A Structural Cosmological Prediction with Zero Free Parameters: $\Omega_{\rm MO}/\Omega_M = \sqrt{3}\,\pi$

Manuel Alberto Celedón Mejía*

May 21, 2026

Abstract. We derive the cosmological abundance ratio between the dark-matter mode (MO) and the baryonic-matter mode (M) of a saturated physical substrate (the Pleno) strictly from its discrete geometric topology, obtaining the exact structural quotient $\Omega_{\rm MO}/\Omega_M = \sqrt{3}\,\pi \approx 5.4414$. The derivation requires zero adjustable parameters and rests on three independent established results: (i) Peierls–Nabarro stability barriers, which pin topological baryonic defects to the local Voronoi cell of the diamond lattice (regular tetrahedron) and leave non-topological dark-matter defects to expand isotropically (sphere); (ii) the theory of crystal surface modes of Maradudin, Wallis, and Mills (1971), which establishes that the thermodynamic weight of saturated extended defects is strictly proportional to their surface area; and (iii) the exact ratio of Haar volumes $\mathrm{Vol}(SO(3))/\mathrm{Vol}(SU(2)) = 1/2$ given by the Peter–Weyl theorem (1927). The identification of the topologically protected M mode with fermionic baryonic matter and of the non-topological MO mode with scalar dark matter is a derivational closure of the PIU corpus via Finkelstein–Rubinstein (1968) and Giulini (1993). Under this identification, the predicted ratio coincides with the Planck 2018 cosmic microwave background determination $\Omega_c/\Omega_b = 5.375 \pm 0.077$ at $0.86\,\sigma$, providing a structural alternative to fine-tuning in the dark sector.

1. Introduction

The standard cosmological model (ΛCDM) parameterizes the universe's energy budget so successfully that cold dark matter is determined to outmass baryonic matter by a factor of roughly five [1]. The theoretical origin of this specific ratio, however, $\Omega_c/\Omega_b \approx 5.375$, remains one of the most striking open problems in fundamental physics. In the standard paradigm both abundances depend on decoupled high-energy initial conditions, freeze-out mechanisms and independent free parameters (WIMP cross-sections, baryogenesis asymmetries, etc.) that must be tuned a posteriori to match observations.

The Principle of Universal Integrity (PIU) postulates that the universe is governed by a continuous hyperelastic substrate (the Pleno) structured as a bipartite diamond lattice with coordination number $Z = 4$ at the Planck scale [12]. Within PIU, the fundamental matter modes are not independent entities but distinct classes of topological and non-topological defects of a single continuous medium:

- the *baryonic mode* M, carrying a conserved topological charge ($B = 1$);
- the *dark-matter mode* MO, a non-topological scalar condensate.

The identification of M with the fermionic baryonic sector and of MO with the scalar dark-matter sector is itself a derivational closure of the PIU corpus (closure C-PP.3a, V31.9), supported by the Finkelstein–Rubinstein theorem on the spin-statistics of topological solitons [9, 10, 11]. Consequently, the cosmological ratio of these two modes is not a historical accident of the Big Bang but a rigid geometric consequence of the substrate's discrete structure.

In this Letter we prove the *Topological Asymmetry by Lattice Frustration Theorem* (Theorem T5 of the PIU corpus, V31.9), deriving the abundance ratio $\Omega_{\rm MO}/\Omega_M = \sqrt{3}\,\pi$ strictly from first principles without a single free parameter, and we compare the prediction with the Planck 2018 cosmic microwave background determination.

2. Defect Geometry and Peierls–Nabarro Pinning

In a saturated, post-cascade continuous substrate mapped onto a discrete diamond lattice with coordination number $Z = 4$, macroscopic localized defects interact with the underlying discrete geometry through Peierls–Nabarro (PN) stability barriers [2, 3]. The PN mechanism dictates how an extended defect minimizes its energy configuration with respect to the lattice periodicity.

The Pleno admits two distinct classes of macroscopic localized states, distinguished by their topological charge:

Baryonic mode M. Topologically protected defects carrying baryon number $B = 1$. The conserved topological charge forces the core of the defect to lock into the specific symmetries of the local lattice structure, in order to minimize the PN barrier. In a $Z = 4$ diamond lattice, the local Wigner–Seitz (Voronoi) cell is a regular tetrahedron. Therefore baryonic defects are strictly pinned to a tetrahedral boundary condition.

Dark-matter mode MO. Non-topological defects (scalar Q-balls in the sense of Coleman [5]). Lacking a conserved topological charge to enforce lattice alignment, these scalar condensates bypass the directional PN barriers and expand isotropically to minimize surface tension, adopting a purely spherical geometry.

The energy-minimizing geometric forms for the two sectors are therefore the sphere (MO) and the regular tetrahedron inscribed in a sphere of radius R (baryonic M). Their respective surface areas are

$$A_S = 4\pi R^2 \quad \text{(sphere, MO)}, \tag{1}$$

$$A_T = \tfrac{8\sqrt{3}}{3} R^2 \quad \text{(tetrahedron inscribed in sphere, } M\text{)}. \tag{2}$$

3. Thermodynamic Weights from Surface Modes

To compute the cosmological abundance ratio we must evaluate the statistical thermodynamic weight $\mathcal{Z}_i$ of each defect class. For extended saturated defects in a continuous substrate, the internal volume modes are locked (saturated) and the relevant low-energy collective excitations are strictly surface vibrational modes (Category C dynamics in the Manton–Sutcliffe classification of topological-defect dynamics [4]).

Following the theory of crystal surface modes of Maradudin, Wallis, and Mills [6, 7], the two-dimensional density of states $g^{2D}(\omega)$ of acoustic surface modes on a defect boundary of area A is given by

$$g^{2D}(\omega) = \frac{A}{2\pi\, s^2}\,\omega, \tag{3}$$

where s is the transverse sound velocity of the substrate. The total thermodynamic weight $\mathcal{Z}_i$ (and consequently the asymptotic mass density Ω_i) of a defect class in the thermalized post-cascade regime is proportional to the integral of this density of states up to the Debye frequency ω_D:

$$\Omega_i \propto \mathcal{Z}_i = \int_0^{\omega_D} g^{2D}(\omega)\, f(\omega, T)\, d\omega = A_i \cdot \phi(T, s, \omega_D). \tag{4}$$

Since both defect classes reside in the same background substrate (the Pleno) and the thermodynamic factor $\phi(T, s, \omega_D)$ depends only on the bulk substrate parameters (and not on the defect class), ϕ is strictly universal and cancels exactly in any relative quotient. The abundance ratio is therefore dictated entirely by the ratio of boundary areas:

$$\frac{\Omega^{\text{geom}}_{\text{MO}}}{\Omega^{\text{geom}}_{M}} = \frac{A_S}{A_T} = \frac{4\pi R^2}{(8\sqrt{3}/3)R^2} = \frac{\sqrt{3}\,\pi}{2}. \tag{5}$$

4. Spinorial Penalty via Haar Volumes

The purely geometric area ratio evaluates to $\sqrt{3}\,\pi/2$. The baryonic and dark-matter modes, however, reside in distinct representations of the rotational group manifold, and this distinction introduces a fundamental phase-space statistical penalty.

By the closure C-PP.3a of the PIU corpus [12], supported by the Finkelstein–Rubinstein theorem on the spin-statistics of topological solitons [9, 10, 11], the topologically protected baryonic defects of M realize half-integer-spin fermionic statistics with internal degrees of freedom mapping to the covering group $SU(2)$. The non-topological scalar Q-balls of MO, lacking such a topological charge, realize spin-0 bosonic statistics with internal degrees of freedom mapping directly to $SO(3)$.

When the macroscopic phase space accessible to each defect class is integrated over its rotation-group manifold, the Peter–Weyl theorem [8] dictates that the total accessible Haar measure of a compact Lie group fixes its statistical weight. The covering map $SU(2) \to SO(3)$ is the canonical 2-to-1 homomorphism, implying the manifold relation $SO(3) \cong SU(2)/\mathbb{Z}_2$ and the exact ratio

$$\frac{\text{Vol}(SO(3))}{\text{Vol}(SU(2))} = \frac{\pi^2}{2\pi^2} = \frac{1}{2}. \tag{6}$$

Consequently, when the macroscopic scalar geometry (the tetrahedron) is populated by fermionic degrees of freedom in projection onto observable $SO(3)$-valued macroscopic quantities, the baryonic sector incurs a strict factor 1/2 phase-space penalty relative to the scalar dark-matter sector.

5. The Cosmological Ratio

Combining the thermodynamic surface-mode weighting (Eq. (4)) with the topological phase-space penalty (Eq. (6)), the cosmological density ratio between the dark-matter mode MO and the baryonic mode M of the Pleno is

$$\frac{\Omega_{\text{MO}}}{\Omega_M} = \frac{A_S}{A_T} \times \left[\frac{\text{Vol}(SO(3))}{\text{Vol}(SU(2))}\right]^{-1}. \tag{7}$$

Substituting the exact geometric values $A_S/A_T = \sqrt{3}\,\pi/2$ and $\text{Vol}(SO(3))/\text{Vol}(SU(2)) = 1/2$:

$$\boxed{\frac{\Omega_{\text{MO}}}{\Omega_M} = \sqrt{3}\,\pi \approx 5.441398.} \tag{8}$$

This constitutes the formal closure of Theorem T5 of the PIU corpus [12] (V31.9, Block E), modulo three minor structural sub-caveats (dominance of surface modes in the saturated regime, exact universality of the thermodynamic factor ϕ, transient behaviour at finite cosmic time), each of low severity and not affecting the derivational backbone.

6. Observational Agreement

The Planck 2018 cosmic microwave background anisotropy measurements provide the most precise empirical constraints on the cosmological energy budget. The reported physical density parameters are [1]

$$\Omega_c h^2 = 0.1200 \pm 0.0012, \qquad \Omega_b h^2 = 0.02237 \pm 0.00015,$$

yielding the derived ratio

$$\left(\frac{\Omega_c}{\Omega_b}\right)_{\rm Pl} = 5.375 \pm 0.077. \tag{9}$$

Under the identification of the PIU modes MO and M with the observational components Ω_c (cold dark matter) and Ω_b (baryons), respectively—a closure derived in the PIU corpus via Peierls–Nabarro pinning and Finkelstein–Rubinstein spin-statistics—the theoretical prediction $\sqrt{3}\,\pi \approx 5.4414$ falls within 1.23% of the central observational value, corresponding to a statistical distance of

$$z = \frac{5.375 - \sqrt{3}\,\pi}{0.077} = -0.86\,\sigma.$$

A complete Bayesian comparison of this structural prediction (together with the analogous PIU prediction $\Omega_{\rm EO} \equiv f_c \approx 0.6869$ for the dark-energy fraction) against the null hypothesis of accidental numerological coincidence is carried out in [12] (Block G, V31.10), yielding a cosmological Bayes factor $K_c \approx 6.71 \times 10^5$ ($\log_{10} K_c = 5.83$), placing the evidence in Jeffreys' *decisive* category.

7. Conclusion

The longstanding "cosmic coincidence" between dark and baryonic matter densities is resolved within the PIU framework as a topological necessity rather than a numerical accident. By combining Peierls–Nabarro lattice pinning, Maradudin's surface-mode thermodynamics, and the exact Peter–Weyl ratio of Haar volumes—with the identification of the baryonic mode M with fermionic statistics derived via Finkelstein–Rubinstein—the PIU framework predicts the cosmological dark-to-baryonic abundance ratio with zero free parameters and zero observational input, achieving sub-σ agreement with Planck 2018. The companion paper [13] establishes the axiomatic foundations and derives the fundamental constants G and $\hbar$ from the same structural framework, with combined error below 0.1% and again no adjustable parameters.

References

[1] N. Aghanim *et al.* (Planck Collaboration), "Planck 2018 results. VI. Cosmological parameters," Astron. Astrophys. **641**, A6 (2020).

[2] R. E. Peierls, "The size of a dislocation," Proc. Phys. Soc. **52**, 34 (1940).

[3] F. R. N. Nabarro, "Dislocations in a simple cubic lattice," Proc. Phys. Soc. **59**, 256 (1947).

[4] N. Manton and P. Sutcliffe, *Topological Solitons*, Cambridge University Press (2004).

[5] S. Coleman, "Q-balls," Nucl. Phys. B **262**, 263 (1985).

[6] A. A. Maradudin, E. W. Montroll, G. H. Weiss, and I. P. Ipatova, *Theory of Lattice Dynamics in the Harmonic Approximation*, 2nd ed., Academic Press (1971); see also A. A. Maradudin, R. F. Wallis, and D. L. Mills, *Surface Phonons and Polaritons*, Phys. Rev. B **4**, 962 (1971).

[7] M. G. Cottam and D. R. Tilley, *Introduction to Surface and Superlattice Excitations*, Cambridge University Press (1989).

[8] F. Peter and H. Weyl, "Die Vollständigkeit der primitiven Darstellungen einer geschlossenen kontinuierlichen Gruppe," Math. Ann. **97**, 737 (1927).

[9] D. Finkelstein and J. Rubinstein, "Connection between spin, statistics, and kinks," J. Math. Phys. **9**, 1762 (1968).

[10] D. Giulini, "On the configuration-space topology in general relativity," Helv. Phys. Acta **68**, 86 (1993).

[11] E. Witten, "Current algebra, baryons, and quark confinement," Nucl. Phys. B **223**, 433 (1983).

[12] M. A. Celedón Mejía, "PIU V31.10—Resumen Técnico Extendido y Autocontenido," `piuniversal.com` (2026).

[13] M. A. Celedón Mejía, "PIU: A Universal Physical Substrate Proposal—Axiomatic Foundations and Derivation of G and $\hbar$," Companion Paper, `piuniversal.com` (2026).