Quantum Mechanics and General Relativity as Effective Limits of a Single Scalar Field

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Abstract

We demonstrate that the Einstein field equations of General Relativity (GR) and the Schrödinger equation of Quantum Mechanics (QM) emerge as two effective descriptions of a single underlying complex scalar field Ψ (the *Pleno*, the continuous hyperelastic scalar substrate postulated by PIU), operating under complementary limits. Instead of quantising the metric or geometrising the quantum state, we show that both are macroscopic derivative phenomena. GR emerges in the coherent infrared limit via Sakharov's induced-gravity mechanism, where the elastic properties of the substrate yield the Einstein–Hilbert action and an emergent effective metric. QM arises in the non-relativistic local hydrodynamic limit via the Madelung–Bohm correspondence, where the phase gradient dictates momentum and the amplitude curvature yields the Bohmian quantum potential. We are careful throughout to distinguish what is genuinely *derived* from F1 (an algebraic identity between the F1 kinetic coefficient and the Einstein–Hilbert coefficient, and an exact mathematical equivalence between the Madelung–Bohm equations and Schrödinger) from what is an *ontological identification* (the Pleno as the substrate carrying both descriptions). Consequently, the spacetime metric and the quantum wavefunction are not distinct physical entities, but complementary effective modalities of the same continuous substrate.

1. Introduction

The most profound foundational tension in contemporary physics is the conceptual and mathematical incommensurability between General Relativity (GR) and Quantum Mechanics (QM). Traditional approaches to quantum gravity attempt either to quantise the gravitational field (Loop Quantum Gravity, String Theory) or to formulate quantum mechanics on a curved background. Both approaches implicitly assume that the metric $g_{\mu\nu}$ and the quantum state $|\psi\rangle$ belong to fundamentally distinct ontological categories.

The *Principle of Universal Integrity* (PIU) proposes a radical ontological simplification: neither the metric nor the wavefunction is fundamental. Instead, a single, continuous, hyperelastic physical substrate—the *Pleno*—is postulated and described by a complex scalar field $\Psi(x^\mu)$. In this paper we demonstrate, with explicit epistemic labelling, the two analytical pathways by which GR and QM emerge as effective macroscopic limits of the same fundamental Lagrangian, $\mathcal{L}_{F1}$, anchoring every step against the PIU technical corpus V31.10 [6].

A central commitment of PIU is the *Ontological Protocol* [6]: before any equation is written, three questions must be answered. Is it derivable from the five axioms? Is the Pleno the agent, or is something external acting as mechanism? Does standard physics appear as an *effective limit* (correct) or as *origin* (incorrect)? The derivations below pass this protocol: the Einstein–Hilbert action and the Schrödinger equation appear strictly as effective descriptions of the underlying scalar dynamics.

2. The fundamental field and Lagrangian

The Pleno is a complex scalar field $\Psi = S\, e^{i\theta}$ characterised by three independent Planckian scales: density ρ_P, length ℓ_P, and characteristic wave velocity c. The canonical, Lorentz-invariant Lagrangian density $\mathcal{L}_{F1}$ is

$$\mathcal{L}_{F1} = \tfrac{1}{2}\,\rho_P c^2 \ell_P^2\,(\partial_\mu\Psi)^*(\partial^\mu\Psi) - V(|\Psi|) - \frac{\rho_P c^2 \ell_P^4}{40}\,|\Box\Psi|^2, \quad (1)$$

where $V(|\Psi|)$ contains the topological confinement potential V_{topo} and the Klauder affine quantum wall V_q guaranteeing a stable minimum structural density $S_{\min}$ [6].

Structural identity of the kinetic coefficient. A key algebraic identity follows from the F3 derivation in the corpus:

$$\tfrac{1}{2}\,\rho_P c^2 \ell_P^2 \;\equiv\; \frac{c^4}{2G}, \quad (2)$$

exhibiting that the F1 kinetic coefficient *is* the Einstein–Hilbert coefficient. This is not an adjustment: it is the consequence of $G = c^2/(\rho_P \ell_P^2)$, which the corpus V31.10 derives at status `[derived]` with error below 0.1% [6]. Likewise, $\hbar \equiv \rho_P c\,\ell_P^4$ is a derived identity (F4), and the UV coefficient satisfies $\gamma_\kappa = \hbar c/40$ [6], manifesting that the higher-derivative term is genuinely quantum in nature, not a classical regulator.

Epistemic status: Eq. (1) is the canonical `[derived]` form of F1 in V31.10; Eq. (2) is a `[derived]` identity (theorem $\gamma_G = 1$, V30.7 / Appendix CC of the corpus).

3. Quantum Mechanics as hydrodynamic emergence

To recover quantum mechanics, we analyse the local, non-relativistic limit of the Pleno dynamics through the Madelung decomposition [1, 2]. Writing $\Psi = S\, e^{i\theta}$ with S, θ real and expanding the kinetic term gives

$$(\partial_\mu\Psi)^*(\partial^\mu\Psi) = (\partial_\mu S)^2 + S^2(\partial_\mu\theta)^2. \quad (3)$$

In the hydrodynamic reading, $\rho \equiv S^2$ is the structural fluid density and $\partial_\mu\theta$ the velocity-potential gradient. Under the *Madelung–Bohm correspondence*, the canonical momentum is identified as $p_\mu = \hbar\,\partial_\mu\theta$, with the native quantum of action being precisely $\hbar \equiv \rho_P c\,\ell_P^4$.

Effective mass. The inertial parameter m entering the velocity field $\mathbf{v} = (\hbar/m)\nabla\theta$ is *not* a primitive of F1; it is the effective inertial mass of the localised perturbation, set by the curvature of the effective potential at the structural minimum, $m_{\text{eff}} = \rho_P \ell_P^4\, V''(S_{\min})/c^2$ [6]. Particles are localised topological perturbations $\delta S\, e^{i\theta}$ upon the background $S_{\min}$; their masses are properties of the substrate's response, not external inputs.

Continuity and quantum Euler equations. Applying the Euler–Lagrange equations of $\mathcal{L}_{F1}$ to S and θ separately in the non-relativistic limit yields the coupled system

$$\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = 0, \quad (4)$$

$$\partial_t\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{m_{\text{eff}}}\nabla(V_{\text{ext}} + Q)\,, \quad (5)$$

with $\mathbf{v} = (\hbar/m_{\text{eff}})\nabla\theta$. These are exactly the continuity equation and the quantum Euler equation of the Bohmian formulation. The emergent quantum potential is

$$Q = -\frac{\hbar^2}{2m_{\text{eff}}}\frac{\nabla^2 S}{S}, \quad (6)$$

intrinsically determined by the amplitude curvature.

From hydrodynamics to Schrödinger. The pair (4)–(5) is mathematically equivalent to the Schrödinger equation for Ψ in the same non-relativistic regime [1, 2]. Importantly:

Epistemic status: The Schrödinger equation is *not* "derived from more fundamental principles" inside PIU. It is recovered as the *exact mathematical restatement* of the macroscopic hydrodynamics of Ψ under the Madelung–Bohm correspondence, plus the ontological identification of Ψ as the state of the Pleno (Axiom A1). This is the explicit status declared in V31.10 §4.bis [6].

The "wavefunction" is therefore not an element of an abstract configuration space but the direct field state $\Psi(x,t)$ of the Pleno; observed quantum correlations are hydrodynamic consequences of incompressible phase linkages in the substrate (Bell/EPR analysis in V31.10 §15.quinquies is at status `[strong logical deduction]` pending fully closed formalisation).

4. General Relativity as elastic emergence

The macroscopic geometry of spacetime emerges from the coherent infrared limit of the same field Ψ. The effective metric is not a primitive but a function of the Pleno's perturbations:

$$g_{\mu\nu}^{\text{eff}} = \eta_{\mu\nu} + h_{\mu\nu}(\Psi). \tag{7}$$

To recover the dynamics of $g_{\mu\nu}^{\text{eff}}$, we use Sakharov's induced-gravity mechanism [3, 4]. Integrating out the high-energy modes of Ψ in the presence of a background deformation produces the generic curvature expansion

$$\Gamma_{\text{ind}}[g] = \int d^4x\,\sqrt{-g}\,\Big[\Lambda_{\text{ind}} - \frac{c^4}{16\pi G_{\text{ind}}}\,R + \mathcal{O}(R^2)\Big]. \tag{8}$$

Because the kinetic coefficient of $\mathcal{L}_{F1}$ is exactly $c^4/(2G)$ by Eq. (2), the induced Newton constant G_{ind} coincides with the macroscopic Newton constant *without* any ad-hoc renormalisation. This is the content of the $\gamma_G = 1$ theorem (V30.7, Appendix CC of [6]): an internal corollary of F1, not an external assumption.

Variation of the induced action with respect to $g^{\mu\nu}$ recovers the Einstein equations

$$R_{\mu\nu} - \tfrac{1}{2}R\,g_{\mu\nu} + \Lambda\,g_{\mu\nu} = \frac{8\pi G}{c^4}\,T_{\mu\nu}^{(\text{Pleno})}, \tag{9}$$

where $T_{\mu\nu}^{(\text{Pleno})}$ is the energy–momentum tensor built entirely from configurations of Ψ: localised topological defects (baryons), extended non-topological scalar profiles (dark matter, MO), and the background tension of $S_{\min}$ (dark energy, EO).

Dark energy under the EO-Curtain reading (V31.10). The V31.10 reformulation [6] sharpens the ontology of the Λ term: dark energy is the *permanent structural state* of the Pleno-ground, not a dynamical component that can be diluted or attracted by gravity. Cosmological "expansion" is the dispersion of solitonic perturbations against this invariant background. The Einstein equation (9) remains formally unchanged; what changes is the physical interpretation of the Λ contribution, which is now read as a structural property of $S_{\min}$ rather than a separately dynamical fluid. The quantitative prediction $\Omega_{\text{EO}} = f_c \approx 0.6869$ is preserved [5].

Epistemic status: Eq. (9) is `[derived]` from F1 + Sakharov induction + the algebraic identity (2), with $\gamma_G = 1$ a theorem of the corpus, not a fit [6].

5. Ontological synthesis

The derivational pathways above force a deliberate conceptual reorganisation.

1. **The nature of the wavefunction.** $\psi(x,t)$ is not a probability amplitude residing in an abstract configuration space; it is the direct physical state $\Psi(x,t)$ of a continuous fluid-like substrate. Quantum correlations (Bell/EPR) are hydrodynamic consequences of the underlying incompressible phase linkages at the topological level. We emphasise that V31.10 [6] carries the formal Bell derivation at `[strong logical deduction]` status, with full closure pending.

2. **The nature of the metric.** $g_{\mu\nu}(x,t)$ is not a primitive geometrical entity that "curves"; it is a macroscopic measure of the elastic strain and density variations within the Pleno. Gravity is an induced interaction, not a fundamental force.

3. **The false dichotomy.** The incompatibility between GR and QM arises from attempting to quantise an emergent elastic property (the metric) or geometrise a hydrodynamic state. They are distinct mathematical limits—one describing local circulation and density (QM), the other describing macroscopic strain and coarse-grained energy transport (GR)—of the same complex scalar field Ψ.

6. Epistemic ledger

Following the PIU validation protocol [6], each principal result is labelled below according to its corpus-canonical status.

Result	PIU corpus status
$\frac{1}{2}\rho_P c^2 \ell_P^2 \equiv c^4/(2G)$ (2)	`[derived]` (F3, $\gamma_G = 1$ theorem)
$\hbar \equiv \rho_P c \ell_P^4$, $\gamma_\kappa = \hbar c/40$	`[derived]` (F4, Nelson–IS bridge)
Madelung–Bohm $\Leftrightarrow$ Schrödinger	`[derived]` as equivalence; identification of Ψ with the Pleno is via Axiom A1
Sakharov induction of Einstein equations	`[derived]` (V31.10 §4.4.bis)
m_{eff} from $V''(S_{\min})$	`[derived]` (V31.10 §4.bis.bis)
Bell / EPR as hydrodynamic incompressibility	`[strong logical deduction]` (V31.10 §15.quinquies)

7. Conclusion

By postulating a single, hyperelastic complex scalar field parameterised by three independent Planckian scales, we have shown that both the Schrödinger equation and the Einstein field equations arise as complementary effective limits of the same dynamics. The Madelung–Bohm hydrodynamic limit produces the quantum of action $\hbar$ and the Bohmian potential; Sakharov's elastic-fluctuation induction produces the gravitational constant G and the Einstein–Hilbert action. Unification, in this picture, does not require higher-dimensional geometry: it requires recognising a common material substrate. The status declarations above keep the claim epistemically transparent—what is derived is derived, and what remains an ontological identification is named as such.

References

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