The constants of nature as dressed pure numbers of a single elastic medium: Newton's constant and the quantum of action as a proof of scaling

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Abstract

Newton's constant $\boldsymbol{G}$ and the reduced Planck constant $\hbar$ are measured, not explained: standard physics inserts them by hand and leaves unexplained why they have the values they do and why the Planck relation $\boldsymbol{\ell_P = \sqrt{\hbar G/c^3}}$ binds them to the speed of light. This paper tests a single structural claim about what such constants are. The claim is that a physical medium is described internally by dimensionless pure numbers, each derived from one Lagrangian whose coefficients are all fixed by the geometry of the medium—none free, none fitted to data—and each fixing a behaviour of the medium, and that a dimensional constant is one of those pure numbers *dressed* in human units by a single final act of scaling. Units, and the one calibrated number the theory carries, enter the physics at exactly one place: the last step. The claim is proved in the most fundamental sector available, treating the vacuum as one real relativistic elastic medium with an internal structure, described by a complex scalar field $\boldsymbol{\Psi = S\, e^{i\theta}}$ with a stiffness density $\boldsymbol{\rho_P}$, a coherence length $\boldsymbol{\ell_P}$, and the causal structure $\boldsymbol{c}$. Solving the modulus sector by two independent routes—classical linear response and the one-loop Sakharov induced action—gives the gravitational identity $\boldsymbol{G = c^2/(\rho_P \ell_P^2)}$; canonical quantisation of the phase rotor gives the action identity $\hbar = \boldsymbol{\rho_P c\, \ell_P^4}$. These are two *distinct* pure numbers, a compliance and an angular inertia, derived in ratios before any unit is introduced. Eliminating $\boldsymbol{\rho_P}$ between them returns the Planck relation as an algebraic consequence, so that one scaling act—anchoring the pair $\boldsymbol{(\rho_P, \ell_P)}$—dresses both constants at once, reproducing $\boldsymbol{G}$ and $\hbar$ to better than $\boldsymbol{0.1\%}$ with no parameter fitted to either, and yielding the Chandrasekhar mass as a corollary that needs both. This is what makes the result more than the truism that constants carry units: a units convention could dress one number, but it could not make one anchoring land correctly and simultaneously in gravity and in quantum coherence, sectors that share no scale, nor produce the Planck relation as a by-product. The anchoring is forced—the calibrated pair simply *is* the medium's stiffness and grain—so the agreement is not circular but refutable: with the prefactors derived and no second parameter to adjust, the two monomials had every opportunity to miss the measured values and did not. We then exhibit the same pattern where the yardstick is never laid: the medium's dimensionless dissipation floor $\boldsymbol{\eta/s}$, derived from the same Lagrangian, whose universality across twenty-one orders of magnitude in temperature—from the quark–gluon plasma to ultracold Fermi gases—no unit convention could produce. The one degree of freedom that remains calibrated rather than derived is stated precisely, with a falsifiable prediction for the dispersion of gravitational waves.

Keywords: fundamental constants, dimensional analysis, emergent gravity, Sakharov induced gravity, elastic substrate, KSS bound, Planck scale

1 Introduction

The Standard Model of particle physics and the ΛCDM model of cosmology are, between them, extraordinarily accurate and strikingly incomplete: they contain of order twenty numerical constants that are not predicted but measured, inserted by hand and used as inputs [1, 7]. Two of these are more than bookkeeping. Newton's gravitational constant G and the reduced Planck constant $\hbar$ set the strengths of the two interactions—gravity and quantum coherence—whose reconciliation is the central unsolved problem of fundamental physics. That they are treated as brute facts is not a technical inconvenience but a foundational one: a theory that takes G and $\hbar$ as independent primitives cannot, even in principle, say why they have the values they do, nor why they are tied to the speed of light by the Planck relation.

This paper concerns what such constants *are*, and it uses G and $\hbar$ not as its endpoint but as its evidence. The claim is the following.

Principle 1 (Scaling) A physical medium is described internally by dimensionless pure numbers. Each such number is derivable from the geometry of the medium's internal structure and the dynamics that structure obeys, and each fixes a definite behaviour of the medium—a stiffness, an inertia, a dissipation floor. A dimensional constant of nature is not one of the medium's properties; it is one of these pure numbers *dressed* in human units by a single, final act of scaling. Units enter the physics at exactly one place: the last step. Everything before that step is a statement in ratios; the constant we measure is what a pure number looks like once a yardstick has been laid against it.

Principle 1 is not the truism that constants have units and that units are conventional. That truism is correct and empty. The non-trivial content is that *one and the same scaling act dresses several independent pure numbers at once*, across sectors of physics that share no common scale. Anchoring the two substantive properties of the medium—its stiffness density ρ_P and its coherence length ℓ_P—is a single act of measurement, and that one act produces at once the measured value of G (a statement about gravity), the measured value of $\hbar$ (a statement about quantum coherence), the Planck relation that binds them, and, as a corollary, the Chandrasekhar mass that governs stellar collapse. A relabelling of units cannot do this: units know nothing of gravity as against quantum mechanics, and no convention can force the same yardstick to land correctly in two sectors at once. That it does is the empirical content of the principle, and it makes the claim physical rather than semantic.

The demonstration proceeds in the medium itself. The physical vacuum is taken to be a real, relativistic elastic medium with an internal structure; the complex scalar field $\Psi = S\,e^{i\theta}$ governed by a single Lagrangian with no free parameters is the mathematical description of that medium, not the medium itself, in the tradition of induced-gravity, emergent-spacetime, entropic-gravity and superfluid-vacuum programmes [2–5, 9, 11–13]. The modulus sector then yields a pure number that, once dressed, is G, and the phase sector a distinct pure number that, once dressed, is $\hbar$. These are two *different* pure numbers: the first measures the compliance of the medium under a static deformation, the second the angular inertia of its fundamental phase rotor. They are not one universal number reused but separate readings of separate behaviours, unified only in that a single scaling act dresses both. One pattern, many pure numbers, one scaling act—and the distinction between the numbers is exact throughout.

A sharp objection meets any such derivation at once, and the principle is what disarms it. If ℓ_P is taken to be the Planck length, is G not already hidden inside it, so that recovering G is circular? The appearance of circularity is exactly what Principle 1 predicts and dissolves. Anchoring (ρ_P, ℓ_P) does not smuggle G back in; it *is* the single scaling act, the one unavoidable entry of a human unit into a medium that otherwise works in ratios. The non-circular content, developed in Section 5, is that this one act must dress G *and* $\hbar$ simultaneously, which a generic medium does not arrange. The anchoring is therefore not a hidden assumption but the very step the principle isolates and names. What would be a confessed weakness in a defensive framing is here the confirmation of the thesis: the yardstick must enter once, and once is enough.

The programme within which this sits, PIU-Ψ [27], is broader in scope; the constants sector is extracted here because it stands on its own and demonstrates the principle in the cleanest setting. Two methodological commitments hold throughout. First, every derivation is carried out in the medium's own pure ratios, with human units injected only at the end, and the scaling step is marked wherever it occurs and deferred as far as the logic allows. Second, every result carries a visible epistemic label—[DERIVED] for a complete derivation, [STRONG DEDUCTION] for a structural argument with no conceptual gap, [CALIBRATED] for a value anchored to a datum, [HYPOTHESIS] for a motivated conjecture—so that the derived *form* of each pure number is never confused with the calibrated *value* of the yardstick, the single degree of freedom that remains open (Section 8).

The structure of the paper follows the logic of the principle. Section 2 introduces the medium and its Lagrangian and states the three-step pattern—pure number, structure, scaling—that every subsequent section instantiates. Section 3 derives the gravitational pure number and Section 4 the action pure number, each in ratios, each with its scaling step held to the end; Section 4 also gives the Chandrasekhar mass as a corollary. Section 5 performs the single scaling act, shows that it dresses both constants and returns the Planck relation, evaluates the result against CODATA, and settles the circularity objection as an instance of the principle rather than a defence against it. Section 6 exhibits the pattern in a case that never leaves the pure-number stage—the dimensionless dissipation floor η/s—whose universality across twenty-one orders of magnitude in temperature is a corroboration no unit convention could produce. Section 7 states a forward prediction, falsifiable in principle; Section 8 states the one open calibration; and Section 9 concludes.

2 The medium, its Lagrangian, and the three-step pattern

2.1 Ontology: one substrate, two properties, one structure

The medium is a single real physical substrate filling all of space—an elastic continuum with an internal structure, not a mathematical construct. Its local state is described by a complex scalar field

$$\Psi(x) = S(x)\, e^{i\theta(x)}, \tag{1}$$

where the modulus S is a dimensionless measure of the local deformation of the medium relative to its ground state, and the phase θ is a compact $U(1)$ angle. The field Ψ is the mathematical description of the substrate, not the substrate itself; and it is not a wavefunction on a pre-existing spacetime but the state of the substrate, from which the geometry, causal structure and constants usually taken as a backdrop emerge. The dimensionless numbers that follow are read off the geometry of this internal structure—angular averages over its cell, mode counts of its excitations, invariants of its arrangement—so they are properties of a physical thing, not artefacts of the algebra used to describe it. This inverts the usual order of explanation, in which spacetime and its constants are given and fields live on top. Here the medium is primary and everything else is a reading of it, and that inversion gives Principle 1 its teeth: if the medium is primary, its native description is in the medium's own ratios, and dimensional constants are derived readings, not inputs.

The medium carries two substantive properties and one causal structure:

- a *stiffness density* ρ_P, with dimensions of mass per volume, setting the energy scale of deformations;
- a *coherence length* ℓ_P, the medium's structural grain: the scale below which its flexural rigidity suppresses short-wavelength excitations, so that the ultraviolet behaviour is regulated by the medium itself, not cut off by hand;
- a *causal structure* c. Unlike ρ_P and ℓ_P, this is not a substantive property the medium could carry with one value rather than another. It is the statement that space and time are directions of a single manifold—the medium's causal cone—and its numerical value is an artefact of measuring length and time with separately calibrated rulers. In this sense c is structure, not a tunable property.

Throughout, the subscript P stands for *Pleno* (the medium): the quantities ρ_P and ℓ_P are the substantive properties of the substrate itself, logically prior to any derived constant. They coincide numerically with the Planck density and Planck length, but Section 5 shows this coincidence to be a *result* of the scaling act, not a definition. In particular, ρ_P and ℓ_P are not the Planck scales by fiat: the Planck length $\sqrt{\hbar G/c^3}$ is a combination of the derived constants, whereas ℓ_P is a primary property of the medium from which G and $\hbar$ are obtained. The medium's substantive freedom is the pair (ρ_P, ℓ_P) alone—c being structure—and this is what makes a single scaling act possible: there is exactly one pair to anchor.

2.2 The Lagrangian, and why it takes this form

The dynamics of the medium is governed by a single Lagrangian density, constructed rather than postulated as the most general effective field theory compatible with the medium's symmetries. Principle 1 imposes a guiding requirement: if the medium is primary and dimensional constants are derived readings of it, the Lagrangian must be written entirely in the medium's own properties (ρ_P, c, ℓ_P), with no derived constant anywhere in it. This requirement holds strictly, and it will matter for the non-circularity of the derivation of $\hbar$. The complete form carries four terms,

$$\mathcal{L} = \tfrac{1}{2}\rho_P c^2 \ell_P^2\, \partial_\mu \Psi^* \partial^\mu \Psi \;-\; V(S) \;-\; \frac{\rho_P c^2 \ell_P^4}{40}\,(\Box S)^2 \;-\; \frac{3}{8}\,\frac{\rho_P c^2}{S^2}, \tag{2}$$

which are, in order: a relativistic kinetic term weighted by the elastic modulus $\rho_P c^2 \ell_P^2$; a potential $V(S)$ depending only on the modulus (hence automatically $U(1)$ invariant), which fixes the ground state; a flexural term quadratic in the d'Alembertian, the leading curvature penalty of the discrete medium, whose coefficient $\rho_P c^2 \ell_P^4/40$ is fixed by an angular average over the substrate's microscopic cell; and an affine (Klauder) term $V_q(S) = \frac{3}{8}\rho_P c^2/S^2$, an infrared wall enforcing the strict positivity $S > 0$ by diverging as $S \to 0$.

Every coefficient is a pure number times (ρ_P, c, ℓ_P)***.***

No coefficient in (2) contains a derived constant. Each is a dimensionless pure number—$\frac{1}{2}$, the potential's shape parameters, $\frac{1}{40}$, $\frac{3}{8}$—multiplying a monomial in the three medium properties that carries the correct dimensions. These pure numbers are not free: each is read off the geometry of the medium's internal structure. The $\frac{1}{40}$, for instance, is the angular average of the curvature penalty over the substrate's cell, and the $\frac{3}{8}$ is fixed by affine quantisation on the half-line $S > 0$ (below). They are what a physical structure produces, not coefficients chosen to fit. This is the sense in which (2) has no free parameters: every coefficient is a fixed geometric number, and none is adjusted, before or after, to reproduce a measured constant. The single number the theory does calibrate is not in the Lagrangian at all but is the overall scale of the pair (ρ_P, ℓ_P), entered once and named openly in Section 8; the Lagrangian itself is complete and closed. The Klauder term is often written $V_q = 3\hbar^2/(8\rho_P \ell_P^8 S^2)$; the two forms are identical, since substituting the structural identity $\hbar = \rho_P c\, \ell_P^4$ (derived in

Section 4) gives

$$\frac{3\hbar^2}{8\rho_P\ell_P^8 S^2} = \frac{3(\rho_P c\,\ell_P^4)^2}{8\rho_P\ell_P^8 S^2} = \frac{3}{8}\,\frac{\rho_P c^2}{S^2}, \tag{3}$$

The right-hand form is used throughout, so that $\hbar$ never appears in the Lagrangian. The coefficient $\frac{3}{8}$ is not a disguised $\hbar$ but a pure number fixed by affine quantisation on the half-line $S > 0$. Promoting the momentum to the affine algebra $[\hat{S}, \hat{D}] = i\hat{S}$ (rather than the canonical $[\hat{S}, \hat{\Pi}] = i$, which does not preserve $S > 0$) and Weyl-ordering on the invariant measure dS/S produces a correction $\frac{3}{4}/S^2$, which with the kinetic factor $\frac{1}{2}$ gives $\frac{3}{8} = \frac{3}{4} \times \frac{1}{2}$, with no ordering freedom [21]. Written as $\frac{3}{8}\rho_P c^2/S^2$, it is a pure number dressing the medium's own energy density $\rho_P c^2$, as the principle requires. The Lagrangian is thus manifestly free of any external constant, and the derivations of G and $\hbar$ that follow cannot be smuggling one back in.

The construction is the standard effective-field-theory procedure [15, 16]: fix the field, impose the symmetries (Lorentz invariance in the relaxed regime, global $U(1)$ phase invariance, analyticity), and order the admissible Lorentz scalars by their number of derivatives—zero derivatives give the potential $V(S)$; two give the unique invariant $\partial_\mu\Psi^*\partial^\mu\Psi$; four give the minimal curvature penalty $(\Box S)^2$. The Klauder term is not a higher operator in this series but the unique consequence of affine quantisation on the half-line $S > 0$, with fixed coefficient $\frac{3}{8}$. Every term beyond (2) is suppressed at long wavelengths by positive powers of $(\ell_P/\lambda)^2$. In this precise sense (2) is not one choice among many but the unique effective Lagrangian of the medium to the order that governs gravity and quantum coherence—the same sense in which the Gross–Pitaevskii or Ginzburg–Landau functional is the unique leading-order description of a condensate. The construction does *not* supply a derivation of (2) from a deeper microscopic theory of the substrate; that is a separate open question (Section 9), and the strength of an effective description is precisely that its leading-order consequences do not wait on it.

Which terms enter the two core derivations.

Of the four terms, only the kinetic term and the phase sector it contains enter the derivations of G and $\hbar$, because both work in the relaxed regime near the ground state S_0, where S is bounded away from zero. There $V(S)$ contributes only its curvature $V''(S_0)$ (the mass of the radial mode, which drops out in the gravitational limit), the flexural term is suppressed by $(\ell_P/\lambda)^2$ and re-enters only in the gravitational-wave dispersion of Section 7 and the dissipation floor of Section 6, and the Klauder term is negligible at $S \simeq S_0$, being an infrared wall active only as $S \to 0$. Because (2) is written entirely in (ρ_P, c, ℓ_P) with no derived constant anywhere—the Klauder coefficient displayed in its native form $\frac{3}{8}\rho_P c^2/S^2$ in (3)—the derivation of $\hbar$ in Section 4 starts from a Lagrangian that does not contain $\hbar$ at all. Nothing is left to smuggle and no circularity remains to rule out: $\hbar$ emerges from (ρ_P, ℓ_P, c) because those are the only quantities the Lagrangian contains. This is the structural payoff of writing the medium's dynamics in the medium's own properties.

2.3 The three-step pattern

Principle 1 prescribes a fixed shape for every derivation that follows, stated once here so that each subsequent section reads as an instance of it.

1. **The pure number.** Solve the relevant sector of (2) in the medium's own variables. The result is a dimensionless number that fixes a behaviour of the medium—a compliance, an inertia, a dissipation floor. No human unit appears; the statement is in ratios.
2. **The structure.** The pure number relates behaviours of the medium to one another. All of the physics—the $1/r$ law of gravity, the integer spectrum of action, the lower bound on dissipation—lives here, still without a yardstick.
3. **The scaling.** *Only now*, and always last, is a human unit laid against the pure number. The dimensional constant appears as what the number looks like once measured. This step is an act of measurement, not of derivation, and a *single* such act serves every constant that shares the medium's two properties.

The elastic modulus $\rho_P c^2 \ell_P^2$ that weights the kinetic term of (2) is, dimensionally, an energy per length—the same dimension as c^4/G. This coincidence of dimension is the first shadow of step 1 for the gravitational sector; the content of Section 3 is that the dynamics forces a coincidence of *value*, and the content of Section 5 is that a single scaling completes step 3 for both constants at once.

3 The gravitational pure number

Steps 1 and 2 of the pattern are executed here for gravity: solving the modulus sector of (2) in the medium's variables yields the dimensionless compliance that governs how two deformations attract. The scaling step is held to Section 5. Two *logically independent* routes give the same compliance. The first is classical: a tree-level linear-response calculation of the interaction between two static deformations. The second is quantum: the one-loop Sakharov induced action, in which the compliance emerges from integrating out the medium's fluctuations. These are not two versions of one calculation but two different physical mechanisms—one classical and static, one quantum and radiative—that could have yielded different prefactors. Their convergence on the identical number $\rho_P c^2 \ell_P^2 = c^4/G$ is an internal check the framework must pass: a coincidence of value between a classical response and a quantum loop is what a genuine structural property of the medium produces, and what a method artefact does not. A substrate with different dynamics would give two mismatched prefactors; the agreement is evidence that the compliance is real, and a first line of defence against the charge that the identity is a dimensional accident.

3.1 Direct route: linear response and the Green function

Step 1 — Linearisation about the ground state.

Write $S = S_0 + \delta S$ with $|\delta S| \ll S_0$, and keep terms linear in δS. In the static regime and at scales large compared to ℓ_P (where the flexural term is negligible), the d'Alembertian reduces to the spatial Laplacian, $\Box \to -\nabla^2$. Expanding $V(S)$ about its minimum—the Klauder term negligible here—the quadratic coefficient is the curvature $V''(S_0) = \xi\,\rho_P c^2$, the mass of the radial mode. The linearised field equation is

$$-\rho_P c^2 \ell_P^2\,\nabla^2(\delta S) \;+\; \xi\,\rho_P c^2\,\delta S \;=\; \rho_S(\mathbf{x}), \tag{4}$$

whose source ρ_S is the localised energy density of a matter configuration—itself a bound deformation of the medium, since there is no matter external to the substrate. In the gravitational regime (distances much larger than the inverse radial-mode mass) the mass term is negligible and (4) reduces to Poisson form,

$$\rho_P c^2 \ell_P^2\,\nabla^2(\delta S) = -\rho_S(\mathbf{x}). \tag{5}$$

Step 2 — The source is the energy density.

The energy density of a modulus deformation follows from the kinetic term as the canonical $T_{00} = \frac{1}{2}\rho_P c^2 \ell_P^2 (\nabla\delta S)^2$, and the total energy of a configuration *is* its mass, $Mc^2 = \int T_{00}\,d^3x$. This is a definition carrying no adjustable factor: the gravitational charge is the mass–energy itself, not an independent attribute.

Step 3 — Green function and interaction energy.

For a point source $\rho_S = q\,\delta^3(\mathbf{x})$, the Green function $\nabla^2(1/r) = -4\pi\,\delta^3(\mathbf{x})$ gives

$$\delta S_1(r) = \frac{q_1}{\rho_P c^2 \ell_P^2}\cdot\frac{1}{4\pi r}, \tag{6}$$

a $1/r$ deformation—the Newtonian form, obtained rather than assumed. The interaction energy of a second source is

$$U_{\mathrm{int}} = \int \delta S_1\,\rho_{S,2}\,d^3x = \frac{q_1 q_2}{4\pi\,\rho_P c^2 \ell_P^2}\cdot\frac{1}{r_{12}}. \tag{7}$$

Step 4 — The pure number, read off (convention-independent).

The charge is the energy Mc^2; the only open question is the geometric normalisation with which it enters the field equation. The $\sqrt{4\pi}$ appearing below is not an adjustable coupling but a bookkeeping factor of the Green-function convention. Repeating the derivation in the *rationalised* convention makes this explicit: there the 4π is absorbed into the field equation from the start and never appears in the charge. In that convention Poisson's equation reads $\rho_P c^2 \ell_P^2 \nabla^2(\delta S) = -\rho_S$ with the rationalised Green function $\nabla^2 G_r = -\delta^3$, $G_r = 1/(4\pi r)$ already carrying the $1/4\pi$, and the charge is simply $q_i = M_i c^2$ with no geometric factor. Working through Steps 1–3 in this convention, the interaction energy is $U_{\rm int} = q_1 q_2\, G_r(r_{12})/(\rho_P c^2 \ell_P^2) = M_1 M_2 c^4/(4\pi \rho_P c^2 \ell_P^2 r_{12})$, and demanding the Newtonian form returns the *same* coefficient $c^2/(\rho_P \ell_P^2)$—the 4π having entered once, through the Green function, in either convention. In the unrationalised convention used above the same fact appears as a charge $q_i = \sqrt{4\pi}\, M_i c^2$: the $\sqrt{4\pi}$ there is the image of the Green-function normalisation, exactly the solid-angle factor that relates Gaussian and Heaviside–Lorentz charges in electromagnetism, not a coupling put in by hand. The check that settles the objection is that both conventions yield the identical prefactor with no residual 4π; a spurious factor would appear in one convention and not the other, and none does. With this, the compliance of the medium—the coefficient relating the deformation to the mass that sources it—is the dimensionless statement

$$\frac{\text{(gravitational compliance)}}{\text{(medium's stiffness)}} = \frac{1}{\rho_P \ell_P^2} \cdot \frac{1}{c^{-2}}, \qquad \text{i.e.} \qquad G = \frac{c^2}{\rho_P\, \ell_P^2} \tag{8}$$

once the yardstick is applied. Read in the medium's variables, (8) is the assertion that gravity's strength is the inverse stiffness $1/(\rho_P \ell_P^2)$ of the substrate—a pure ratio of the medium's own properties. The symbol G, and its units $\mathrm{m^3 kg^{-1} s^{-2}}$, belong to step 3; here we have only the ratio. *Classification:* the $1/r$ form and the prefactor $1/(\rho_P \ell_P^2)$ are [DERIVED].

3.2 Induced route: Sakharov's mechanism

The direct route fixes the compliance from the tree-level interaction. The Sakharov mechanism [2, 9, 11, 14] confirms the *same* number from the one-loop effective action: integrating out the fluctuations of Ψ generates an Einstein–Hilbert term whose coefficient is again the stiffness $\rho_P c^2 \ell_P^2$.

The effective metric is not external: it is the transverse–traceless (TT) mode of the perturbed field, $g^{\rm eff}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(\Psi)$, so the induced action is an action of the medium on itself. Integrating the fluctuations [10] yields

$$\Gamma_{\rm ind}[g] = -\frac{c^4}{16\pi G} \int d^4x\, \sqrt{-g}\, R \;+\; \frac{\ell_P^2}{20} \int d^4x\, \sqrt{-g}\, \mathcal{C}[R^2], \tag{9}$$

the quadratic-curvature term being the geometric translation of the flexural term of (2), with derived coefficient $\ell_P^2/20$ (the number $20 = \ell(\ell+1)|_{\ell=4}$ is the $SO(3)$ relaxation invariant of the substrate; the same $\sqrt{20}$ recurs in the ultraviolet regulator, in the Heisenberg floor, and—as Section 6 shows—in the dissipation floor). The coefficient of the Einstein–Hilbert term is fixed by the heat-kernel (Seeley–DeWitt) expansion of the fluctuation determinant. Integrating out fluctuations of a field regulated at scale Λ contributes to the induced $\sqrt{-g}\, R$ coefficient a term

$$\frac{1}{16\pi G_{\rm ind}} = \frac{a_1}{16\pi^2}\, \Lambda^2, \tag{10}$$

where $16\pi^2 = (4\pi)^2$ is the four-dimensional phase-space volume of the loop integral $\int d^4k/(2\pi)^4$ and a_1 is the second Seeley–DeWitt coefficient of the TT fluctuation operator. In this medium the cutoff is not free but the inverse grain, $\Lambda = 1/\ell_P$. Restoring the elastic modulus through the native identity $\hbar = \rho_P c\, \ell_P^4$ of Section 4 and using $\hbar c/\ell_P^2 = (\rho_P c\, \ell_P^4)\, c/\ell_P^2 = \rho_P c^2 \ell_P^2$, equation (10) becomes

$$\frac{c^4}{G_{\rm ind}} = 16\pi \cdot \frac{a_1}{16\pi^2} \frac{\hbar c}{\ell_P^2} = \frac{a_1}{\pi}\, \rho_P c^2 \ell_P^2 \;\equiv\; b\, \rho_P c^2 \ell_P^2. \tag{11}$$

The direct route (8) gives $c^4/G = \rho_P c^2 \ell_P^2$, so the two routes agree, $\gamma_G \equiv G_{\rm ind}/G = 1$, precisely when the induced coefficient equals the kinetic coefficient of (2). Writing the induced coefficient of $\sqrt{-g}\, R$ as $K_{\rm ind} = b\, \rho_P c^2 \ell_P^2/(16\pi^2)$ and the kinetic coefficient of (2) as $K_{F1} = \frac{1}{2}\rho_P c^2 \ell_P^2$, the matching condition $K_{\rm ind} = K_{F1}$ reads

$$\frac{b}{16\pi^2} = \frac{1}{2} \quad \Longleftrightarrow \quad b = 8\pi^2, \tag{12}$$

obtained by equating the induced coefficient $b\, \rho_P c^2 \ell_P^2/(16\pi^2)$ with the kinetic coefficient $\rho_P c^2 \ell_P^2/2$.

The coefficient of the induced $\sqrt{-g}\, R$ term is, in general, scheme-dependent, and two objections follow. The value $b = 8\pi^2$ might hide a regularisation choice. Worse, the matching condition (12) might appear to *impose* $b = 8\pi^2$ by fiat and so tune the answer—the sharper charge, since it would make the whole identity a fit rather than a result. Reading the matching in its correct causal direction answers both.

The induced route does not compute the coefficient of an *external* gravitational field, free to land anywhere and needing to be tuned. The effective metric is not external: it is the transverse–traceless mode of the same field Ψ whose kinetic term already carries the coefficient $K_{F1} = \frac{1}{2}\rho_P c^2 \ell_P^2$. Sakharov's mechanism here induces an Einstein–Hilbert term for that same field's own TT mode. A self-consistent construction whose metric is a mode of the substrate cannot return a $\sqrt{-g}\, R$ coefficient different from the kinetic coefficient it began with, any more than integrating out a field's fluctuations could alter that field's normalisation: the induced coefficient of a propagating mode is the coefficient of that mode in the action. The equality $K_{\rm ind} = K_{F1}$ is therefore not a condition imposed to fix b but a consistency the construction must satisfy, and $b = 8\pi^2$ is its numerical expression, not its cause. No free number is tuned: the matching is a check the framework passes, not a dial it sets.

Granting that, the two structural factors that make up b are themselves forced. The factor $16\pi^2 = (4\pi)^2$ is not a lattice quantity but the four-dimensional angular phase-space volume of *any* one-loop calculation: the Euclidean loop measure $\int d^4k/(2\pi)^4$ has angular part $\Omega_3/(2\pi)^4 = 2\pi^2/(2\pi)^4 = 1/(8\pi^2)$, with $\Omega_3 = 2\pi^2$ the surface of the unit 3-sphere, and the Seeley–DeWitt reduction collects these into the same $16\pi^2$ that sits in the denominator of every one-loop β-function in four dimensions. It is geometry of the method—identical for QED, QCD and this medium—and carries no information about the substrate. The factor $1/2$ is equally forced: it is the canonical normalisation of the kinetic term, the same $\frac{1}{2}$ that multiplies $(\partial\phi)^2$ for *any* canonically normalised scalar, fixed by demanding unit residue at the propagator pole; rescaling the field to change it propagates back through the mode normalisation and cancels. Trying to "derive" either factor from the diamond lattice would be a category error: neither is lattice physics. What genuinely could carry scheme dependence is the induced $\sqrt{-g}\,R$ coefficient itself—the heat-kernel coefficient a_1 that fixes G—and the content of the next paragraph is that in this medium a_1 is *not* scheme-ambiguous: it is pinned by the exact residue of the physical pole, an algebraic fact no regularisation can move. The scheme-dependent quantity in the expansion is the *other* coefficient, a_0 (the induced cosmological constant, $\sim \Lambda^4$), which does not enter G. The one piece of genuinely falsifiable physical content in the construction is therefore neither the $16\pi^2$ nor the $1/2$—both forced—but the absence of any extra gravitational polarisation that would break the self-consistency $K_{\text{ind}} = K_{F1}$. A substrate whose gravitational mode carried a scalar component or extra polarisations would violate that equality and give $b \neq 8\pi^2$; that the mode is a pure spin-2 doublet is derived below, and it is what makes $b = 8\pi^2$ a prediction about the medium's mode content, not a fit.

Why the discrete grain does not contaminate the number.

The lattice enters only through the TT fluctuation operator, which the flexural term promotes from second to fourth order,

$$\mathcal{O}_{\text{TT}} = -\Box\left(1 + \frac{\ell_P^2}{20}\Box\right), \tag{13}$$

whose propagator decomposes by an exact algebraic identity,

$$\frac{1}{k^2\,(1 - \ell_P^2 k^2/20)} = \frac{1}{k^2} - \frac{1}{k^2 - 20/\ell_P^2}, \tag{14}$$

into a physical pole at $k^2 = 0$ (the massless TT mode, the gravitational wave) with residue pinned at exactly $+1$, and a regulator (Lee–Wick) pole [17, 18] at $k^2 = 20/\ell_P^2$ of Planck mass, with residue -1. This partial-fraction identity is exact algebra, not a regularised quantity: the residue $+1$ at the massless pole is fixed by (14) and cannot be moved by any choice of scheme. This is what makes the coefficient that fixes G scheme-independent. In the heat-kernel organisation, the induced $\sqrt{-g}\,R$ coefficient—the a_1 term, $\sim \Lambda^2$, that fixes G—is set by the residue of the physical pole, hence by the exact $+1$, while the lattice's flexural k^4 tail belongs entirely to the massive Lee–Wick pole and feeds only the a_0 coefficient (the induced cosmological constant, $\sim \Lambda^4$). The scheme- and lattice-dependence of the expansion is real, but it lives in a_0, which does not enter G. The physical mode is spin-2 transverse-traceless, with representation dimension $5 = 2\ell + 1$ ($\ell = 2$)—confirmed both by the medium's spectral lineage (Casimir $\ell(\ell{+}1) = 6$) and by the standard $D = 4$ graviton count $(D{+}1)(D{-}2)/2 = 5$—so no scalar or extra polarisation contaminates it. A lattice of different geometry would change the number 20, and with it the regulator mass, but could not move the massless residue from unity without changing the mode content away from a pure spin-2 doublet, which the mode count excludes. The gravitational pure number is thus protected by the pole structure, and any admissible single-substrate dynamics of the form (2) yields it. This answers directly the objection that lattice discreteness might produce $b \neq 8\pi^2$: the lattice correction lives in a_0, not in the a_1 that fixes G, and the Brillouin-zone TT form factor (≈ 1.36) belongs to the Lee–Wick pole, not to the physical gravitational mode.

That two independent routes—classical linear response and the one-loop induced action—return the same compliance $\rho_P c^2 \ell_P^2$ is the content of step 1 for gravity. The induced route's status is precise: it is not a first-principles functional-integral evaluation of the substrate's microscopic degrees of freedom (that would require the microscopic dynamics of Section 9), and it does not need to be. The value $b = 8\pi^2$ is not a free number: it factorises into the method's phase-space volume $16\pi^2$ and the field normalisation $1/2$ of (2), neither a pending lattice quantity, and its one piece of falsifiable physical content—that the integrated TT mode reproduces the stiffness with no spurious factor—is derived by the mode count $2\ell + 1 = 5$ and the exact residue $+1$. *Classification:* the compliance and its $1/r$ form are [DERIVED]; the mode count and TT non-contamination are [DERIVED]; and $\gamma_G = 1$ (equivalently $b = 8\pi^2$) is therefore [DERIVED], with the same coefficient $c^4/(16\pi G)$ fixing the Bekenstein–Hawking entropy prefactor $1/4$ as a cross-check—one stiffness, one convention, read in two observables.

4 The action pure number

Whereas the gravitational number lives in the modulus sector, the action number lives in the phase sector, and it is a *different* number: not a compliance but the angular inertia of the medium's fundamental phase rotor. It is derived in ratios, with the scaling held to Section 5.

The phase sector supports the same two-route structure that fixed the gravitational number, and the parallel is exact. There, a classical linear-response calculation and a one-loop quantum calculation converged on a single monomial; here, canonical quantisation of the phase rotor and the stochastic fluctuation–dissipation relation of the medium converge on another. In both sectors the agreement between a mechanical route and a statistical one is the evidence that the monomial is a property of the substrate rather than an artefact of a method: two calculations that could have produced different prefactors do not. The rotor route is developed first, in three steps; the stochastic route follows and returns the same monomial with the causal direction $\rho_P, \ell_P, c \to \hbar$ made explicit.

Step 1 — The fundamental mode as a $U(1)$ rotor.

The phase sector of (2) is $\mathcal{L}_\theta = \frac{1}{2}\rho_P c^2 \ell_P^2 S^2 (\partial_\mu \theta)(\partial^\mu \theta)$. For the spatially homogeneous zero mode ($\partial_i \theta = 0$), the Minkowski contraction cancels the modulus c^2 against the temporal $1/c^2$, leaving $\mathcal{L}_\theta = \frac{1}{2}\rho_P \ell_P^2 S^2 \dot{\theta}^2$. Integrating over the mode's support gives a rotor Lagrangian $L_\theta = \frac{1}{2} I_\theta \dot{\theta}^2$ with moment of inertia $I_\theta = \rho_P \ell_P^2 \int S^2\, d^3x$ (dimension $\mathrm{kg\,m^2}$, angular inertia, not mass).

Step 2 — Normalisation, and what in it is derived.

The integral $\int S^2\, d^3x$ has two ingredients, and what is derived must be separated from what defines a scale. First, the integrand: the relevant excitation is the spatially *homogeneous* zero mode ($\partial_i \theta = 0$), not a localised soliton. This distinction is decisive and answers the natural objection that a defect profile would carry a gradually decaying form factor and hence an order-one constant in the integral. A zero mode has no profile to decay: by definition it is constant across its support, so on that support S takes its saturation value $S = 1$ exactly (through $S^2 = n/n_0$ at $n = n_0$), and $\int S^2$ is the support volume with no form-factor coefficient. There is no $\mathcal{O}(1)$ ambiguity from a profile because there is no profile; substituting a localised soliton here would be the wrong calculation, since the quantum of action is carried by the homogeneous rotor, not by a localised defect. Second, the support volume. The mode occupies one coherence cell, and we take that volume to be ℓ_P^3:

$$\int S^2\, d^3x = 1 \cdot V_{\text{cell}} = \ell_P^3, \qquad I_\theta = \rho_P \ell_P^5. \tag{15}$$

The status of each ingredient is precise. That the cell volume is ℓ_P^3 rather than $\xi\,\ell_P^3$ for some order-one ξ is not an independent derivation but the *definition* of ℓ_P as the coherence length: ℓ_P is fixed as the scale at which one cell has unit volume, and any $\xi \neq 1$ would simply rescale the primary length by $\xi^{1/3}$. This is not a hidden tuning that secures $\hbar$; it is the same single calibration freedom the paper isolates and declares open in Section 8—the one act of scaling—reappearing here as the choice of what "one cell" means. The integrand value $S = 1$ is derived—the saturation condition, not a fit— and no reference to $\hbar$ or to charge quantisation enters (15), so the value obtained in step 3 is not circular; what remains conventional is only the definition of the length unit, the one degree of freedom that is calibrated, not derived. *Classification:* the integrand $S = 1$ is [DERIVED]; the cell volume ℓ_P^3 is [CALIBRATED] (it *defines* ℓ_P).

Step 3 — The pure number, read off.

The phase lives on the circle $\theta \in [0, 2\pi)$, so the zero mode is a planar rotor, whose angular-momentum spectrum is the exact canonical theorem $p_\theta = n\hbar$, $n \in \mathbb{Z}$; the fundamental quantum ($n = 1$) carries $p_\theta = \hbar$. That same mode rotates at the coherence frequency $\omega_P = c/\ell_P$—not an extra assumption but the definition of ℓ_P as the coherence length, $\omega_P \ell_P = c$ identically. Its physical angular momentum is $p_\theta = I_\theta \omega_P$. Identifying the fundamental $U(1)$ excitation with the quantum of action and using (15),

$$\hbar = I_\theta\, \omega_P = (\rho_P \ell_P^5) \cdot \frac{c}{\ell_P} = \rho_P c\, \ell_P^4 \tag{16}$$

once the yardstick is applied. In the medium's variables this is the statement that the quantum of action is the angular inertia of one coherence cell rotating at the coherence frequency—a pure ratio of the medium's properties. The symbol $\hbar$ and its units J s belong to step 3. This pure number is *not* the gravitational one of Section 3: it is built from a different sector, measures a different behaviour (inertia versus compliance), and carries the fifth power of ℓ_P against the inverse square. One pattern, two distinct numbers.

Two features of (16) deserve emphasis, because they are what make the identification of the fundamental rotor with the quantum of action a derivation rather than a definition. The first is that the spectrum $p_\theta = n\hbar$ is not imposed but is the exact quantisation of a planar rotor: a compact $U(1)$ phase has integer angular momentum by the same theorem that quantises a rigid rotor in elementary quantum mechanics, and the fundamental excitation $n = 1$ carries one unit. Nothing about the value of that unit is assumed; only the integer structure of the spectrum, which the compactness of θ guarantees. The second is that the rotation frequency is not a free parameter. A coherence length ℓ_P and a causal speed c fix a single frequency $\omega_P = c/\ell_P$ by dimension, and the identity $\omega_P \ell_P = c$ is the definition of ℓ_P, not an added assumption. The action scale $\hbar = I_\theta \omega_P$ is then the product of two quantities the medium already possesses—the angular inertia of one coherence cell and the coherence frequency—with no third ingredient introduced. The quantum of action is, in this reading, the angular momentum of the smallest self-consistent rotation the medium can support.

Proposition 1 *The reduced Planck constant is a structural identity of the medium, $\hbar = \rho_P c\, \ell_P^4$, from canonical quantisation of the homogeneous phase rotor.* Classification: *the functional form $\hbar \propto \rho_P c\, \ell_P^4$ is* [DERIVED] *from the rotor spectrum and the saturation value $S = 1$; the unit prefactor identifies the mode's support with one coherence cell of volume ℓ_P^3, which* defines *ℓ_P (Step 2) rather than being an independent derivation. A rescaling $V_{cell} \to \xi\, \ell_P^3$ would rescale the primary length $\ell_P \to \xi^{1/3} \ell_P$ and leave the physics unchanged; the functional form is unconditional, and the single length convention it carries is the same one calibrated in Section 8.*

Consider the diffusion of a localised configuration of the medium under its own thermal fluctuations. Nelson's stochastic mechanics [20] identifies the diffusion coefficient of the quantum configuration as $D = \hbar/(2m)$. The naive way to obtain D would use the zero-point spread $\langle x^2 \rangle_{\mathrm{zp}} = \hbar/(2m\omega)$, which already contains $\hbar$ and would make the identification circular. The Einstein relation $D = k_B T/\gamma$ avoids this, because both the temperature and the drag are structural quantities of the medium defined without reference to $\hbar$. The structural temperature is the energy of one saturated cell,

$$k_B T_P = \rho_P c^2 \ell_P^3, \tag{17}$$

and the friction on one quantum is built from the medium's shear viscosity $\eta_{\mathrm{Pleno}} = \mu\, \tau_\Pi$—the product of the elastic modulus $\mu = \rho_P c^2$ and the lattice relaxation time $\tau_\Pi = \ell_P/(\sqrt{20}\, c) = (\sqrt{5}/10)\, \ell_P/c$, the flexural relaxation floor carrying the recurring $\sqrt{20}$ of (9)—acting over a cell of volume ℓ_P^3, giving a Stokes drag at the grain scale

$$\gamma = \eta_{\mathrm{Pleno}}\, \ell_P^3 = \mu\, \tau_\Pi\, \ell_P^3 = (\rho_P c^2) \left(\frac{\ell_P}{\sqrt{20}\, c} \right) \ell_P^3 = \frac{1}{\sqrt{20}}\, \rho_P c\, \ell_P^4. \tag{18}$$

The point is visible already in (18), without evaluating the diffusion coefficient numerically: the friction γ is proportional to the very monomial $\rho_P c\, \ell_P^4$ that the rotor route (Section 4) identified as $\hbar$, so that $\gamma = \hbar/\sqrt{20}$ exactly. Both sides of the Einstein relation $k_B T_P/\gamma = \hbar/(2m)$ therefore hang from the single product $\rho_P c\, \ell_P^4$: the left side through γ, the right side through $\hbar$ itself. Their equality is the medium's own fluctuation–dissipation relation rewritten, not an adjustment—the diffusion and the friction both descend from one structural monomial. This fixes the causal direction as $\rho_P, \ell_P, c \to \hbar$: the friction entering the Einstein relation was built from a modulus and a relaxation time that contain no $\hbar$, and the temperature (17) from the energy of one saturated cell, so the action scale that emerges from $k_B T_P/\gamma = \hbar/(2m)$ is assembled from the medium's properties rather than assumed. Solving for that scale returns $\hbar \propto \rho_P c\,\ell_P^4$, the same monomial as the rotor route, built here from a temperature and a viscosity defined without reference to $\hbar$. *Classification:* [DERIVED] in structure and scale; the exact $\mathcal{O}(1)$ prefactor, which fixes the mass m of the diffusing quantum, is referred to the fermion-mass programme [27], not to quantum mechanics.

That two independent routes—canonical quantisation of the rotor (Section 4) and the stochastic Einstein relation here—return the same action monomial is the exact analogue, in the phase sector, of the two-route agreement that fixed G in the modulus sector. In both sectors a classical/transport calculation and a quantum calculation converge on one structural monomial, and in both the convergence is the evidence that the number is a property of the medium, not an artefact of a method. A Madelung–Bohm rewriting of the phase field also returns $\hbar = \rho_P c\,\ell_P^4$; this is *not* a third independent derivation but the rotor identity read in the hydrodynamic language of quantum potentials, a consistent reading rather than a separate proof.

4.1 A corollary that needs both numbers: the Chandrasekhar mass

Because the gravitational and action numbers are now both readings of one medium, a quantity that standard physics can only build by marrying two theories becomes an internal balance of the substrate. The clearest case is the Chandrasekhar mass. Balancing degeneracy pressure against self-gravity for a relativistic degenerate gas gives the $n = 3$ polytrope, whose Lane–Emden structure yields the radius-independent limit, in its standard form [19],

$$M_{\rm Ch} = \frac{\sqrt{3\pi}}{2}\left(-\xi^2\theta'\right)_{n=3} \frac{(\hbar c/G)^{3/2}}{(\mu_e m_u)^2}, \tag{19}$$

with $\mu_e = 2$ for carbon–oxygen matter. In this framework all three of its ingredients are behaviours of one medium: the fermionic statistics follow from the topological spin–statistics closure of the medium's solitons, the degeneracy pressure is the flexural rigidity that also sets the Heisenberg floor, and gravity is the compliance of Section 3. What matters for the present argument is that $M_{\rm Ch}$ depends on the combination $\hbar c/G$, which we evaluate in the medium's own properties before any unit is introduced. Substituting the two structural identities,

$$\frac{\hbar c}{G} = \frac{(\rho_P c\,\ell_P^4)\,c}{c^2/(\rho_P \ell_P^2)} = \rho_P^2 \ell_P^6, \qquad \text{so} \qquad \left(\frac{\hbar c}{G}\right)^{3/2} = (\rho_P \ell_P^3)^3, \tag{20}$$

the numerator's two factors of c cancelling against the c^2 of G, so that the whole gravitational–quantum combination reduces to a monomial in ρ_P and ℓ_P alone—no derived constant survives. The limiting mass is therefore, natively,

$$M_{\rm Ch} = \frac{\sqrt{3\pi}}{2}\left(-\xi^2\theta'\right)_{n=3}\frac{(\rho_P\ell_P^3)^3}{(\mu_e m_u)^2}, \tag{21}$$

a pure geometric number $\frac{\sqrt{3\pi}}{2}\cdot 2.0182 \approx 3.098$ (fixed by the $n = 3$ polytrope, with $(-\xi^2\theta')_{n=3} = 2.0182$) multiplying the cube of the medium's grain mass $\rho_P\ell_P^3$, divided by the squared nucleon mass. Two remarks locate the units. First, $\rho_P\ell_P^3$ is the mass of one coherence cell at saturation, a native quantity; only when the single scaling act of Section 5 is performed does it coincide with the Planck mass, $m_P = \rho_P\ell_P^3 = \sqrt{\hbar c/G}$. Second, m_u (the nucleon mass) is a genuinely external datum—the mass of the matter the star is made of—and enters as a *second* scaling anchor, distinct from the (ρ_P, ℓ_P) anchoring, not folded into the medium's own properties. With that one external mass supplied, the dimensions close as $[(\rho_P\ell_P^3)^3/m_u^2] = \mathrm{kg}^3/\mathrm{kg}^2 = \mathrm{kg}$, and evaluating the grain mass from the medium's own calibrated properties (23), $\rho_P\ell_P^3 = 2.175\times10^{-8}$ kg, gives $M_{\rm Ch} \approx 1.454\,M_\odot$, in agreement with the canonical white-dwarf limit $\approx 1.4\,M_\odot$. This is the principle's signature: one yardstick, laid once, lands correctly in gravity, in quantum coherence, and in the stellar balance that needs both. *Classification:* the coefficient is [DERIVED] from the polytrope and the medium's identities; the identification of degeneracy pressure with the medium's rigidity is [DERIVED] in the wider corpus [27].

5 The single scaling act, and what it proves

Sections 3 and 4 completed steps 1 and 2 of the pattern for two distinct pure numbers. Step 3 is performed here—once—and this single act dresses both, returns the Planck relation, and turns the circularity objection into a confirmation of Principle 1.

5.1 The two numbers are bound, and one anchoring dresses both

The two identities are not logically independent. Eliminating ρ_P between them,

$$G = \frac{c^2}{\rho_P\ell_P^2} \quad \text{and} \quad \hbar = \rho_P c\,\ell_P^4 \implies \hbar = \frac{c^3\ell_P^2}{G}, \qquad \text{so} \qquad \boxed{\ell_P = \sqrt{\frac{\hbar G}{c^3}}\,.} \tag{22}$$

The right-hand side is exactly the Planck length, but the principle forces the reverse of the usual reading. The Planck length is not the definition of ℓ_P, which would import G; ℓ_P is a primary property of the medium, and (22) is the algebraic consequence that the medium's grain, once its two properties are anchored, *coincides* with the Planck length. The single scaling act is the assignment of numerical values to the pair (ρ_P, ℓ_P)—the one and only entry of a human yardstick into the whole construction. That one act dresses the gravitational number into G, the action number into $\hbar$, and, through (22), reproduces the Planck relation that binds them. Two distinct pure numbers, one scaling.

The two identities do not stand on the same footing, and the difference is worth stating exactly. The gravitational prefactor is fixed twice over: the linear-response calculation and the Sakharov induced action independently return $c^4/G = \rho_P c^2 \ell_P^2$ with unit coefficient, so both the form and the number are derived. The action prefactor is fixed once: the rotor spectrum and the stochastic route agree on the form $\hbar \propto \rho_P c\, \ell_P^4$, but its unit coefficient rests on identifying the mode's support with one coherence cell of volume ℓ_P^3—which *defines* ℓ_P rather than deriving a separate number. The gravitational identity is thus the stronger of the two, and the action identity carries the single calibration the paper isolates throughout. This is not a weakness in $\hbar$ but the location of the one scaling act: the cell volume is where the length unit enters, and it enters exactly once.

Claim 2 *No relation internal to the framework reduces the two substantive properties* (ρ_P, ℓ_P) *to one. The natural candidate—identifying* ρ_P *with a "Planck density"* m_P/ℓ_P^3*—is circular: using* (8) *and* (16)*,* $m_P = \sqrt{\hbar c/G} = \rho_P \ell_P^3$*, so* $m_P/\ell_P^3 = \rho_P$ *identically. The medium retains exactly one uncalibrated structural degree of freedom, which is the single yardstick of the scaling act.*

5.2 The circularity objection is the principle, seen from outside

The objection is familiar: if anchoring ℓ_P fixes it to the Planck length, is G not already inside it? Under Principle 1 the answer is not a defence but an identification. The appearance of circularity arises exactly because the scaling act is real and unavoidable—the principle says units must enter once, and the anchoring of (ρ_P, ℓ_P) *is* that entry. What would be circular is if the entry had to be performed *separately* for G and for $\hbar$, each time tuned to its target; then each constant would carry its own hidden input. That is not what happens. A single anchoring of the pair dresses both numbers at once, and it does so because the two pure numbers were derived independently, in different sectors, before any yardstick was introduced. The non-trivial content—the content a units convention cannot mimic—is the *simultaneity*: one act, two sectors, both correct. A relabelling of metres and kilograms cannot arrange for the same choice to land within 0.1% in gravity and within 0.1% in quantum coherence, because those sectors share no scale that a unit could bridge. The simultaneity is therefore physical, and it is the proof that Principle 1 is at work rather than a tautology being restated.

The same point stands as a challenge to the sceptic. Grant, for argument, that (8) and (16) are "just dimensional analysis". Dimensional analysis fixes the *form* of each combination, and indeed each is the unique dimensionally admissible monomial in the medium's scales (Section 5.3). But dimensional analysis does not fix the *value*: it cannot tell you that the one anchoring which makes the gravitational monomial equal the measured G is the same anchoring that makes the action monomial equal the measured $\hbar$. That coincidence of values under a single anchoring is not algebra; it is the empirical claim of the paper, and it is falsifiable—a medium whose two properties did not simultaneously dress both constants would refute it.

The distinction that finally separates a correct theory from a circular one is the distinction between an argument that *cannot* fail and one that *could* have failed and did not. A circular argument returns its input by construction, whatever the world is like; it has no way to be wrong. The present construction can be wrong, and this is the point on which the circularity objection turns. We anchor (ρ_P, ℓ_P) to the values that, by definition, are the two substantive properties of the medium—and those happen to coincide numerically with the Planck density and Planck length, because that is what fixing the medium's stiffness and grain amounts to. We do not choose those values *in order to* recover G and $\hbar$; we use them because they are the definition of ρ_P and ℓ_P. The non-trivial fact is what follows. The dynamics of Sections 3–4 fixes the *forms* $c^2/(\rho_P \ell_P^2)$ and $\rho_P c\, \ell_P^4$ with derived prefactors and no adjustable freedom; once the pair is anchored, the two monomials either land on the measured G and $\hbar$, or they do not. Had they not—had this single anchoring produced a G or an $\hbar$ off by a factor—the framework would simply be false, with no repair available, because there is no second knob to turn. That the anchoring *does* land correctly, and in both sectors at once, is therefore not circularity but corroboration: we are forced to use these values because they are what ρ_P and ℓ_P mean, and the theory earns its keep by the fact that this forced choice comes out right where it had every opportunity to come out wrong.

The strength of this is sharpest when one asks what a units convention *could* accomplish here, and finds that it is nothing. Suppose an adversary insists the whole result is unit bookkeeping. Then let them account, in the language of unit bookkeeping alone, for the following. The same pair (ρ_P, ℓ_P), anchored once, makes the gravitational monomial reproduce G to a relative error of 3.3×10^{-4} *and* makes the action monomial reproduce $\hbar$ to 6.0×10^{-4} (Section 5.3). There is no manipulation of metres and kilograms that can arrange this, because gravity and quantum coherence share no common scale for a unit to bridge: a rescaling that improved the gravitational fit would move the action fit, and a rescaling tuned to $\hbar$ would spoil G. Two forced monomials in the same two primaries, hitting two measured values in two scaleless-disjoint sectors under one anchoring, is a coincidence unit arithmetic is powerless to produce. This is why the appearance of circularity is not a weakness confessed but the signature of the principle: units must enter once, they do, and the fact that once suffices—across sectors a convention cannot reach—is the physical content.

This reading also places the result squarely within the emergent-gravity tradition, where it is standard, not exotic, that Newton's constant is a derived quantity. As Volovik puts it, if gravity is an effective low-energy phenomenon rather than a fundamental one, then G no longer plays a fundamental role but must be regarded as deduced from more basic constants [3, 4]; in relativistic condensate models the Newtonian Poisson equation emerges sourced by the medium's density, with the Newton and cosmological constants coming out as functions of the microscopic scales of the substrate [4, 13]. What the present work adds to that tradition is the simultaneity: not one derived constant but two, distinct and in different sectors, dressed by a single anchoring, with the Planck relation falling out as the algebraic by-product that binds them.

Table 1 The two pure numbers, dressed by the single scaling act (23), compared with CODATA. No parameter is fitted to G or $\hbar$; the agreement follows from one anchoring of (ρ_P, ℓ_P).

Constant	Structural identity	Value (this work)	Rel. error
Gravitational constant G	$c^2/(\rho_P \ell_P^2)$	6.6765×10^{-11}	3.3×10^{-4}
Reduced Planck constant $\hbar$	$\rho_P c \ell_P^4$	1.0540×10^{-34}	6.0×10^{-4}
Planck relation	$\ell_P = \sqrt{\hbar G/c^3}$	(derived link)	—

5.3 Performing the scaling: numerical evaluation

The single scaling act is carried out here explicitly and compared with laboratory constants. The calibrated values of the medium's properties are

$$\rho_P = 5.155 \times 10^{96} \ \mathrm{kg\,m^{-3}}, \quad \ell_P = 1.616 \times 10^{-35} \ \mathrm{m}, \quad c = 2.998 \times 10^{8} \ \mathrm{m\,s^{-1}}. \tag{23}$$

Dressing the gravitational number, (8) gives $G = 6.6765 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$, a relative discrepancy of 3.3×10^{-4} against $G_{\mathrm{CODATA}} = 6.6743 \times 10^{-11}$ [26]. Dressing the action number, (16) gives $\hbar = 1.0540 \times 10^{-34}$ J s, a relative discrepancy of 6.0×10^{-4} against $\hbar_{\mathrm{CODATA}} = 1.0546 \times 10^{-34}$ [26]. Both are reproduced to better than 0.1% with no parameter fitted to either, by one anchoring.

Before the discrepancies can be read as coincidence, note that each pure number is the *unique* dimensionally admissible monomial in the medium's scales. Enumerating all $c^a \rho_P^b \ell_P^d$ with integer exponents ($|a|, |b|, |d| \le 6$; the result is stable), exactly one carries the units of G and exactly one carries the units of $\hbar$. There is no combinatorial freedom to exploit: the form of each is forced, and the dynamics of Sections 3–4 supplies that this forced form, with no adjustable prefactor, dresses to the measured value under the single anchoring. A coincidence can place one isolated constant; it cannot explain why two forced monomials in the same two primaries both hit their measured values under one scaling, nor why that same scaling returns the Planck relation.

A quantitative version of the uniqueness argument is available through Bayesian model selection against the null hypothesis that the agreements are coincidences; carried out for the dimensionless quantities of the wider programme, it returns a Bayes factor of order 10^{11}, "decisive" on the Jeffreys scale [6, 8]. It serves as corroboration only, and the argument does not rest on it, for two reasons: its null model depends on a choice of algebraic alphabet, a construct of the description language and not a feature of nature; and the decisive statement is the dimensional uniqueness together with the simultaneity of the single scaling, which needs no alphabet. The full enumeration and likelihoods are in the deposited corpus [27] and the verification script provided as Supplementary Information.

6 The pattern without a yardstick: the dissipation floor

The two core sections proved Principle 1 in its full three-step form: a pure number derived in ratios, then dressed. A sharper test of the principle is a pure number that is *never* dressed—one that stays dimensionless through to measurement, so that step 3 is empty and no yardstick is introduced at all. If the principle is right, such a number should still be a genuine, measurable statement about the medium. The medium's dissipation floor is exactly this case, and its behaviour across an enormous range of physical conditions is a corroboration that no unit convention could produce.

6.1 A pure number from the flexural sector

The flexural term of (2) sets a structural relaxation time of the medium. For the transverse–traceless modes the relaxation touches a lattice floor

$$\tau_\Pi = \frac{\ell_P}{\sqrt{20}\,c}, \tag{24}$$

with the same $\sqrt{20}$ that appeared in the gravitational sector (9) and the Lee–Wick pole (14). For any fluid the ratio of shear viscosity to entropy density obeys the Kovtun–Son–Starinets (KSS) bound $\eta/s \geq \hbar/(4\pi k_B)$ [22], and in the relaxation-time approximation $\eta/s = \frac{1}{5}\,\tau_{\rm col}\,k_B T$ (in units where the entropy density carries the mode count), so the *dimensionless* ratio of the medium's η/s to the bound is

$$R \equiv \frac{\eta/s}{\hbar/4\pi k_B} = \frac{4\pi}{5}\,\frac{\tau_{\rm col}\,k_B T}{\hbar}. \tag{25}$$

In the relativistic regime the dissipative modes are the massless TT modes, whose collision time reaches the lattice floor $\tau_{\rm col} = \tau_\Pi$, and the relevant temperature is the medium's saturation temperature $T_{\max}$. We derive $T_{\max}$ first, then substitute.

The saturation temperature.

The TT-mode gas reaches its physical maximum temperature when its energy density equals the maximum density the medium can carry (the saturation bound of the Lagrangian). The radiation density of a gas of g_* massless polarisations is the Stefan–Boltzmann form

$$\rho_{\rm rad}(T) = \frac{\pi^2}{30}\,g_*\,\frac{(k_B T)^4}{(\hbar c)^3}, \tag{26}$$

and the saturation condition is $\rho_{\rm rad}(T_{\max}) = S^2_{\rm max,geom}\,\rho_P c^2$, with the derived polarisation count $g_* = 2$ (the two TT polarisations, from $2\ell + 1 = 5$ reduced to physical modes) and the geometric saturation modulus $S^2_{\rm max,geom} = 8\sqrt{6}/9$. Solving (26) for

$(k_BT_{\max})^4$ and using $(\hbar c)^3 = (\rho_P c^2 \ell_P^3)^3$ in the medium's units,

$$(k_BT_{\max})^4 = \frac{30}{\pi^2 g_*} S^2_{\max,\mathrm{geom}} (\rho_P c^2 \ell_P^3)^4 = \frac{15\, S^2_{\max,\mathrm{geom}}}{\pi^2} (\rho_P c^2 \ell_P^3)^4 = \frac{40\sqrt{6}}{3\pi^2} (\rho_P c^2 \ell_P^3)^4, \tag{27}$$

where the last step inserts $g_* = 2$ and $S^2_{\max,\mathrm{geom}} = 8\sqrt{6}/9$ (so $15 \cdot \frac{8\sqrt{6}}{9}/1 = \frac{40\sqrt{6}}{3}$). Taking the fourth root,

$$k_BT_{\max} = \left(\frac{40\sqrt{6}}{3\pi^2}\right)^{1/4} \rho_P c^2 \ell_P^3 = \frac{5^{1/4}\, 6^{7/8}}{3\sqrt{\pi}} \rho_P c^2 \ell_P^3 = 1.34874\, \rho_P c^2 \ell_P^3. \tag{28}$$

The floor.
Substituting $\tau_{\mathrm{col}} = \tau_\Pi$ from (24), $k_BT = k_BT_{\max}$ from (28), and $\hbar = \rho_P c\, \ell_P^4$ into (25),

$$R_{\mathrm{floor}} = \frac{4\pi}{5} \frac{\dfrac{\ell_P}{\sqrt{20}\, c} \cdot \dfrac{5^{1/4} 6^{7/8}}{3\sqrt{\pi}} \rho_P c^2 \ell_P^3}{\rho_P c\, \ell_P^4} = \frac{4\pi}{5} \cdot \frac{5^{1/4} 6^{7/8}}{3\sqrt{20}\,\sqrt{\pi}}, \tag{29}$$

where every factor of ρ_P, c and ℓ_P cancels between numerator and denominator—the ratio is a pure number. Simplifying $\sqrt{20} = 2\sqrt{5}$ and collecting the powers of 5, 6 and π,

$$\boxed{R_{\mathrm{floor}} = \frac{2 \cdot 5^{3/4} \cdot 6^{7/8} \cdot \sqrt{\pi}}{75} = 0.7580\ .} \tag{30}$$

This is a pure number: substituting $\hbar = \rho_P c\, \ell_P^4$ into (30) removes every constant—$\hbar$ by substitution, the medium's properties by dimensional cancellation, and k_B by self-annihilation, since it enters (25) once in $T = \varepsilon/k_B$ and once in the bound $\hbar/4\pi k_B$ and cancels. The Boltzmann constant is thus not a property of the medium at all but a conversion ratio between two human yardsticks (joule and kelvin) for the same substantive quantity—energy per phase degree of freedom—and it disappears from every physical observable of the medium. The number 0.7580 contains only the medium's own structure: the Stefan–Boltzmann factor $2\pi^2/45$ (itself reproduced natively by the phase-configuration count to thirteen figures), the derived polarisation count $g_* = 2$, the geometric modulus $S^2_{\max,\mathrm{geom}} = 8\sqrt{6}/9$, and the lattice $\sqrt{20}$. *Classification:* [STRONG DEDUCTION], the floor being a derived structural number awaiting only a precise collision time within the relativistic window.

6.2 The corroboration: one floor across twenty-one orders of magnitude

The strategic value of the floor is not a single predicted number but a universality that a unit artefact cannot fake. Under the medium's ontology there are no systems distinct from the medium: the quark–gluon plasma (QGP) is the medium in a relativistic plasma configuration, a strange metal is the medium in strongly coupled condensed matter, and a unitary ultracold Fermi gas is the medium at a Feshbach resonance. All three are the same substrate in different regimes, and all three saturate the same dimensionless dissipation bound to the same order of magnitude. The measured values span

$$\underbrace{\sim 10^{-9}\,\mathrm{K}}_{\text{Fermi gas}} \quad \text{to} \quad \underbrace{\sim 10^{12}\,\mathrm{K}}_{\text{QGP}} \qquad = \text{ twenty-one orders of magnitude in } T, \tag{31}$$

with the QGP best fit near T_c at $\eta/s \sim 1$–2.5 in units of the bound [22], strange metals showing Planckian dissipation [23], and the unitary Fermi gas reaching a minimum $\eta/s \approx 5$ times the bound near $T/T_F \approx 0.27$ [24, 25]. None violates the bound.

Two points mark the limit of the claim. First, the coefficient is *not* the same number across the three regimes: the relativistic (massless-mode) window is $[1, \sim 2.5]$ and contains the QGP best fit, whereas the cold Gross–Pitaevskii regime of F1 populates a gapped mode that closes energy-transfer channels and raises η/s toward ~ 5 for the Fermi gas. The *sign* of this difference is derived—a gap can only lengthen the collision time and so raise η/s—while its exact magnitude mixes the derived existence of the gap with an imported datum for its size; 0.7580 is not a universal prediction for all fluids but the floor of the relativistic regime. Second, what *is* universal, and is the anti-tautology content, is the order of magnitude of the bound across twenty-one orders of magnitude in temperature and the derived sign of its regime dependence. A number produced by unit algebra cannot predict that three systems separated by twenty-one orders of magnitude in temperature saturate the same dimensionless bound: units know nothing of temperature, yet the bound is saturated at every temperature probed. This is the pure-number stage of Principle 1 standing entirely on its own, with step 3 empty.

There is, moreover, a structural reason the bound exists at all, which the standard holographic derivation leaves as a conjecture. In the medium there is no vacuum: every mode is coupled to the rest of the continuous medium at every point, so no mode can be isolated and the viscosity cannot vanish. The bound $\eta/s \geq \hbar/4\pi k_B$ is, in this framework, the statement that the medium has no decoupled regions—the impossibility of vacuum, read as a transport bound. The universality of the bound is thus the universality of a single substrate obeying a single no-vacuum principle. *Classification:* [STRONG DEDUCTION] for the identification of the bound with the no-vacuum principle; [DERIVED] for the sign of the regime dependence; [CALIBRATED] for the magnitude of the cold-regime coefficient.

7 A forward prediction, falsifiable in principle

The framework is not merely retrodictive. The flexural term of (2) modifies the propagation of the TT modes—the gravitational waves—giving the derived dispersion relation

$$\omega^2 = c^2 k^2 \left[1 + \frac{(k\ell_P)^2}{20}\right], \tag{32}$$

where the coefficient $1/20$ is the flexural phase coefficient $1/40$ of (2) times the factor 2 from varying $(\Box h_+)^2$—the same $\sqrt{20}$ once more. The TT mode is massless at zeroth order, so its speed is exactly c; the sole departure from luminal propagation is the Planck-scale correction in (32). For frequencies accessible to current detectors ($k\ell_P \sim 3 \times 10^{-41}$ at ~ 100 Hz) the correction is $\sim 6 \times 10^{-83}$, some sixty-seven orders of magnitude below the present bound $\sim 10^{-15}$ on anomalous gravitational-wave dispersion, and consistent with GW170817. This is beyond any foreseeable experiment and is not a near-term test. Its value is of a different kind: the framework fixes the sign and magnitude of the correction with no freedom, so the identification of gravitational waves as flexural phase modes is falsifiable in principle—a measured dispersion larger than (32), or a TT mode with rest mass, would refute it—and the smallness is itself a consistency check, since a substrate model predicting a correction near current bounds would already be excluded. *Classification:* [DERIVED] for the dispersion relation; [HYPOTHESIS] for the primordial-spectrum suppression it implies at $k \sim \ell_P^{-1}$.

8 What remains open: the one uncalibrated number

Principle 1 states the open problem with unusual precision, because it isolates exactly where a datum must enter. Everything before step 3 is derived: the form of each pure number, the dynamics that fixes its prefactor, the uniqueness of each dimensional monomial, and the binding relation (22). What is calibrated, not derived, is the single scaling act itself—the numerical value of the pair (ρ_P, ℓ_P). This is not a hidden circularity but the visible, named, once-only entry of a human yardstick that the principle requires; the framework works in ratios up to that point and cannot, from within, manufacture a unit.

The sharp open question is therefore not "is the derivation circular?" (the simultaneity of Section 5.2 settles that) but: *can the scaling act be performed from outside the gravitational and quantum sectors?* At present (ρ_P, ℓ_P) is anchored to the Planck scale, which is built from G and $\hbar$ themselves. What would remove the last trace of the objection is an independent anchoring—a measurement or derivation of the medium's stiffness or grain from physics that is neither gravitational nor quantum, which would then *predict* G and $\hbar$ rather than being anchored alongside them. Section 6 is the first evidence that such an external handle exists in principle: the dissipation floor is a pure number of the same medium, anchored not at the Planck scale but in heavy-ion collisions and cold-atom traps—territory untouched by G and $\hbar$. It does not yet fix (ρ_P, ℓ_P) numerically, but it shows that the medium speaks in the same pure numbers in sectors where no gravitational or quantum yardstick is available, which is exactly the direction an independent calibration must come from. Until that calibration exists, the status is: the *form* of every identity is derived and falsifiable; the single *scaling* rests on a one-number anchor. This is the boundary of the present claim, and the sharpest target for future work.

9 Conclusion

The paper tested a claim about what the constants of nature are: that a real physical medium works internally in dimensionless pure numbers, each read off the geometry of the medium's internal structure through one Lagrangian with no free parameters, and that a dimensional constant is one of those numbers dressed in human units by a single final act of scaling. The claim was proved in the most fundamental sector available. Solving the modulus sector of (2) by two independent routes gave one pure number, the gravitational compliance; quantising the phase rotor gave a *different* pure number, the angular inertia of the fundamental mode. Collected, the results of the paper are the three structural identities

$$G = \frac{c^2}{\rho_P \ell_P^2}, \qquad \hbar = \rho_P c\, \ell_P^4, \qquad \ell_P = \sqrt{\frac{\hbar G}{c^3}}, \tag{33}$$

of which the first two are derived independently, in ratios, and the third is their algebraic consequence—not an input. A single scaling act, the anchoring of the pair (ρ_P, ℓ_P) in (23), dresses the first two at once into the measured G and $\hbar$ to better than 0.1% with no parameter fitted to either, makes the third coincide numerically with the Planck length, and, through the combination $\hbar c/G = \rho_P^2 \ell_P^6 = m_P^2$, reproduces the Chandrasekhar mass $M_{\mathrm{Ch}} \approx 1.454\, M_\odot$ that needs both. Two distinct pure numbers, one scaling.

This is why the result is not the empty observation that constants have units. A units convention could dress one number; it could not make a single anchoring land within 0.1% simultaneously in gravity and in quantum coherence, sectors that share no scale, nor produce the Planck relation and a stellar mass limit as by-products. The simultaneity across sectors is the physical signature of a single substrate beneath, and it converts the circularity objection into the confirmation the principle predicts: units must enter once, and once is enough.

The pattern was then shown to hold even where the yardstick is never laid—the dimensionless dissipation floor of the medium, saturated to the same order of magnitude by systems separated by twenty-one orders of magnitude in temperature, and given, by the medium's no-vacuum structure, a foundation the standard holographic bound leaves as a conjecture. A pure number that stays undressed and is nonetheless measurable across that range is the principle standing on its own. The same Lagrangian also fixes, with no further freedom, the dispersion of gravitational waves as flexural modes of the medium: a correction beyond present reach, but one whose sign and magnitude the framework sets, so the identification is falsifiable in principle.

Two boundaries remain. The medium is described here at the effective, order-parameter level, and we do not derive its Lagrangian (2) from a deeper microscopic theory—nor need the leading-order identities wait on that, just as the Gross–Pitaevskii description of a condensate was correct before its derivation from the many-body Hamiltonian. And the single scaling act is, at present, calibrated rather than derived: anchored to the Planck scale for want of an independent handle, though Section 6 indicates where such a handle lies. Supply that one external number, and the identities of this paper stop being retrodictions of G and $\hbar$ and become predictions of them from the properties of the medium alone. That is the sense in which the constants of nature may not be constants at all, but readings—dressed pure numbers—of a single elastic thing.

The constants sector treated here is one part of a considerably broader programme. PIU-Ψ applies the same medium and the same Lagrangian (2) to the emergence of the Einstein equations, the Standard-Model sector, the topological origin of spin and statistics, and the cosmological sector, with the derivations, the verification scripts, and the full compendium deposited openly [27]. Readers who wish to examine the wider framework, reproduce its numbers, or test its further predictions will find the complete corpus on Zenodo (10.5281/zenodo.21205881) and an overview at piuniversal.com. Scrutiny of those results, and of the identities established here, is invited.

Supplementary information. A self-contained Python script (standard library only) is provided as Supplementary Information. It reproduces, in four parts: the physical identities of Section 5, including the Chandrasekhar mass; the dissipation-floor number of Section 6, evaluated three ways to exhibit the symbolic cancellation of ρ_P, c, ℓ_P and k_B; the dimensional-uniqueness counts that establish exactly one admissible combination of (ρ_P, ℓ_P, c) for each of G and $\hbar$; and, kept separate and clearly labelled as corroboration rather than proof, the enumerative Bayesian test with all of its heuristic inputs printed for inspection.

Declarations

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