$\sqrt{3}\,\pi$ in the dark sector: PIU-Ψ, a $w{=}{-1}$ background with no freedom, and the DESI DR2 dark-energy signal

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Abstract

DESI has reported up to 3–4 σ in favour of dynamical dark energy over a cosmological constant; an independent empirical analysis shows that this signal is the tension in the matter density Ω_m between probes, not a dynamics of the background. This work places that diagnosis inside PIU-Ψ, a theory that treats the vacuum as an elastic medium —the Pleno— of which ΛCDM, gravity and the constants of nature emerge as effective limits, and adds a prediction the empirical analysis does not contain. The dynamics of the medium fixes an *exact* equation of state $w = -1$ with no freedom to depart from it: not a fitted value, but the $T_{\mu\nu} \propto g_{\mu\nu}$ symmetry of the medium's floor, which kinetic positivity shields against the phantom region. Confronted with the thirteen DESI DR2 BAO points, that background fits the raw data with $\chi^2/\nu = 0.96$, the preference for $w \neq -1$ is $1.67\,\sigma$, and information criteria localize the signal in the anchoring of Ω_m. The same floor $S_{\min}$ that fixes $w = -1$ fixes, with no parameters, the ratio of physical densities $\omega_{\rm dm}/\omega_b = \sqrt{3}\,\pi = 5.44$, at $0.86\,\sigma$ from Planck; it is derived here in full from the substrate geometry —a sphere/tetrahedron area ratio with a Peter–Weyl factor, on a lattice whose coordination follows from the isotropy and isostaticity of the medium. The signature $dw_0/d\Omega_m \approx 1.2$ ($R^2 = 0.994$) gives the test: measuring Ω_m without the template must return $w_0 = -1$.

Keywords: dark energy, DESI, BAO, ΛCDM, elastic medium, Ω_m tension, dark sector, physical densities, diamond lattice

1 Introduction

In March 2025 the DESI collaboration presented, with its second data release (DR2) of baryon acoustic oscillations, evidence that dark energy might not be a cosmological constant [1]. Fitting the Chevallier–Polarski–Linder (CPL) parametrization —$w(a) = w_0 + w_a(1 - a)$— to the combination of BAO with the cosmic microwave background (CMB) and type Ia supernovae, DESI infers a preference for $w_0 > -1$ and $w_a < 0$ reaching, depending on the supernova compilation used, between 2.5 and $3.9\,\sigma$ over ΛCDM [1, 2]. The claim reopened the question of whether the dark sector has dynamics of its own.

A recent empirical diagnosis answers that it does not. Ó Colgáin and Sheikh-Jabbari [3], reconstructing the ΛCDM parameter Ω_m from the w_0w_aCDM cosmologies that the DESI collaboration prefers, identify the dynamical signal as the tension in the matter density Ω_m between probes —a rising Ω_m trend at high redshift and a sharp deviation at low redshift, the latter driven by supernovae discrepant with the DESI full-shape analysis in overlapping redshift ranges—, not a dynamics of the background. Their diagnosis is atheoretical: it is obtained by parameter recon-

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struction over the data, with no hypothesis about the nature of the background, and concludes that dynamical dark-energy claims are premature and point to systematics or a break of ΛCDM.

This work places that diagnosis inside a substrate theory and adds a prediction it does not contain. The theory is PIU-Ψ, which describes the vacuum as a real elastic medium —the Pleno— of which gravity, quantum mechanics and the cosmology of ΛCDM emerge as effective limits [4, 5]. Its background dynamics fixes an *exact* equation of state $w = -1$, and does so structurally, prior to the data: it follows from the Lagrangian of the medium, it is not fitted to DESI (§2).

The status of that $w = -1$ must be stated precisely, because the weight of the argument depends on it. That exact $w = -1$ is a prediction with content is shown by the very conflict that motivates this work: cosmology currently holds a several-σ dispute over whether dark energy has $w = -1$ or $w \gtrsim -1$. That conflict would not exist if $w = -1$ were a forced consequence of every reasonable theory —there would be nothing to measure. The space of dark-energy theories the community takes seriously does not land generically on exact $w = -1$: quintessence gives dynamical $w > -1$, phantom models $w < -1$, k-essence almost any trajectory, interacting dark energy a varying $w(z)$. What is generic is the *bound* $w \geq -1$ of a canonical scalar, not the *realized value* exact $w = -1$, which is a special and falsifiable point within that space.

What PIU-Ψ claims about that point is stronger than sharing it with ΛCDM. PIU-Ψ *does not have the freedom* to produce anything else: its $\delta = 0$ is structural, not a parameter that could have been tuned to -0.9 had the data asked for it. A quintessence model with a free potential can accommodate the DESI signal if it were real; PIU-Ψ cannot, and that rigidity is what makes $w = -1$ a prediction at risk. Two features sharpen it: the sign of the apparent deviation is derived —the signal can only push toward $w > -1$, never toward the phantom region $w < -1$, by kinetic positivity—, and the same floor $S_{\min}$ that fixes $w = -1$ also fixes, with no parameters, the proportion of the dark sector (§8), a link a quintessence model does not have. The convergence between the empirical diagnosis and this derived background is real, and its value lies in the *methodological independence* of the two routes —one is data analysis, the other deduction from first principles— reaching the same special point of the theory space.

The result proper to PIU-Ψ, the one the empirical analysis does not contain, lives in the layer the Ω_m tension does not touch. The tension inhabits a *derived* parameter of the ΛCDM fit; the layer of near-model-independent quantities —the physical densities read from the acoustic peak heights— does not show it. On that robust layer PIU-Ψ predicts, with no free parameter, that the ratio of dark to baryonic matter is $\omega_{\rm dm}/\omega_b = \sqrt{3}\,\pi$, at $0.86\,\sigma$ from the Planck value, an observable that ΛCDM describes with two independently fitted parameters. That number is derived in full in this work (§8), without deferring its core to the larger programme: from the substrate geometry —a ratio of areas between the sphere and the tetrahedron of the lattice, with a group factor— on a lattice whose tetrahedral coordination is derived from the isotropy and isostaticity of the medium. The value of that coincidence lies not in the $0.86\,\sigma$, which alone would be one more agreement: it lies in that a number from pure geometry, obtained without looking at the data, lands there against a robust target the rival theory does not predict.

The exposition first documents the convergence on the background —the background prediction (§2), the raw fit (§4), the Bayesian analysis (§5) and the model comparison that localizes the signal in the anchoring of Ω_m (§6)— and situates which part of the diagnosis is empirical and external (§7). It then develops the proper prediction and its complete derivation (§8), the falsifiable signature that separates the artefact from a real dynamics (§9), the reading of the whole as convergence (§10) and the conditional status of the result (§11). A verification script that recomputes every quantity —including the geometric chain up to $\sqrt{3}\,\pi$— accompanies the work as supplementary material.

2 The background prediction of PIU-Ψ

In PIU-Ψ the late cosmological sector is a phase condensate of the medium on the spectral floor $S_{\rm min}$. The modulus of the field $\Psi = S\,e^{i\theta}$ rests on that floor with its value frozen —the rigidity scale of the medium is the Planck scale, so modulus excursions are suppressed by an ultra-Planckian factor, exponentially small at cosmological energies— and the phase has no mean velocity, $\dot\theta_0 = 0$. The energy–momentum tensor of a complex scalar field in that state is

$$T_{\mu\nu} = \rho_P\,\ell_P^2\left[\partial_\mu S\,\partial_\nu S + S^2\,\partial_\mu\theta\,\partial_\nu\theta\right] - g_{\mu\nu}\,\mathcal{L}, \tag{1}$$

with $\mathcal{L}$ the Lagrangian density of the medium. With the modulus frozen ($\partial_\mu S \to 0$) and the phase static ($\partial_\mu\theta \to 0$), the kinetic terms vanish and one is left with $T_{\mu\nu} = -g_{\mu\nu}\,\mathcal{L} = -g_{\mu\nu}\,V(S_{\rm min})$, that is, $T_{\mu\nu} \propto g_{\mu\nu}$. A tensor proportional to the metric has, by definition, pressure $p = -\rho$: the equation of state is *exactly* $w = -1$. It is not a fitted value nor the limit of a slow-roll trajectory, but the symmetry of the ground state: dark energy here is the density of the medium's own floor, with $w = -1$ as the tensorial signature of that state. This identification is [DERIVED] from the Lagrangian; the form of V and the stability of the floor are developed in the corpus [4].

This $w = -1$ shares the bound $w \geq -1$ with any canonical scalar sector, but what distinguishes it is not the bound but that the background cannot depart from it: the distinction lies in the absence of freedom, in the sign of the deviation, and in the link to the dark sector. The density and pressure of the background are read from the tensor,

$$\rho = \tfrac{1}{2}\rho_P\ell_P^2\dot S^2 + V_{\rm eff}, \qquad p = \tfrac{1}{2}\rho_P\ell_P^2\dot S^2 - V_{\rm eff}, \tag{2}$$

so that $w + 1 = 2\delta/(\delta+1)$ with $\delta \equiv (\text{kinetic})/(\text{potential}) \geq 0$. On the floor ($\delta = 0$, $\dot S = 0$) one recovers exactly $w = -1$; any excitation ($\delta > 0$) gives $w > -1$. By the positivity of the canonical kinetic term, every residue pushes the inferred w above -1 —toward quintessence— and never below, into the phantom region $w < -1$: a condensate gaining physical density with time would have $w + 1 = -\frac{1}{3}\,d\ln\rho/dN < 0$, forbidden, and a rolling phase mode ($\dot\theta_0 \neq 0$) would dilute as dust, incompatible with a $w = -1$ component. The sign of the artefact is thus derived, and coincides with the direction DESI reports [DERIVED]. What separates PIU-Ψ from a quintessence model is not the bound, but that the same floor $S_{\rm min}$ that fixes $w = -1$ also fixes the dark-matter ratio $\sqrt{3}\,\pi$ and the dark-energy fraction f_c (§8): a quintessence model does not tie its equation of state to the proportion of the dark sector, because its potential is free; PIU-Ψ does, because the three quantities come from the same potential on the same lattice.

The observational consequence is precise. The normalized background expansion history,

$$E(z) \equiv \frac{H(z)}{H_0} = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}, \tag{3}$$

coincides with that of flat ΛCDM to linear order, with the same source $\rho_{m,0}(1+z)^3 + \rho_{\rm DE}$ and a single shape freedom, the matter density Ω_m, tied to the cosmological initial condition of the field [5]. The correction to this form appears only in the nonlinear regime of the potential walls, not yet accessible in BAO, and constitutes a declared open thread [OPEN THREAD: NONLINEAR CORRECTION TO $H(z)$]. To linear order the PIU-Ψ background is indistinguishable from ΛCDM with $w = -1$: there is no free w_0 or w_a, but fixed $w = -1$. If that background is correct, fitting a two-parameter template (w_0, w_a) to data generated by it cannot, without additional information, return anything other than $(-1, 0)$; any apparent deviation must come from the inference procedure. That is the hypothesis the rest of the work examines.

3 Data and method

We use the thirteen public DESI DR2 BAO measurements [1]: the ratios D_M/r_d and D_H/r_d for six anisotropic tracers in the range $0.51 \leq z \leq 2.33$, and D_V/r_d for the bright galaxy tracer at $z = 0.295$, with r_d the sound horizon at the drag epoch. For the anisotropic tracers the reported correlation $\rho(D_M, D_H)$ is incorporated, building the corresponding 2×2 covariance block. The statistic is

$$\chi^2 = \sum_i \mathbf{r}_i^\top \mathsf{C}_i^{-1} \mathbf{r}_i, \qquad \mathbf{r}_i = \mathbf{d}_i - \mathbf{m}_i(\boldsymbol{\theta}). \tag{4}$$

Distances are computed from $E(z)$: $D_H/r_d = [c/(H_0 r_d)]/E(z)$, $D_M/r_d = [c/(H_0 r_d)] \int_0^z dz'/E(z')$ and $D_V/r_d = [z\,(D_M/r_d)^2 (D_H/r_d)]^{1/3}$. Each fit has two classes of parameter: the *shape* parameters of $E(z)$ —Ω_m for $w = -1$; (Ω_m, w_0) for wCDM; (Ω_m, w_0, w_a) for CPL— and a global scale factor $N \equiv c/(H_0 r_d)$, dimensionless, common to all models, treated as a nuisance parameter. Marginalizing N leaves the fit on the shape of $E(z)$ and not on H_0 or r_d separately, which makes the test independent of the sound-horizon calibration. Thus the $w = -1$ model fits two parameters (Ω_m and N) against thirteen measurements, with eleven degrees of freedom. Table 1 summarizes the runs.

Table 1: Runs of the analysis: data, shape parameters (besides the scale factor N, common), and external anchoring. The three anchor values of Ω_m are public: 0.315 ± 0.007 from the CMB (Planck), 0.296 ± 0.007 from the DESI full-shape analysis (CMB-independent), and 0.295 ± 0.015 from BAO geometry.

Run	Shape parameters	Ω_m anchor
Raw fit (§4)	$w{=}-1$; wCDM; CPL	none
MCMC (§5)	CPL (Ω_m, w_0, w_a)	none / 0.315 ± 0.007
Models (§6)	$w{=}-1$; wCDM; CPL	none; 0.315; 0.296
Signature (§9)	CPL, Ω_m fixed	sweep 0.297–0.321

For CPL the dark-energy density evolves as $\rho_{\rm DE}(z)/\rho_{\rm DE,0} = (1+z)^{3(1+w_0+w_a)} \exp[-3w_a z/(1+z)]$, inserted into $E(z)$. The case $w = -1$ corresponds to $(w_0, w_a) = (-1, 0)$. The external Ω_m anchor, when used, enters as an additional Gaussian term in (4); Section 9 and the appendix discuss its form and its more realistic version in terms of the physical density $\omega_m = \Omega_m h^2$ and the acoustic scale.

4 The $w = -1$ background against the raw data

The PIU-Ψ background, equation (3) with fixed $w = -1$ and Ω_m as the only shape freedom, fits the thirteen DESI DR2 measurements with $\Omega_m = 0.297$ and $\chi^2 = 10.53$ for eleven degrees of freedom, that is $\chi^2/\nu = 0.96$ (Fig. 1). The $w = -1$ background describes the raw BAO data without tension.

The three-shape-parameter CPL template improves the fit to $\chi^2 = 5.81$, with $(w_0, w_a) = (-0.18, -2.70)$. The improvement is $\Delta\chi^2 = 4.72$ at the cost of two additional parameters. Under Wilks' theorem, $\Delta\chi^2 = 4.72$ with two degrees of freedom corresponds to $p = 0.095$, a significance of $1.67\,\sigma$: below the $2\,\sigma$ threshold ($\Delta\chi^2 > 6.18$) and far from the $3\,\sigma$ one ($\Delta\chi^2 > 11.83$). The raw BAO data do not require $w \neq -1$. The result matches that of the collaboration itself for BAO alone, $w = -0.99^{+0.15}_{-0.13}$ in the constant-w model [1]. The best CPL fit also falls in a physically strange region —$w_0 = -0.18$ describes a dark energy that today behaves almost like matter—, a sign that the minimum reflects not a physical preference but the slack of the template on data that do not constrain it.

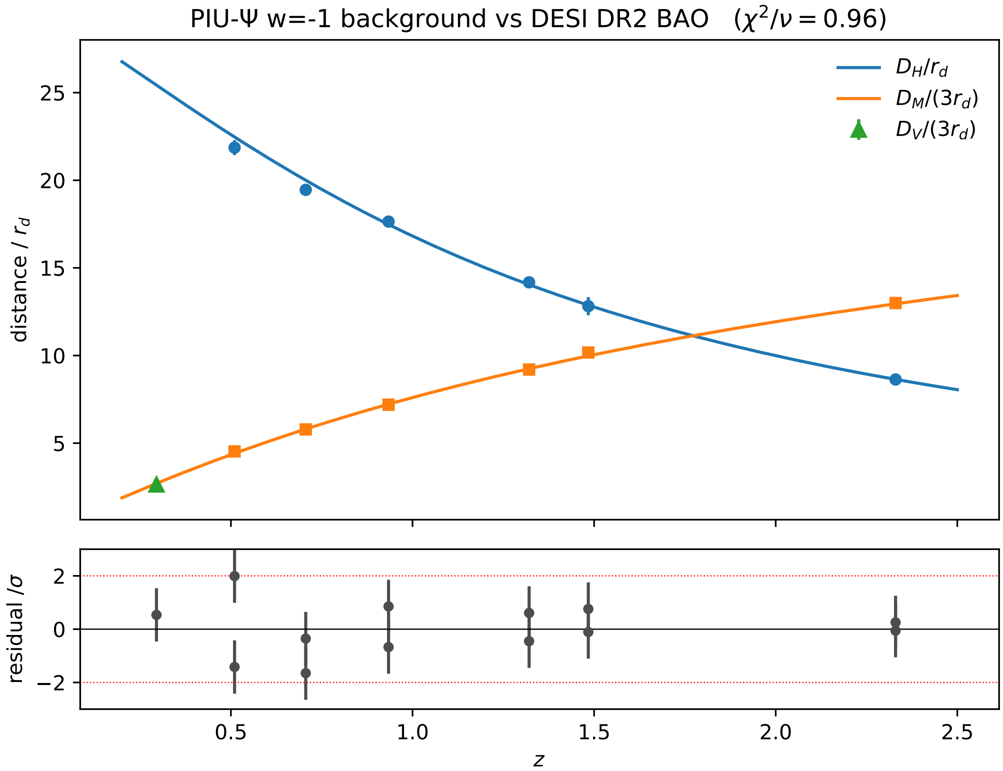

Figure 1: PIU-Ψ $w = -1$ background (curves) against the thirteen DESI DR2 BAO measurements (points with error bars; circles D_H/r_d, squares $D_M/(3r_d)$, triangle $D_V/(3r_d)$). Bottom: normalized residuals, all within $2\,\sigma$. The fit gives $\chi^2/\nu = 0.96$.

5 The Bayesian analysis with the full covariance

The least-χ^2 fit locates the best point; the position of $(w_0, w_a) = (-1, 0)$ relative to the full credibility region requires sampling the posterior. With an ensemble sampler [6] of forty walkers —fifteen hundred burn-in steps and six thousand of production— the posterior of (N, Ω_m, w_0, w_a) is sampled under the CPL template with the full covariance of (4). The chains converge: the Gelman–Rubin statistic is $\hat{R} < 1.015$ for the four parameters, with acceptance fraction 0.56 and autocorrelation time $\tau \approx 90$ steps.

With BAO alone, the posterior of (w_0, w_a) is markedly non-Gaussian: an elongated, degenerate band, reflecting that BAO by itself constrains the pair weakly. Its marginals are $w_0 = -0.17^{+0.42}_{-0.45}$ and $w_a = -2.78^{+1.57}_{-1.47}$. The position of $(-1, 0)$ is evaluated by the density credibility level, not by a Gaussian distance the shape of the posterior would invalidate: the point $(-1, 0)$ lies on the boundary of the 91 % contour, equivalent to $2.2\,\sigma$ in two dimensions, that is *inside* the $2\,\sigma$ (95.4 %) contour. In the one-dimensional marginals, $w_0 = -1$ lies $2.0\,\sigma$ away and $w_a = 0$ lies $1.7\,\sigma$ away. The $w = -1$ background is consistent with raw BAO in the Bayesian analysis too; the blue band of Fig. 2 encloses it.

6 Model comparison and the role of the Ω_m anchor

The consistency of $w = -1$ with BAO is not enough to locate the origin of the dynamical signal: one must compare models over data that include the information generating it. The signal reported by DESI comes from combining BAO with the CMB and supernovae; its decisive ingredient, isolated in the recent literature [12, 8], is the discrepancy in the matter density: BAO geometry prefers $\Omega_m \approx 0.297$, while the CMB, through the acoustic scale, prefers $\Omega_m \approx 0.315$ [7, 1]. The dynamical template with $w = -1$ is therefore compared through information criteria —Akaike (AIC) and Bayesian (BIC), which penalize additional parameters— over three configurations: BAO alone, and BAO with a Gaussian anchoring of Ω_m at each of the two disputed values.

Table 2: Model comparison by information criteria. Δ refers to CPL minus $w{=}-1$; ΔBIC > 0 favours $w{=}-1$. The Jeffreys scale reads $\Delta\mathrm{BIC} \in (2, 6)$ as positive preference and $(6, 10)$ as strong.

Configuration	$\chi^2_{w=-1}$	χ^2_{CPL}	ΔBIC
BAO alone	10.53	5.81	+0.4
BAO $+\Omega_m = 0.315$ (CMB)	13.10	8.16	+0.3
BAO $+\Omega_m = 0.296$ (CMB-indep.)	10.54	9.31	+4.0

The result (Table 2) is clean. With BAO alone no model is preferred ($\Delta\mathrm{BIC} = +0.4$): BAO does not distinguish. Anchoring Ω_m to the high CMB value, CPL improves the χ^2 but does not overcome the parameter penalty ($\Delta\mathrm{BIC} = +0.3$; the AIC barely favours it): the $3\,\sigma$ signal that DESI reports requires, besides the CMB, the supernova information. And anchoring Ω_m to the low value, $w = -1$ is positively preferred ($\Delta\mathrm{BIC} = +4.0$), with the best CPL collapsing to $w_a = -0.05$, that is, back to $w = -1$.

The low value is not an arbitrary choice. Two CMB-independent determinations of Ω_m agree on it: BAO geometry gives $\Omega_m = 0.295 \pm 0.015$ [13], and the full-shape analysis of DESI galaxy clustering —which measures Ω_m from the shape of the transfer functions, without using the CMB— gives $\Omega_m = 0.296 \pm 0.007$ [9]. Against both, the CMB value, 0.315 ± 0.007, is the outlier. The preference for dynamical dark energy is, in this reading, the Ω_m tension between probes translated to the (w_0, w_a) plane: it emerges when the high CMB Ω_m is imposed and vanishes when the value preferred by the CMB-independent probes is used.

7 The signal as an Ω_m tension: the empirical route

The model comparison shows that the signal tracks the Ω_m anchor; this fact should be placed within the empirical diagnosis that established it independently of any background theory. Reconstructing the ΛCDM parameter Ω_m from the $w_0 w_a$CDM cosmologies that DESI prefers, Colgáin and Sheikh-Jabbari showed that the dynamical signal translates into an Ω_m tension dependent on effective redshift, with two features: a rising trend of Ω_m at high redshift and a sharp deviation at low redshift [3]. The CMB cannot originate the signal —it constrains the integrated quantity $D_M(z_*)$, which leaves $H(z)$ free at intermediate redshifts—, and indeed DESI BAO combined with the CMB and a nucleosynthesis prior, without supernovae, yields only $\sim 2\,\sigma$. The low-redshift deviation is driven by the supernovae: the DES sample is in tension with the DESI full-shape analysis at the same effective redshift, up to $3.4\,\sigma$. The conclusion of that work is that the signal points to unexplored systematics or a break of ΛCDM, and that claims of dynamical dark energy are premature.

This is the empirical diagnosis, and it is not from PIU-Ψ: it is obtained by parameter reconstruction over the data, with no hypothesis about the nature of the background. Its relation to the present work is one of convergence, not dependence. The direct mechanism is seen thus: a universe with *exact* $w = -1$ —the PIU-Ψ background, with $\Omega_m = 0.297$— generates observables at the DESI redshifts that, fitted with CPL using BAO alone, return $(w_0, w_a) = (-1, 0)$ with $\chi^2 = 0$; the template does not invent dynamics from pure BAO. The signal arises upon adding the external Ω_m anchor. The (w_0, w_a) direction is so degenerate in BAO that the official DESI signal, $(w_0, w_a) = (-0.42 \pm 0.21, -1.75 \pm 0.58)$ for the BAO+CMB combination [1], worsens the χ^2 of the $w = -1$ universe by just 4.2 —the same $\Delta\chi^2$ scale that in §4 made the CPL improvement insignificant: BAO does not separate one background from another—; and forcing the pair toward that signal, the fitted Ω_m slides from 0.297 to 0.36 to compensate. When the CMB anchor instead fixes Ω_m on its own, that lever engages: the posterior of a $w = -1$ universe shifts to $w_0 = -0.78^{+0.09}_{-0.10}$, $w_a = -0.59^{+0.31}_{-0.34}$, with 97 % of its mass in the $w_0 > -1$, $w_a < 0$ quadrant (red band of Fig. 2). This is the sign and approximate magnitude of the signal, generated from a background that never stopped being $w = -1$.

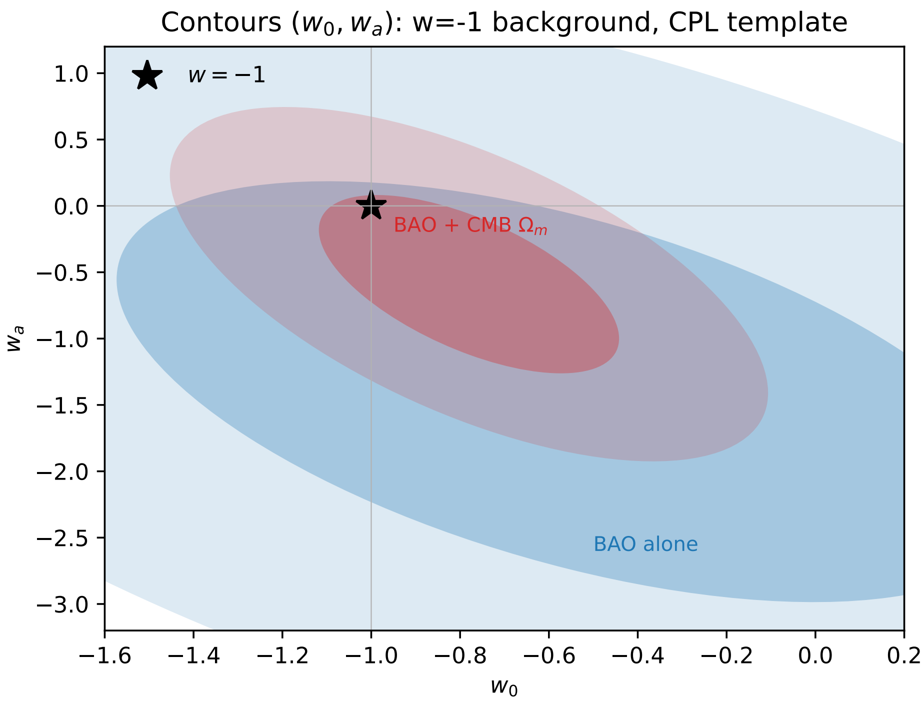

Figure 2: Credibility contours 68 % and 95 % in the (w_0, w_a) plane for a $w = -1$ background under the CPL template. In blue, DESI BAO alone: the degenerate band encloses the $w = -1$ point (star). In red, BAO with the CMB Ω_m anchor: the posterior contracts and shifts to the $w_0 > -1$, $w_a < 0$ quadrant, even though the true background is $w = -1$.

That the shift is not an artefact of the flat Ω_m anchor is checked by redoing it with a realistic CMB anchor —two Gaussian terms, in the physical density $\omega_m = \Omega_m h^2$ and in the Planck shift parameter R—, under which the best CPL still falls in the signal quadrant ($w_0 = -0.59$, $w_a = -0.90$): the effect is physical, not from the simplified prior (Appendix A).

8 The robust prediction of the dark sector

That the signal is an Ω_m tension has a consequence the empirical route does not pursue but that PIU-Ψ exploits, and that leads to a prediction of its own. The parameters Planck reports are not all of the same type. One layer is *near-model-independent*: the physical densities $\omega_b = \Omega_b h^2$ and $\omega_{\rm dm} = \Omega_{\rm dm} h^2$, and the acoustic angle θ_*, are read from the relative acoustic peak heights almost without assuming the late cosmology. The other layer —H_0, Ω_m, Ω_Λ, σ_8— are *outputs* of the six-parameter ΛCDM fit, not direct readings [7, 3].

The tension that sustains the dynamical dark-energy signal lives entirely in the second layer. Indeed, Ω_m is not an observable: $\Omega_m = \omega_m/h^2$, and h is itself a derived parameter, so a single physical density $\omega_m \approx 0.143$ splits into different pairs (Ω_m, h) depending on the probe or the template. The "Ω_m tension" is a tension of that partition, not of the physical density: two CMB-independent probes —BAO geometry, $\Omega_m = 0.295 \pm 0.015$ [13], and the full-shape analysis, $\Omega_m = 0.2962 \pm 0.0095$ [9]— agree on the low value, and the CMB, 0.315, is the outlier. In the robust layer there is no such tension.

On that robust layer, PIU-Ψ makes a prediction ΛCDM cannot make. The ratio of the depleted excitations of the condensate —dark matter over baryonic matter— is a ratio of the substrate geometry, and is derived with no free parameter:

$$\frac{\omega_{\rm dm}}{\omega_b} = \frac{\Omega_{\rm dm}}{\Omega_b} = \sqrt{3}\,\pi = 5.44, \tag{5}$$

where the factor h^2 cancels in the ratio, so the prediction is confronted against physical densities and not against fit parameters. The Planck value, read from the peak heights, is $\omega_{\rm dm}/\omega_b = 5.375 \pm 0.077$; the deviation is $0.86\,\sigma$, with not a single fitted parameter nor observational input. In ΛCDM this ratio is the quotient of two free parameters, fixed independently by the data.

The derivation of (5) is given here in full, link by link, for the reader to judge without recourse to another document. In PIU-Ψ baryonic matter (M) and dark matter (DM) are two classes of solitonic defects of the same medium: M carries conserved topological charge (baryon number, a winding of the phase, with fermionic statistics by the Finkelstein–Rubinstein theorem [15, 16]); DM carries global Noether charge without winding (Q-ball class [17], scalar). The ratio of their cosmological densities is obtained from the quotient of the semiclassical partition functions of the two classes, evaluated in the saturated regime after crystallization. Three pieces fix it, each with its own origin.

Piece 1: the weight is linear in the area of the defect boundary. The excess energy of each soliton over the sea $S_{\rm min}$ is the energy of its domain wall. In the saturated regime, the hard bound of the medium's potential makes the internal curvature diverge, $V''_{\rm eff}(S) \propto (S_{\rm max} - S)^{-2}$, and confines any modulus deformation to a boundary shell of vanishing thickness; the soliton mass is then wall energy, $m_i = \Sigma\, A_i$, with

$$\Sigma = \int \sqrt{2\, V_{\rm eff}(S)}\; dS \tag{6}$$

the surface tension of the wall, which depends *only* on the potential $V_{\rm eff}$ —the same for both classes. The tension Σ is therefore universal and cancels in the mass ratio, leaving $m_{DM}/m_M = A_S/A_T$ exact and linear in the area. (The Gaussian fluctuation determinant is not the source of this linearity: its volume term —common to both classes, sharing the same floor— cancels identically in the ratio, so no bulk contribution contaminates the area ratio.)

Piece 2: the shape of each boundary is forced by the defect's topology. Linearity in the area is not enough; one needs which area. And the geometry of each boundary is not postulated: it is the minimum of the medium's surface functional under the topological constraint of each class.

- **Uncharged defect (DM): the sphere is forced.** A Q-ball has no topological anchoring to the lattice —its phase does not wind, its charge is Noether, global. Its boundary is free to relax, and the surface energy the Lagrangian assigns it —the gradient term $\frac{1}{2}\rho_P c^2 \ell_P^2 |\nabla S|^2$, manifestly isotropic— is minimized at fixed volume by the sphere, by the isoperimetric theorem. The spherical geometry is the minimum of the functional, not a choice:

$$A_S = 4\pi R^2. \tag{7}$$

- **Charged defect (M): the tetrahedron is forced.** A soliton with baryon number $B = 1$ has its phase wound, and that winding anchors the defect to the lattice: it cannot slide or round off without unwinding the charge, which topological conservation forbids. Its boundary does not relax to the sphere; it is pinned to the substrate nodes, and the minimal region containing a node with its coordination $Z = 4$ is its Voronoi cell, the tetrahedron —the unique four-point spherical 2-design [18]. A regular tetrahedron of circumradius R has edge $a = R\sqrt{8/3}$ and total area $\sqrt{3}\,a^2$, so that

$$A_T = \sqrt{3}\left(R\sqrt{8/3}\right)^2 = \sqrt{3}\cdot\frac{8}{3}R^2 = \frac{8\sqrt{3}}{3}R^2. \tag{8}$$

The sphere/tetrahedron asymmetry is, then, the asymmetry between global charge (free boundary $\to$ isoperimetric) and topological charge (anchored boundary $\to$ Voronoi), both read from the same Lagrangian.

Piece 3: a spinorial factor $1/2$ between the two classes. M is fermionic and its soliton lives in $SU(2) \cong S^3$; DM is scalar and lives in $SO(3) = SU(2)/\mathbb{Z}_2$. Projecting to observables, M undergoes the reduction of the invariant volume

$$\frac{\mathrm{Vol}(SO(3))}{\mathrm{Vol}(SU(2))} = \frac{\pi^2}{2\pi^2} = \frac{1}{2} \qquad \text{(Peter–Weyl)}, \tag{9}$$

and DM does not. The remaining factors of the partition ratio —classical action, asymptotic normalization, and the Finkelstein–Rubinstein statistical sign— cancel to unity: both classes stabilize on the same floor with identical asymptotic boundary conditions, and the statistical sign distinguishes the quantum interpretation (M fermionic, DM bosonic) but not the mean densities, for which $|\chi_{\mathrm{FR}}|^2 = 1$ enters.

The assembly. Gathering the three pieces,

$$\frac{\Omega_{\mathrm{DM}}}{\Omega_{\mathrm{M}}} = \frac{A_S \cdot 1}{A_T \cdot (1/2)} = \frac{2A_S}{A_T} = \frac{2\cdot 4\pi R^2}{(8\sqrt{3}/3)R^2} = \frac{8\pi}{8\sqrt{3}/3} = \frac{3\pi}{\sqrt{3}} = \sqrt{3}\,\pi, \tag{10}$$

a ratio of two areas plus a group factor, with no free scale: the radius R cancels and no parameter remains to fit. Numerically, $\sqrt{3}\,\pi = 1.7321 \times 3.1416 = 5.4414$. The chain is a complete derivation [DERIVED]: each link —linearity in the area from the universal wall tension, the sphere from the isoperimetric minimum, the tetrahedron from the Voronoi anchoring, the Peter–Weyl factor $1/2$— follows from the Lagrangian with no fitted number. A single refinement front remains, of low severity and over an already-derived link: whether Kibble–Zurek nucleation [19, 20] introduces a class-dependent number-density correction that alters the equality $n_{DM}/n_M = 1$ used in the mass→density bridge [OPEN THREAD: CLASS-DEPENDENT NUCLEATION]. It is not a gap in the chain, but a higher-order correction on an already-fixed ratio.

Why the input geometry is not a choice. The strongest objection to (10) is not that the algebra is wrong —it is not— but that the input geometry could have been chosen to land on the observed value. The question localizes precisely: the sphere ingredient is inevitable by the definition of isotropy; all the force rests on whether the substrate lattice is forced to be the diamond $Z = 4$, which is what fixes the tetrahedron. It is, and the primary argument follows from two conditions the saturated medium must satisfy, with no fitted datum.

The first is *second-order isotropy.* By the homogeneity and isotropy of the medium, its dispersion operator must be isotropic at low $\mathbf{k}$, which requires the first-neighbour bond tensor to be isotropic, $\sum_{i=1}^{Z} \delta_i^a \delta_i^b \propto \delta^{ab}$. For the tetrahedral coordination $Z = 4$ with vectors $\boldsymbol{\delta}_i = (\ell_P/\sqrt{3})(\pm 1, \pm 1, \pm 1)$ —the four even-sign-product combinations— the computation gives exactly

$$\sum_{i=1}^{4} \delta_i^a \, \delta_i^b = \frac{4}{3} \, \ell_P^2 \, \delta^{ab}, \tag{11}$$

exact isotropy, with bond angle $\arccos(-\frac{1}{3}) = 109.47°$ between all pairs. The other $Z = 4$ arrangement in three dimensions —the planar-square— gives $\sum_i \delta_i^z \delta_i^z = 0$: it confines to the plane and violates isotropy. The second condition is *Maxwell isostaticity*: the minimum coordination that fixes the degrees of freedom of a mechanical network in d dimensions without leaving soft modes is $Z = 2d$, that is $Z = 4$ for the three-dimensional medium (with the two transverse polarizations of the tensor mode as the effective dimensionality). The intersection of the two —exact isotropy and minimal isostatic Z— selects the tetrahedral coordination uniquely. Direct optimization confirms it: minimizing the anisotropy $\sum_{ab} (T^{ab} - \frac{1}{3} \mathrm{tr}\, T \, \delta^{ab})^2$ over four unit vectors, all random seeds converge to the regular tetrahedron, with residual cost $\sim 10^{-12}$; it is the extremum, not one among several, and coincides with the classical result that four equiangular unit vectors on S^2 are the vertices of the regular tetrahedron. The coordination $Z = 4$ is, then, [DERIVED] from the isotropy of the medium and isostaticity, without invoking the form of the potential or any fit.

This derivation fixes the tetrahedral coordination; that the specific stacking is that of the diamond lattice —and not that of wurtzite, also tetrahedral at first neighbours— follows from three further converging criteria: phase condensation (the field dynamics breaks $U(1) \to \mathbb{Z}_2$ into the A/B bipartition of the lattice, incompatible with the tripartition another structure would require), Finkelstein–Rubinstein statistics (the S^3 topology admitting the fermionic solitons), and the variational selection of the phase transition (the cubic stacking as the minimum of the flexural functional). The full development of these three is in the corpus [4]; the one carrying the weight —the tetrahedral coordination that fixes A_T— is derived here. The spherical and tetrahedral geometries are, then, not two geometries chosen among many: they are the two that the topology of each defect class imposes on a lattice whose coordination is derived from the isotropy and isostaticity of the medium.

What the coincidence is worth. The $0.86\,\sigma$ agreement is not, by its size, spectacular: it is within $1\,\sigma$, like many agreements. Its weight lies not in the number of sigmas, but in the product of two facts. First, the value of $\sqrt{3}\,\pi$ was obtained without looking at the data and with not a single free parameter —it is not a fit that happened to land nearby, but a geometry that happens to be nearby. Second, the target is *robust*: $\omega_{\mathrm{dm}}/\omega_b$ cancels the factor h^2 and is read from the peak heights, so ΛCDM cannot choose it to coincide with PIU-Ψ; the heights fix it with little freedom. Matching a well-constrained quantity by an independent route of one's own, without having sought it, is a cross-validation, not an artefact —and contrasts with ΛCDM, where that ratio is the quotient of two separately fitted parameters. That reproducing $\sqrt{3}\,\pi$ from simple geometry is not generic can be bounded: enumerating the space of algebraic expressions of comparable complexity, the fraction falling within $2\,\sigma$ of the observed value of two cosmological quantities at once is less than 3×10^{-4}; the supplementary script recomputes that enumeration with its assumptions in the open, so anyone can vary them. Its reading is bounded —it rules out that the coincidence is generic, it does not prove the theory— and must not be chained with the Bayes factor the corpus develops separately [4], which additionally weights the fit quality and depends on the full Bayesian construction. [CALIBRATED]

The structure of the argument is, then, the following. The dynamical dark-energy signal lodges in the model-dependent layer, where a derived parameter (Ω_m) differs between probes; the robust layer, where the physical densities live, does not show it; and in that robust layer PIU-Ψ predicts the correct value with no freedom. The total matter density confirms it by the complementary route: $\Omega_m = 1 - f_c = 0.3131$, with f_c the condensed fraction fixed by the geometry, falls at $0.27\,\sigma$ from Planck [4]. This second agreement is weaker than that of $\sqrt{3}\,\pi$, and should be read as such: lacking the factor h^2, its target partially shares the model dependence of Ω_Λ, so it counts in favour of the programme but with less force than the robust ratio. The hierarchy is respected: maximal confidence where h cancels and the anchor is a physical density, declared where the target inherits the ΛCDM assumption.

One point deserves clarification, to avoid reading a contradiction where there is none. This $\Omega_m = 1 - f_c = 0.3131$ that the programme derives falls near the *high* CMB value (0.315), not the low value (0.296) that this work identifies as the one physically preferred by the CMB-independent probes. There is no conflict, because the two numbers are neither the same object nor of the same status. The 0.296 is the Ω_m that fits the BAO *distances* and fixes the shape of $E(z)$; it is the side of the tension that the signature of §9 predicts upon measuring Ω_m without the template. The 0.3131, by contrast, is a *positional* reading —the numerator $S^2_{\mathrm{max,eff}}$ of f_c is a calibrated state quantity, not a closed derived number—, with the same model dependence as its target Ω_Λ; that is why its agreement is declared weaker and is not used to pick a side of the tension. PIU-Ψ's *freedom-free* prediction in the dark sector is neither Ω_m, but the ratio of physical densities $\sqrt{3}\,\pi$, which cancels h^2 and lives in the robust layer the Ω_m tension does not touch: the tension is a disagreement over the partition $\Omega_m = \omega_m/h^2$, i.e. over the split (Ω_m, h), not over the robust physical density. The programme therefore does not commit its closed prediction to the high value or the low one: it places it in the layer where the observable is indifferent to the split.

9 The falsifiable signature

If the signal reflected real dynamical dark energy, the inferred (w_0, w_a) would be a property of the universe, stable against the Ω_m one anchors. If it is the Ω_m tension re-expressed, the inferred w_0 must be a function of the anchored Ω_m. To isolate that dependence we generate the distances of a universe with exact $w = -1$ and $\Omega_m = 0.297$, and over them fit w_0 imposing different point values of the Ω_m anchor, marginalizing the global scale $N \equiv c/(H_0 r_d)$ —the standard BAO nuisance parameter, which without the CMB is not known separately. The inferred w_0 traces a straight line (Fig. 3):

$$\frac{dw_0}{d\Omega_m^{\mathrm{anchor}}} = 1.19 \quad (R^2 = 0.994), \tag{12}$$

over five points in $\Omega_m^{\mathrm{anchor}} \in [0.297, 0.321]$: $(0.297, -1.000)$, $(0.303, -0.991)$, $(0.309, -0.984)$, $(0.315, -0.977)$, $(0.321, -0.971)$. The line crosses $w_0 = -1$ at $\Omega_m = 0.296$, the background value. The sign is positive and has a direct physical reading: imposing more matter, dark energy has to push less, and w_0 rises toward zero —the (Ω_m, w_0) degeneracy of BAO distances. When the anchor coincides with the background Ω_m, exact $w_0 = -1$ is recovered; as it moves away toward the CMB value, w_0 slides toward the signal. A w_0 that is not a property of the background but a linear function of the external prior is the signature that the signal measures the Ω_m discrepancy, not the dynamics of the dark sector. [DERIVED]

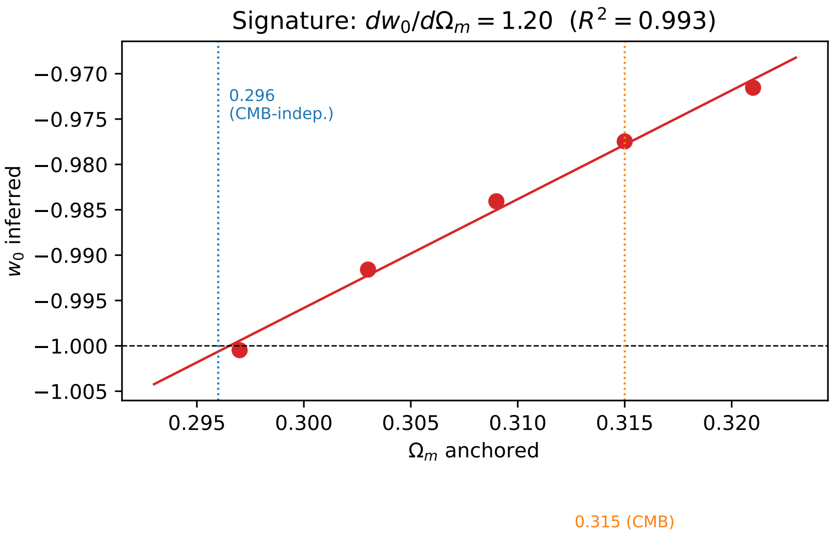

Figure 3: Inferred w_0 as a function of the anchored Ω_m, over a universe with exact $w = -1$. The dependence is linear ($R^2 = 0.994$) with slope 1.2; the positive sign reflects the (Ω_m, w_0) degeneracy of BAO distances. The vertical lines mark the Ω_m of the full-shape analysis (0.296, CMB-independent) and of the CMB (0.315); the line crosses $w_0 = -1$ at the former.

A falsifiable test follows. The assignment of the physical Ω_m decides the question: if it is the low value the CMB-independent probes prefer, the signal is the tension re-expressed and the background is $w = -1$; if it were the high CMB value, there would be real dynamics. A third determination of Ω_m depending on neither BAO nor the CMB —structure growth $f\sigma_8$, or the combination of BAO with primordial nucleosynthesis— breaks the tie. The PIU-Ψ prediction is that this determination will fall on the low side and w_0 will return to -1; if instead w_0 stayed below -1 with Ω_m measured this way, independently of the CPL template, the reading would be refuted. It is a concrete test for DESI DR3 combined with growth data.

10 Convergence: two routes, one conclusion

The value of the background result lies in the methodological independence of the two routes that reach it. The empirical route [3] starts from the data and, with no hypothesis about the background, reconstructs Ω_m from the $w_0 w_a$CDM cosmologies and concludes that the signal is an Ω_m tension attributable to systematics or a break of ΛCDM. The theoretical route starts from the Lagrangian of the Pleno and, prior to the data, derives an exact $w = -1$ background. One is pure data analysis, the other pure deduction from first principles, and they share no assumptions. Their meeting on the same background is not, by itself, a feat —$w = -1$ is generic to a vacuum on its floor— but it is a non-trivial consistency at one point: the theoretical route not only recovers $w = -1$, but derives the *sign* of the artefact (the signal can only push toward $w > -1$) and links that background to a dark-sector prediction the empirical route does not contain.

This convergence is inscribed in a body of independent analyses pointing the same way. The full-shape analysis of DESI galaxy clustering shows no dynamical signal on its own, and its Ω_m determination —derived from the shape of the spectrum, not the geometry— agrees with that of BAO and not with that of the CMB [8, 9]. The BAO signal is dominated by a few tracers [10]. An independent analysis attributes it to an inconsistency between DESI and the Pantheon+ supernovae that violates distance duality, and that when modelled returns compatibility with $w = -1$ [11]. And the significance varies between 2.5 and $3.9\,\sigma$ depending on the supernova compilation [2]. The contribution of this work to that body is twofold: a theoretical route —the derived $w = -1$ background, with the sign of the artefact derived— that none of those references offers, and a prediction of the robust observable —$\omega_{\rm dm}/\omega_b = \sqrt{3}\,\pi$— that none contains.

It is worth separating precisely what each route contributes. That the signal is an Ω_m tension, that the CMB does not originate it and that supernovae drive it at low redshift is an empirical result external to PIU-Ψ [3]. That the background is exactly $w = -1$ by the structure of the condensate, that the sign of the apparent deviation is derived, and that the dark-sector ratio is $\sqrt{3}\,\pi$, are deductions of PIU-Ψ external to the empirical analysis. The strength of the whole lies in neither route separately, but in that routes with nothing in common reach the same place and in that the theoretical route additionally predicts a number the empirical one cannot.

11 Conclusion

Two independent routes converge on a cosmological background with $w = -1$. The empirical route reconstructs Ω_m from the cosmologies DESI prefers and finds that the dynamical dark-energy signal is the Ω_m tension between probes, not a dynamics of the background. The theoretical route derives exact $w = -1$ from the dynamics of the Pleno, structurally prior to the data, and additionally derives the sign of the artefact —the signal can only push toward $w > -1$, never toward the phantom region. This work documents the convergence with the public DESI DR2 data: the $w = -1$ background fits the raw data with $\chi^2/\nu = 0.96$; the preference for $w \neq -1$ in raw observables is $1.67\,\sigma$; and the information criteria localize the signal in the anchoring of Ω_m, present with the high CMB value and absent with the low value that two CMB-independent probes prefer.

To that convergence PIU-Ψ adds what the empirical route does not contain: a parameter-free prediction of the dark sector's robust observable, $\omega_{\rm dm}/\omega_b = \sqrt{3}\,\pi$, at $0.86\,\sigma$ from Planck, derived in full in this work from the substrate geometry —the sphere of the uncharged defect against the Voronoi tetrahedron of the topologically charged defect, with a Peter–Weyl spinorial factor, on a lattice whose tetrahedral coordination follows from the isotropy and isostaticity of the medium, not from a choice. The value of that coincidence lies not in its size in sigmas, but in that a number from pure geometry, obtained without fitting, lands on a robust target —a layer the Ω_m tension does not touch and that ΛCDM describes with two free parameters. It is the same floor $S_{\rm min}$ that fixes $w = -1$ in the background and $\sqrt{3}\,\pi$ in the dark sector: that link, absent in a quintessence model, is what distinguishes PIU-Ψ from any $w = -1$ background.

The status of the result is conditional, and should be stated as such. If the physical Ω_m is the low value on which BAO geometry and the full-shape analysis agree —both CMB-independent—, the dynamical signal is the tension re-expressed and the background is $w = -1$, as PIU-Ψ predicts and as the empirical diagnosis concludes. If the physical Ω_m were the CMB one, there would be real dynamics. The question is resolved by measuring Ω_m by a route independent of BAO and the CMB —structure growth $f\sigma_8$ or the combination of BAO with nucleosynthesis—; the signature

$dw_0/d\Omega_m \approx 1.2$ ($R^2 = 0.994$, §9) predicts that, measured this way, w_0 will return to -1. The refutation threshold is declared: the reading falls if that independent measurement places Ω_m on the CMB side, or if in raw model-independent observables a preference for $w \neq -1$ above $3\,\sigma$ appears that does not dissolve upon removing the CPL parametrization, or if the robust ratio $\omega_{\rm dm}/\omega_b$ moves away from $\sqrt{3}\,\pi$ with better data. That a background with no freedom to depart from $w = -1$ could have fallen at any of those three tests, and today falls at none, is the precise way this result supports PIU-Ψ: not by asserting a victory, but by exposing itself to a defeat that did not occur.

A The realistic CMB anchor

The simplified anchor of §7 imposes a Gaussian term in Ω_m. The CMB does not constrain Ω_m directly, but the physical density $\omega_m = \Omega_m h^2$ (through the relative acoustic peak heights) and the angular acoustic scale (to one part in 3000). The realistic anchor replaces the Ω_m term by two terms: one in $\omega_m = 0.1430 \pm 0.0011$ and one in the shift parameter $R = \sqrt{\Omega_m}\, H_0\, D_C(z_*)/c = 1.750 \pm 0.005$, both from Planck 2018 [7], with the sound horizon $r_d(\omega_m, \omega_b)$ evaluated by the Aubourg et al. fitting formula [14] and $\omega_b = 0.02237$. The fit is over (Ω_m, h, w_0, w_a). The best CPL gives $w_0 = -0.59$, $w_a = -0.90$, $\Omega_m = 0.316$: the shift to the signal quadrant persists, confirming that it is a physical effect of the degeneracy and not of the flat prior.

Under this anchor the $w = -1$ background takes $\Omega_m = 0.271$, in tension with the late-time probes. It is worth being explicit about why that value is forced and not physical, so as not to dismiss the discrepancy as a mere "artefact". The mechanism is the $\omega_m = \Omega_m h^2$ degeneracy within the fit. The CMB fixes the physical density $\omega_m \approx 0.143$ precisely, and fixes the acoustic scale R, which involves the integrated distance $D_C(z_*)$; that distance, in turn, depends on $H(z)$ over the whole range, and hence on h and the equation of state. With the freedom of (w_0, w_a), the fit reconciles ω_m, R and the BAO distances by sliding $w(z)$; without that freedom —in the $w = -1$ background— the only lever left to accommodate fixed ω_m and the BAO distances at once is the (Ω_m, h) partition of ω_m: the fit pushes Ω_m down and h up to $\Omega_m = 0.271$. The 0.271 is not, then, the prediction of the background —which is the 0.296 of the CMB-independent probes— but the value the $w = -1$ fit is forced to take to absorb the CMB tension in the only direction left free to it. That neither the $w = -1$ background nor CPL achieves an Ω_m consistent with the late-time probes under the CMB anchor is, precisely, the Ω_m tension seen from the fit: the conflict resides in the partition of ω_m, not in the equation of state.

B Convention of epistemic labels

The bracketed labels declare the status of each statement: [DERIVED] for a result with a complete derivation from the Lagrangian of the medium, [STRONG DEDUCTION] for a strong logical deduction with no conceptual gap but no closed proof, [CALIBRATED] for a value confronted with data with its error declared, and [OPEN THREAD:] for a declared open thread whose resolution is deferred to future work. The convention comes from the PIU-Ψ programme [4].

Data and code availability

The DESI DR2 BAO measurements are public [1]; the thirteen points used and the D_M–D_H correlation matrix are included as supplementary material in two files (`desi_dr2_bao.csv` and `desi_dr2_corr.csv`), with their provenance documented in each header. The fitting, Bayesian sampling, model comparison and figure scripts accompany the work as supplementary material. A verification script (`verificar_desi.py`, `numpy`/`scipy` only) recomputes every numerical quantity of the work —including the full geometric chain that produces $\sqrt{3}\,\pi$ from the areas of the sphere and the tetrahedron and the Peter–Weyl factor— and aborts with a non-zero exit code on any out-of-tolerance discrepancy; each quantity carries its origin label (derived, calibrated or observational input), and a results table summarizes the checks. The derivation of the diamond lattice and the Bayes factor against numerical coincidence are developed in the corpus [4]; the reader can reproduce them with the open code deposited there.

Conflict of interest

The author declares no conflicts of interest.

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