded by A2, since $S_{\min} > 0$), $U(1)$-breaking terms (forbidden by A5), and external curvature couplings $\xi R|\Psi|^2$ (incompatible with P-FUND, since the metric is not postulated independently), the canonical Lagrangian $\mathcal{L}_{F1}$ is uniquely fixed up to three structural stiffnesses:

$$\mathcal{L}_{F1} = \tfrac{1}{2}\rho_P c^2 \ell_P^2 (\partial_\mu \Psi)^* (\partial^\mu \Psi) - V(|\Psi|) - \tfrac{\rho_P c^2 \ell_P^4}{12} |\Box\Psi|^2, \quad (1)$$

where $\Box = \partial_\mu \partial^\mu$ is the d'Alembertian and the effective potential $V(|\Psi|)$ incorporates a bare interaction term, an affine quantum constraint (in the sense of Klauder [5]), and a topological confinement term, as established by the PIU-1-M Theorem of the corpus [7].

3.1 Native structural identities

The coefficients in Eq. (1) are not free; they are strictly determined by the Planckian scales (ρ_P, ℓ_P, c). Two native identities are of paramount importance.

Kinetic coefficient. The prefactor of the canonical kinetic term is dimensionally and numerically identical to the Einstein–Hilbert coefficient of the Ricci scalar:

$$\tfrac{1}{2}\rho_P c^2 \ell_P^2 = \tfrac{c^4}{2G}. \quad (2)$$

This is the structural identity that enables Sakharov's induced-gravity mechanism to recover Newtonian and general-relativistic gravitation as effective macroscopic limits.

Bending stiffness and its quantum nature. The higher-order operator $|\Box\Psi|^2$ in $\mathcal{L}_{F1}$ enters the Lagrangian with coefficient $\rho_P c^2 \ell_P^4/12$. The associated *bending stiffness* appearing in the linearized equation of motion—i.e., the coefficient that controls the dispersion relation of long-wavelength fluctuations after the standard SO(3) isotropy average is performed (cf. [7], V30.1.1)—is

$$\gamma_\kappa = \frac{\rho_P c^2 \ell_P^4}{40}. \quad (3)$$

The numerical prefactor 1/40 relative to the Lagrangian prefactor 1/12 is a fixed consequence of the SO(3) angular average over higher-derivative dispersion modes and is not an adjustable factor.

Using the structural identity $\hbar = \rho_P c\, \ell_P^4$ (cf. Sec. 4.2 below), the bending stiffness γ_κ admits an exact native rewriting:

$$\boxed{\gamma_\kappa = \tfrac{\hbar c}{40}.} \quad (4)$$

This identity proves that the higher-derivative term in $\mathcal{L}_{F1}$ is not a classical regularization device but a structural quantum operator, since its coefficient is exactly proportional to $\hbar$. It also provides an independent triple-bridge interpretation (structural / Nelson stochastic / Israel–Stewart hydrodynamic) of the corresponding fakeon mode, including the relaxation time $\tau_\Pi^{\mathrm{PIU}} = t_P/\sqrt{20}$ [7].

4. Derivation of Fundamental Constants

A cornerstone of the PIU framework is its ability to determine macroscopic fundamental constants from the Pleno's intrinsic structural scales. This dramatically reduces the physical parameter space, replacing empirically calibrated constants with derived structural identities.

4.1 Universal Gravitational Constant G

In the PIU framework, gravity is not a fundamental force but an emergent macroscopic phenomenon. Applying Sakharov's induced-gravity mechanism [1] to the canonical kinetic term of $\mathcal{L}_{F1}$, the effective interaction potential between two topological defects (macroscopic masses) of the Pleno, parameterized by spatial perturbations δS_i of the structural density around its ground value $S_{\min}$, is given by

$$U_{\mathrm{int}} = \int (\rho_P c^2 \ell_P^2)\, \nabla(\delta S_1)\cdot\nabla(\delta S_2)\, d^3x = -G\,\frac{M_1 M_2}{r}. \quad (5)$$

The integrand coefficient $\rho_P c^2 \ell_P^2$ coincides with c^4/G via Eq. (2); the Coulomb-like form $1/r$ is the standard Green's function of the Laplacian. The non-trivial structural content of the derivation lies in the *specific value* of G, which is fixed by matching the coefficient:

$$\boxed{G = \frac{c^2}{\rho_P\, \ell_P^2}.} \quad (6)$$

Substituting the postulated independent Planckian scales $\rho_P \approx 5.155 \times 10^{96}\,\mathrm{kg/m^3}$ and $\ell_P \approx 1.616 \times 10^{-35}$ m yields

$$G_{\mathrm{PIU}} = 6.6745 \times 10^{-11}\ \mathrm{N\,m^2/kg^2},$$

which deviates by 0.034% from the CODATA 2018 value [2] ($G_{\exp} = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$), with no adjustable free parameters.

4.2 Reduced Planck Constant $\hbar$

Quantum mechanics enters PIU as an exact mathematical equivalence between the Madelung–Bohm hydrodynamic decomposition of $\mathcal{L}_{F1}$ and the Schrödinger formalism, in the sense of the classical theorems of

Madelung [3] and Bohm [4], applied directly to the Pleno field $\Psi = S\,e^{i\theta}$ (cf. [7], Sec. 4.bis). Identifying the canonical momentum conjugate to the phase θ from $\mathcal{L}_{F1}$ and demanding consistency between the quantization of the circulation and the universal scale of action yields

$$\boxed{\hbar = \rho_P\, c\, \ell_P^4.} \tag{7}$$

Numerically, this structural identity agrees with the exact SI value $\hbar = 1.054571817 \times 10^{-34}\,\mathrm{J\,s}$ within a relative error of 0.058%.

4.3 Joint structural status

The simultaneous agreement of G and $\hbar$ with their experimental values to better than 0.1%, obtained from a single dimensional triplet (ρ_P, ℓ_P, c) with no adjustable parameters, is a structural validation of the canonical form of $\mathcal{L}_{F1}$. The residual errors below 0.1% are well within the tolerance of an uncalibrated structural derivation and are eliminated under a single joint calibration of the pair (ρ_P, ℓ_P); the form of the identities themselves does not depend on such calibration.

5. Discussion: Parameter Reduction and Epistemic Inversion

The simultaneous derivation of G and $\hbar$ from a unified Lagrangian constitutes a structural inversion of the traditional ontology of fundamental physics. Traditional physics defines the Planck scales via macroscopic fundamental constants ($\ell_P = \sqrt{\hbar G/c^3}$, etc.); PIU mathematically inverts this construction: the substrate scales (ρ_P, ℓ_P, c) are taken as ontologically fundamental, and the macroscopic constants G and $\hbar$ emerge as derived structural identities.

This inversion reduces the foundational dimensional input of physics to three Planckian scales, dispensing with the standard set of ~ 19 free parameters of the Standard Model and the ~ 6 of ΛCDM as fundamental quantities. The success of deriving G and $\hbar$ with combined error below 0.1% provides a structurally rigid validation of the axiomatic framework and the tripartite form of $\mathcal{L}_{F1}$. A complete Bayesian analysis comparing this structural hypothesis against the null hypothesis of accidental numerical coincidence is reported in [7] (Block G, V31.10), yielding a decisive global Bayes factor $K_{\mathrm{global}} \sim 10^{11}$ on the Jeffreys scale.

6. Conclusion

The Principle of Universal Integrity offers a mathematically cohesive and ontologically parsimonious foundation for fundamental physics. By defining the universe as a continuous Pleno governed by a single postulate and six strict axioms, the canonically derived Lagrangian $\mathcal{L}_{F1}$ encodes both gravitational and quantum phenomena natively. The derivation of G and $\hbar$ from a single Lagrangian, with no adjustable parameters and with combined accuracy better than 0.1%, constitutes a definitive structural test of the model's rigidity and paves the way for a fully unified description of the cosmos. The companion Letter [8] demonstrates that the same structural framework produces, with zero free parameters and from pure substrate geometry, the cosmological dark-to-baryonic abundance ratio $\Omega_{\mathrm{MO}}/\Omega_M = \sqrt{3}\,\pi$, in sub-$\sigma$ agreement with Planck 2018.

References

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